

Quantitative Methods – I

A.Y. 2021-22

Practice 8

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THEME #1



Continuous Random Variables

Continuous Random Variables

- ✓ Normal distribution $X \sim N(\mu, \sigma^2)$
- ✓ Standard normal distribution $Z \sim N(0,1)$
- ✓ Chi-square distribution $X \sim \chi_n^2$
- ✓ t-Student distribution $X \sim t_n$
- ✓ F distribution $X \sim F_{n,m}$

Probability distributions of Continuous Variables

A continuous random variable X can assume any value over an interval.

2 characteristics (similar to discrete variables):

1. For any interval $[a, b]$, $0 \leq P(a < X < b) \leq 1$
2. The total probability of all the intervals within which X can assume a value is 1

$$\sum P(-\infty < X < +\infty) = 1$$

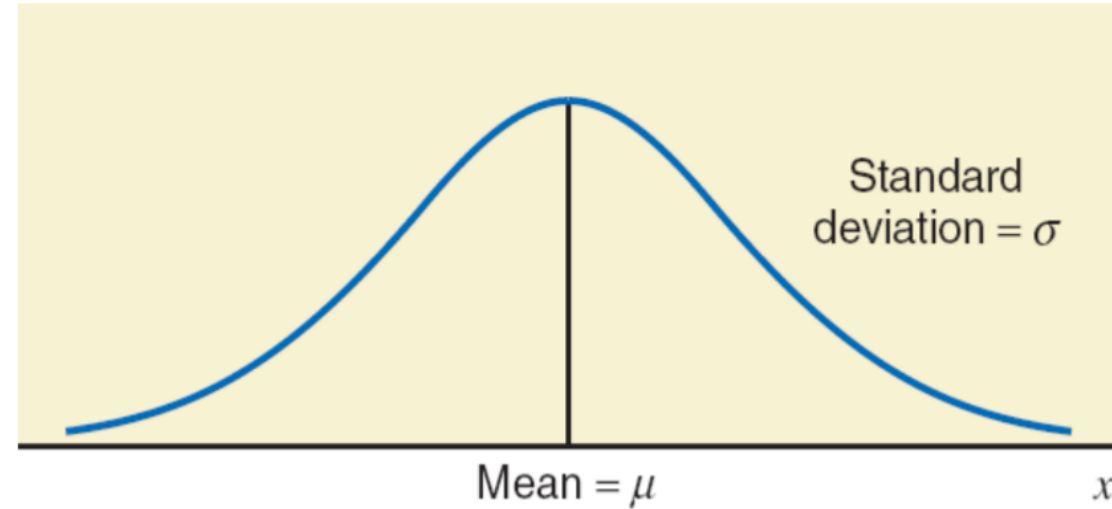
Normal distribution function, $N(\mu, \sigma^2)$

Most widely used continuous probability distribution

$$x \sim N(\mu, \sigma^2)$$

Density function: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2\right\}$

parameters: μ (mean), and σ (the spread)

