Exercise Class in Mathematics

BAE

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Exercise 1.
Describe the domain and find all the horizonal and vertical asymptotes, if any, for each of the following functions:

a) $f(x) = 5x^{11} + 3x^7 - x^4 + 2x^3 + 3$

b) $f(x) = \dfrac{x^3 + 1}{x^2}$

c) $f(x) = \dfrac{x + 3}{x - 2}$

d) $f(x) = \dfrac{x^2 - 2x + 6}{2 - x^2}$

e) $f(x) = \dfrac{x^2 - 9}{x^2 + 4x + 3}$

f) $\dfrac{x^4 + 2x^2 + 1}{x^3 - 4x}$

g) $f(x) = x^2 - 2e^{1/x}$

h) $f(x) = \cos(e^{x^2 + x^5})$

i) $f(x) = \log\left(\dfrac{3 - x}{x - 1}\right)$

j) $f(x) = e^{\frac{8x}{x^2 - 16}} - 2$

k) $f(x) = \sqrt{x^2 - 3x - 4} - x + 3$

l) $f(x) = xe^{\frac{1}{x}}$

Exercise 2.
Determine if the following functions are continuous on their domains:

a) $f(x) = x^2 - e^{x\cos(x)}$

b) $f(x) = |x - 4| - |2 - x|$

c) $f(x) = \sin(\sqrt{x^2 + 1}) + \log(x^2 + x + 1)$

d) $f(x) = \sqrt{\sin(\log(|x| + 1)) + 2}$

e) $f(x) = \begin{cases} x^2 + 3 & \text{if } x \leq 0 \\ 3 - x & \text{if } x > 0 \end{cases}$

f) $f(x) = \begin{cases} \cos(\pi x) + 1 & \text{if } x \leq 1 \\ 6x^2 - 5 & \text{if } x > 1 \end{cases}$,

g) $f(x) = \begin{cases} \frac{1 - \sqrt{1 + \sin(x)}}{x} & \text{if } x < 0 \\ \frac{\sin(x) - 1}{x + 2} & \text{if } x \geq 0 \end{cases}$

h) $f(x) = \begin{cases} \frac{\sqrt{x^4 + x^2}}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$,

i) $f(x) = \begin{cases} \frac{\sin(x)}{x} + \pi & \text{if } x < 0 \\ \pi & \text{if } x = 0 \\ \frac{(e^x - 1)(1 - \cos^2(x))}{x^3} + \pi & \text{if } x > 0 \end{cases}$

j) $f(x) = \begin{cases} \cos(\sqrt{-x})\log(x^2 + 1) & \text{if } x \leq 0 \\ \frac{\sin(\pi x)\log(x + 1)}{x} & \text{if } 0 < x \leq 1 \\ \frac{\cos(x - 1) - 1}{e^{(x-1)} - 1} & \text{if } x > 1 \end{cases}$,

Exercise 3.

Determine for what values of the parameter $a \in \mathbb{R}$ the following functions are continuous on $\mathbb{R}$:

a) $f(x) = \begin{cases} x - a & \text{if } x \leq 2 \\ x^2 + x + 1 & \text{if } x > 2 \end{cases}$

b) $f(x) = \begin{cases} x + a & \text{if } x \leq 0 \\ x^2 - ax + 1 & \text{if } x > 0 \end{cases}$,

c) $f(x) = \begin{cases} e^x + a & \text{if } x \leq 0 \\ x^2 + 2x + 3 & \text{if } x > 0 \end{cases}$

d) $f(x) = \begin{cases} ae^x + a^2 + 1 & \text{if } x \leq 0 \\ \frac{\log(x + 1)}{x} & \text{if } x > 0 \end{cases}$,

e) $f(x) = \begin{cases} x^2 + ae^x \cos(x) + 3 & \text{if } x \leq 0 \\ \frac{3\sin(ax) + e^x - 1}{x} & \text{if } x > 0 \end{cases}$

f) $f(x) = \begin{cases} e^{x + a^2} & \text{if } x < 0 \\ e^{4 - x^2} & \text{if } 0 \leq x \leq 2 \\ \frac{\log(x - 1) + a}{x - 1} & \text{if } x > 2 \end{cases}$,

Exercise 4.

Say if the following functions admit a (global) maximum and a (global) minimum on the given intervals of the real line:

$$a)\ f(x) = x^7 - x^3 + x^5 - 1 \qquad\qquad I := [0, 5]$$

$$b)\ f(x) = x^3 + 5x^3 + 2x + e^x \qquad\qquad I := [-1, 3)$$

$$c)\ f(x) = e^{1/x} \cos(x^2 + x) + 1 \qquad\qquad I := (\pi, 2\pi)$$

$$d)\ f(x) = \sqrt{x^2 + 1} - \log(|x|) \qquad\qquad I := [-2, -1]$$

$$e)\ f(x) = \frac{\sqrt{x-1}}{x-3} \qquad\qquad I := [1, 2]$$

$$f)\ f(x) = \frac{1}{x-2} \qquad\qquad I := [1, 3]$$

Exercise 5.

Say if the following functions admit a (global) maximum and a (global) minimum on the given intervals and in case compute them:

$$a)\ f(x) = x + 3 \qquad\qquad I := [1, 2]$$

$$b)\ f(x) = x^2 + 2 \qquad\qquad I := [-1, 1]$$

$$c)\ f(x) = \frac{1}{x} \qquad\qquad I := [1, 2]$$