

Exercise Class in Mathematics

BAE

Preliminary Exercises

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September 29 2022

Exercise 1.

Describe the domain of each of the following functions:

$$a) f(x) = \log(\log(x))$$

$$b) f(x) = \sqrt{x \log(x)}$$

$$c) f(x) = \log(1 - 3^x)$$

$$d) f(x) = \sqrt{\left(\frac{1}{2}\right)^x - 1}$$

$$e) f(x) = \frac{1}{3^{x^2} - 4}$$

$$f) f(x) = \frac{\sqrt{x}e^{-x}}{2^{x^2+1} - 3}$$

Exercise 2.

For each pair of functions f and g describe the compositions $f \circ g$ and $g \circ f$ explicitly, if they exist, and determine their domains:

$$a) \quad \begin{array}{l} f: \mathbb{R} \longrightarrow \mathbb{R} \\ x \rightarrow x^2 - 1 \end{array}, \quad \begin{array}{l} g: \mathbb{R} \longrightarrow \mathbb{R} \\ x \rightarrow e^x \end{array};$$

$$b) \quad \begin{array}{l} f: \mathbb{R} \longrightarrow \mathbb{R} \\ x \rightarrow x^3 + 2 \end{array}, \quad \begin{array}{l} g: \mathbb{R} \longrightarrow \mathbb{R} \\ x \rightarrow \frac{1}{e^x} \end{array};$$

$$c) \quad \begin{array}{l} f: [0, +\infty) \longrightarrow \mathbb{R} \\ x \rightarrow \sqrt{x} \end{array}, \quad \begin{array}{l} g: (-\infty, 0) \longrightarrow \mathbb{R} \\ x \rightarrow \log(-x) \end{array};$$

$$d) \quad \begin{array}{l} f: \mathbb{R} \longrightarrow \mathbb{R} \\ x \rightarrow \cos(x) + 2 \end{array}, \quad \begin{array}{l} g: [1, +\infty) \longrightarrow \mathbb{R} \\ x \rightarrow \sqrt{x-1} \end{array};$$

Exercise 3.

Compute the following limits:

$$a) \lim_{n \rightarrow +\infty} \frac{n^3 - 6n + 11}{9 - 7n^4},$$

$$b) \lim_{n \rightarrow +\infty} \frac{n^2 + 9n + 3}{4 - n^2},$$

$$c) \lim_{n \rightarrow +\infty} \frac{3n^5 + 6n + 1}{1 - 3n^4},$$

$$d) \lim_{n \rightarrow +\infty} (-1)^n \cdot \frac{n}{n^2 + 1},$$

$$e) \lim_{n \rightarrow +\infty} \frac{\cos(n)}{n^3},$$

$$f) \lim_{n \rightarrow +\infty} \frac{1 + \cos(n) - n\sqrt{n}}{2 + \sin(n) - 3n},$$

$$g) \lim_{n \rightarrow +\infty} (\sqrt{n^2 + n} - n),$$

$$h) \lim_{n \rightarrow +\infty} (\sqrt{n+1} - \sqrt{2n+1}),$$

$$i) \lim_{n \rightarrow +\infty} (\log(n) - n^3 + 2^n - e^n),$$

$$j) \lim_{n \rightarrow +\infty} \frac{3^{2n+1} + 1}{8^n + 1},$$

$$k) \lim_{n \rightarrow +\infty} \frac{7^n n^9}{8^n},$$

$$l) \lim_{n \rightarrow +\infty} \frac{2^n + \log(n^2) + e^n}{n^8 + 8^n + \cos(n) + 1},$$

$$m) \lim_{n \rightarrow +\infty} \frac{\log(n^6)}{n},$$

$$n) \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{5n}\right)^{2n}$$

Exercise 4.

Prove the following limits do not exist:

$$\lim_{n \rightarrow +\infty} \cos(\pi n),$$

$$\lim_{n \rightarrow +\infty} \frac{(-1)^n n}{n+3}$$