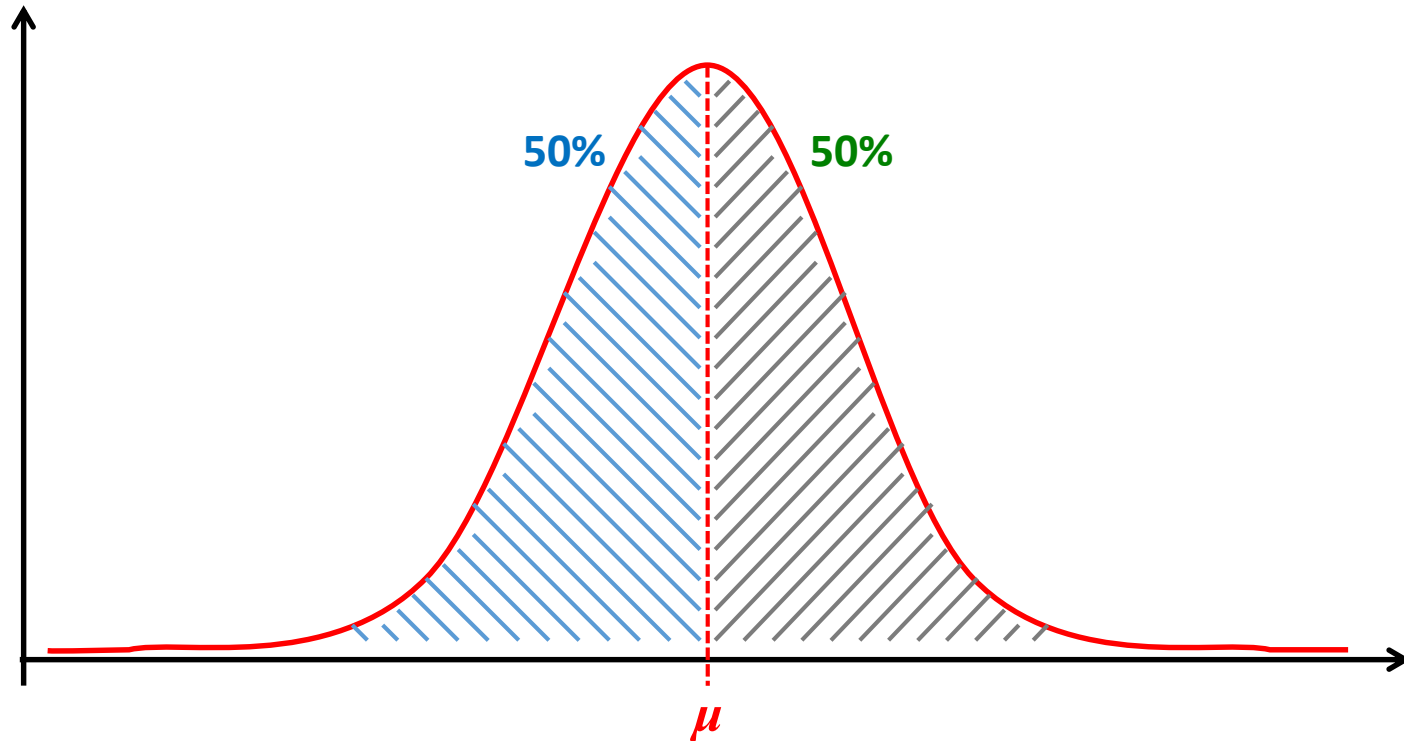


Some examples of things that follow a Normal Distribution:

- Heights of people
- Size of items produced by a machine
- Errors in measurements
- Blood Pressure
- Test Scores
- ...

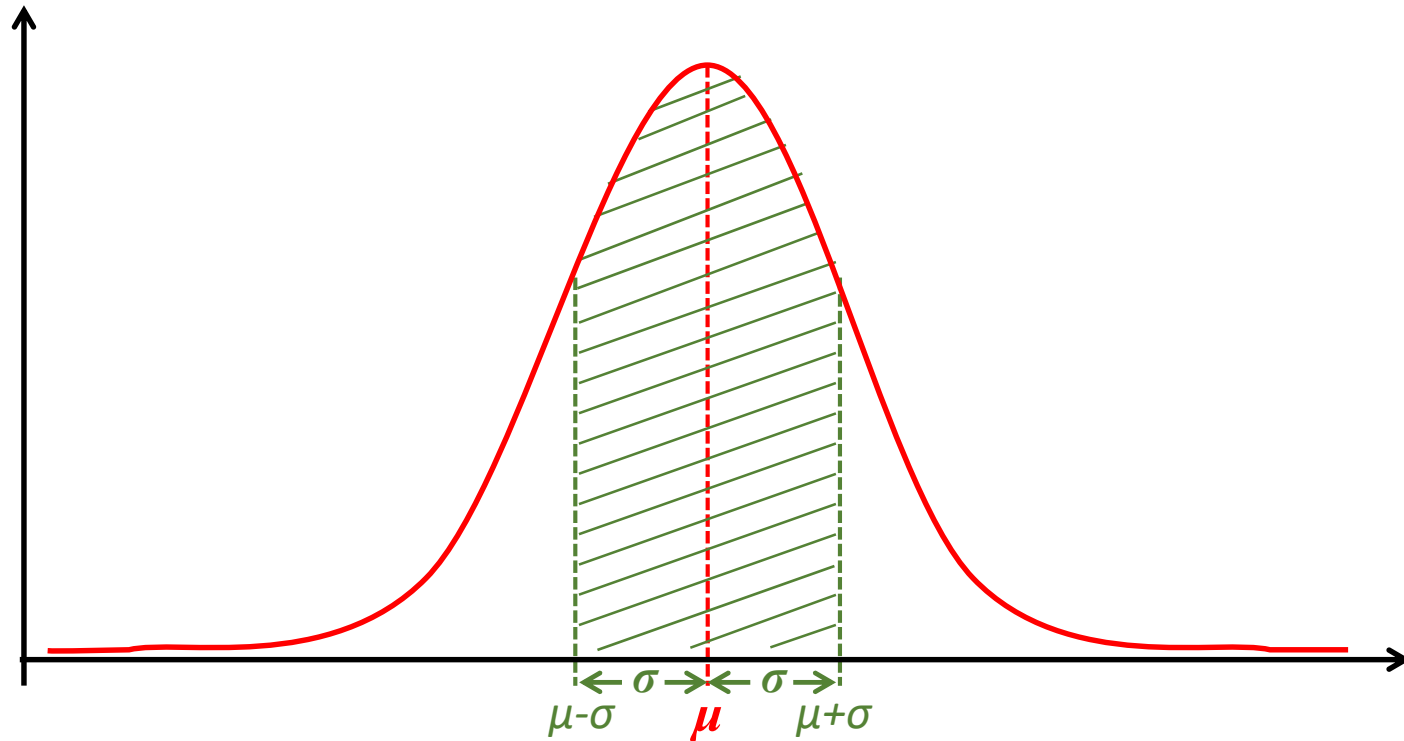
Normal distribution, N :

- It is a bell-shaped curve
- Symmetry about the mean μ (mean = mode = median)
- The total area under the curve is equal to 1 (or 100%)
- 50% of the area is to the left of the mean, and 50% to the right



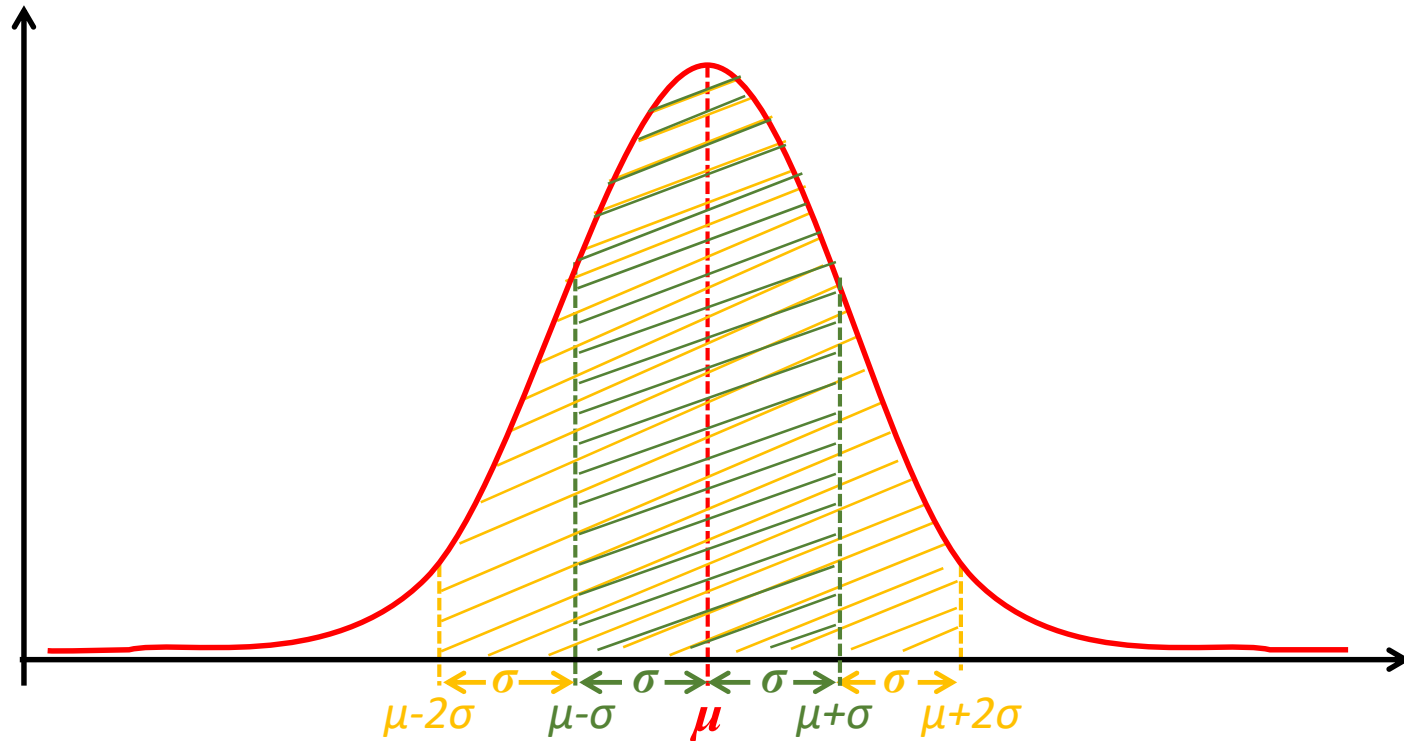
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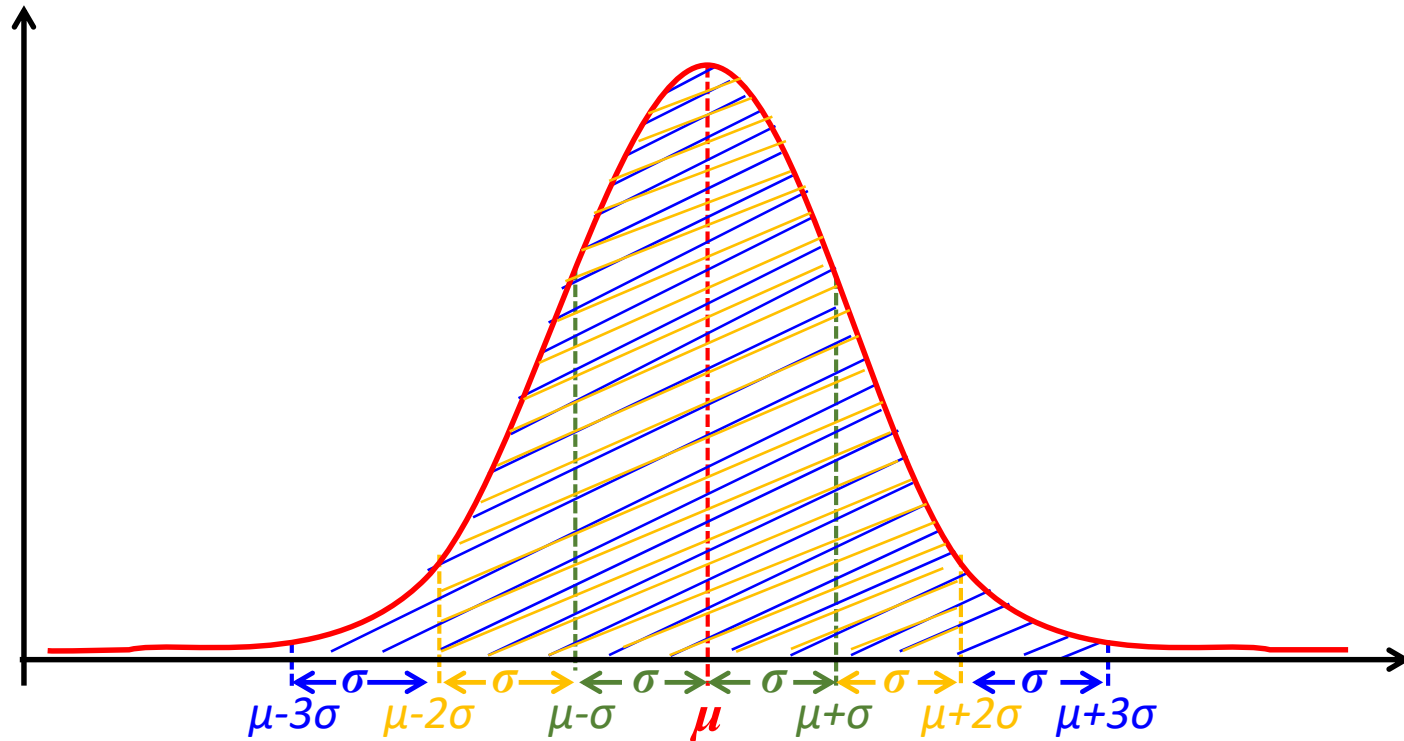
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- Approximately **68.3%** of the area is between $[\mu - \sigma; \mu + \sigma]$
- Approximately **95.4%** of the area is between $[\mu - 2\sigma; \mu + 2\sigma]$
- Approximately **99.7%** of the area is between $[\mu - 3\sigma; \mu + 3\sigma]$



The Standard Normal Distribution, Z , is a Normal Distribution with mean equal to 0 and standard deviation equal to 1, $Z \sim N(0,1)$

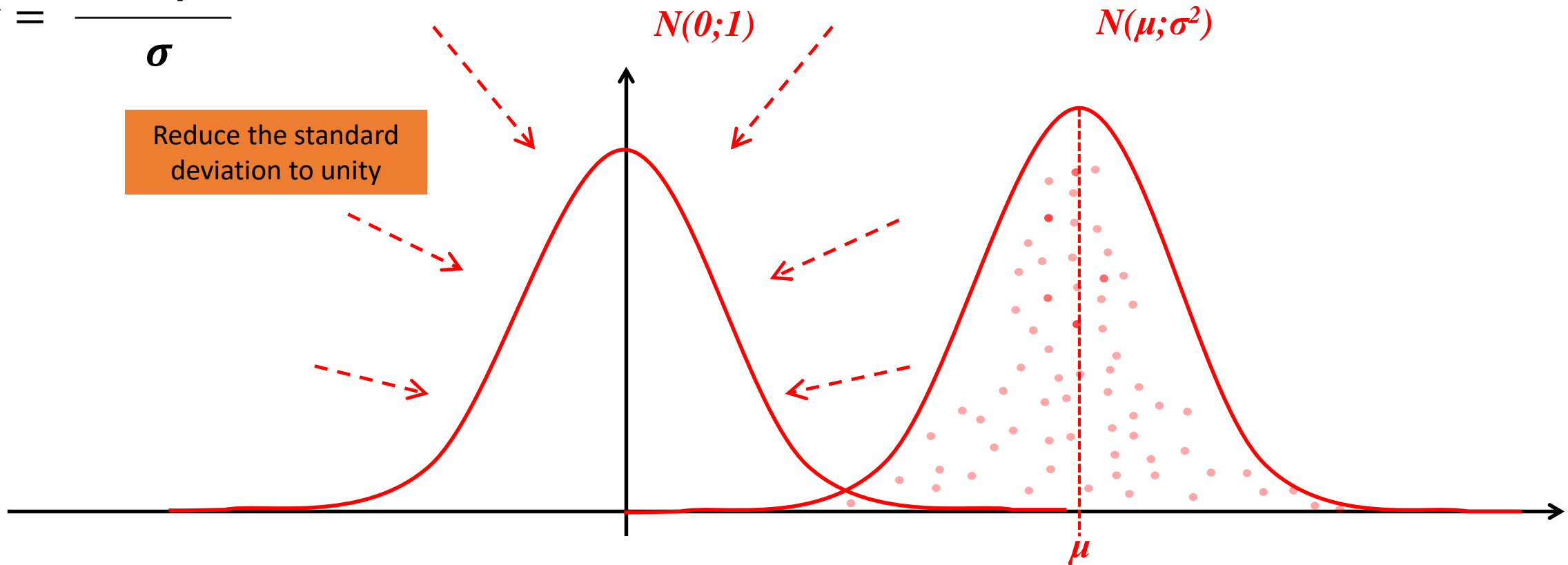
ANY Normal distribution, $X \sim N(\mu, \sigma^2)$, can be converted into a Standard Normal, $Z \sim N(0,1)$, by standardization:

$$Z = \frac{x - \mu}{\sigma}$$

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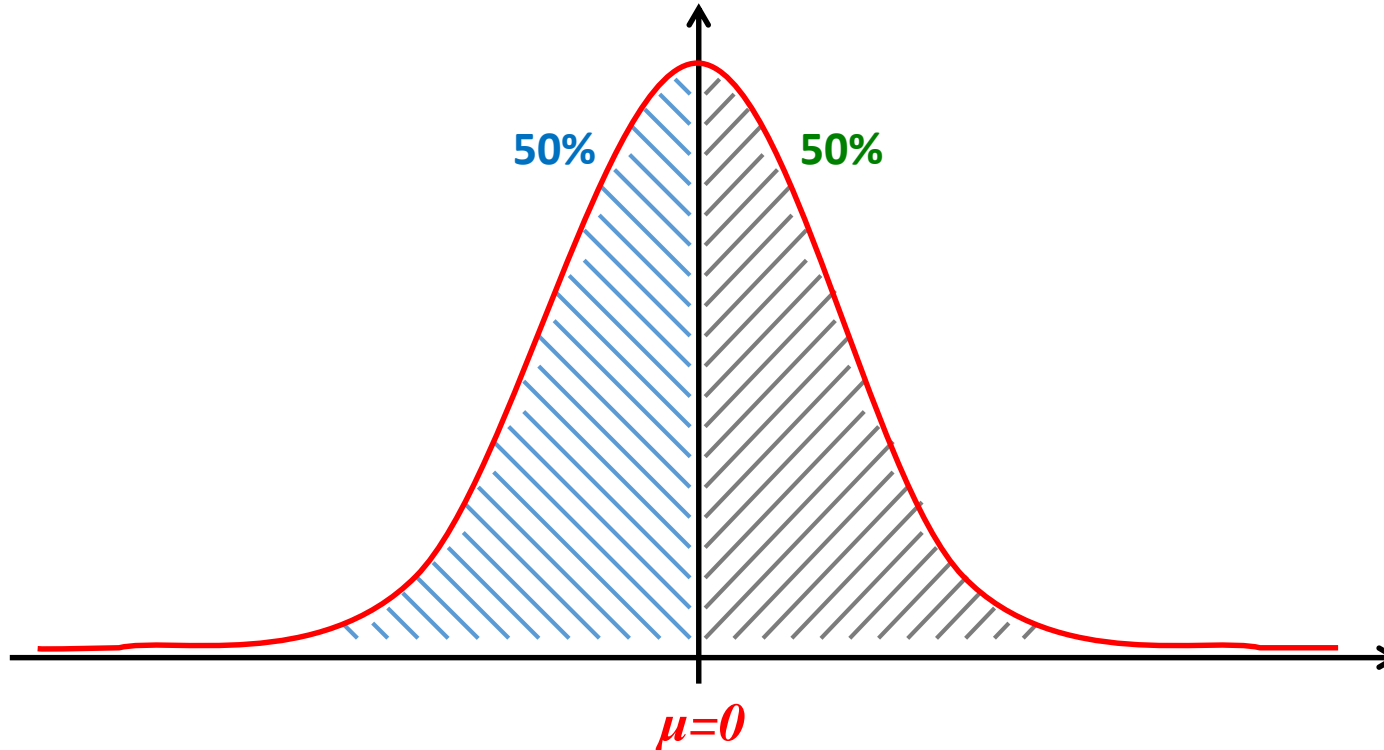
Move the mean to 0

Reduce the standard deviation to unity



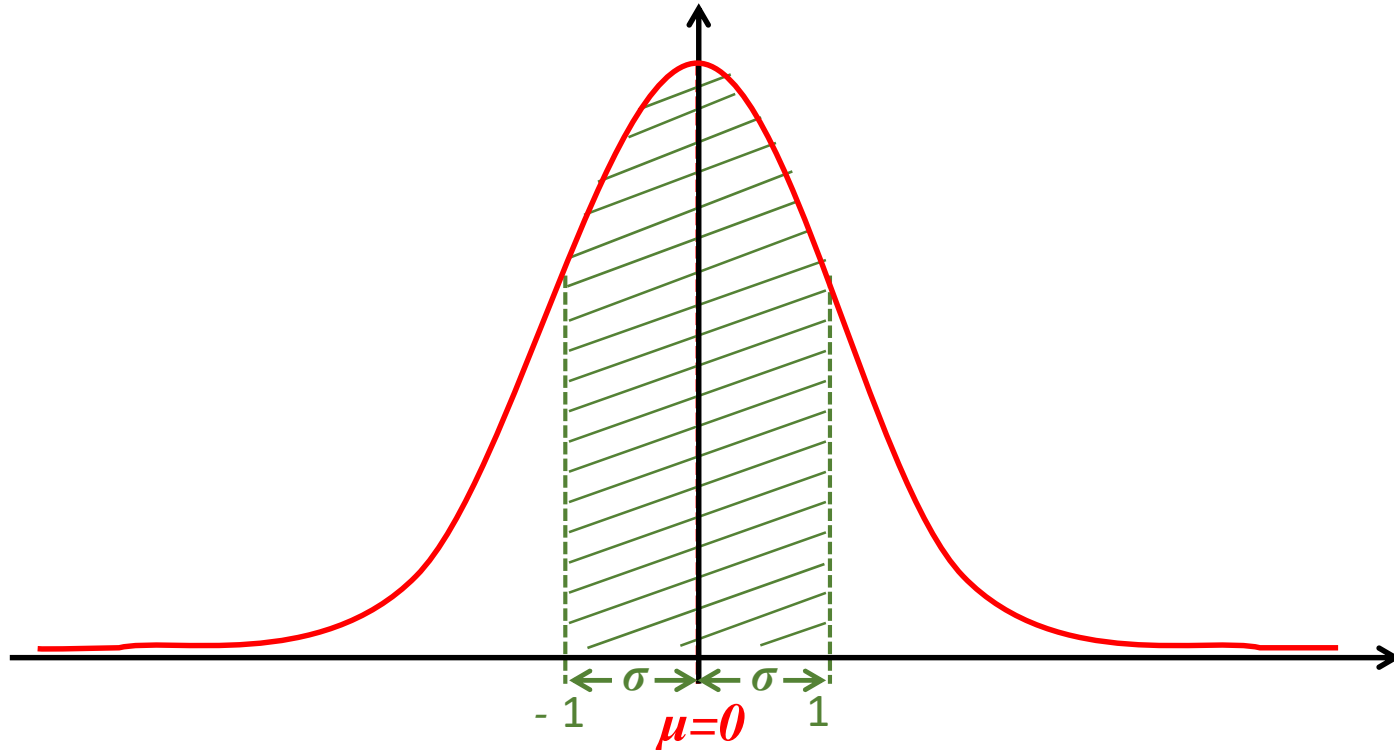
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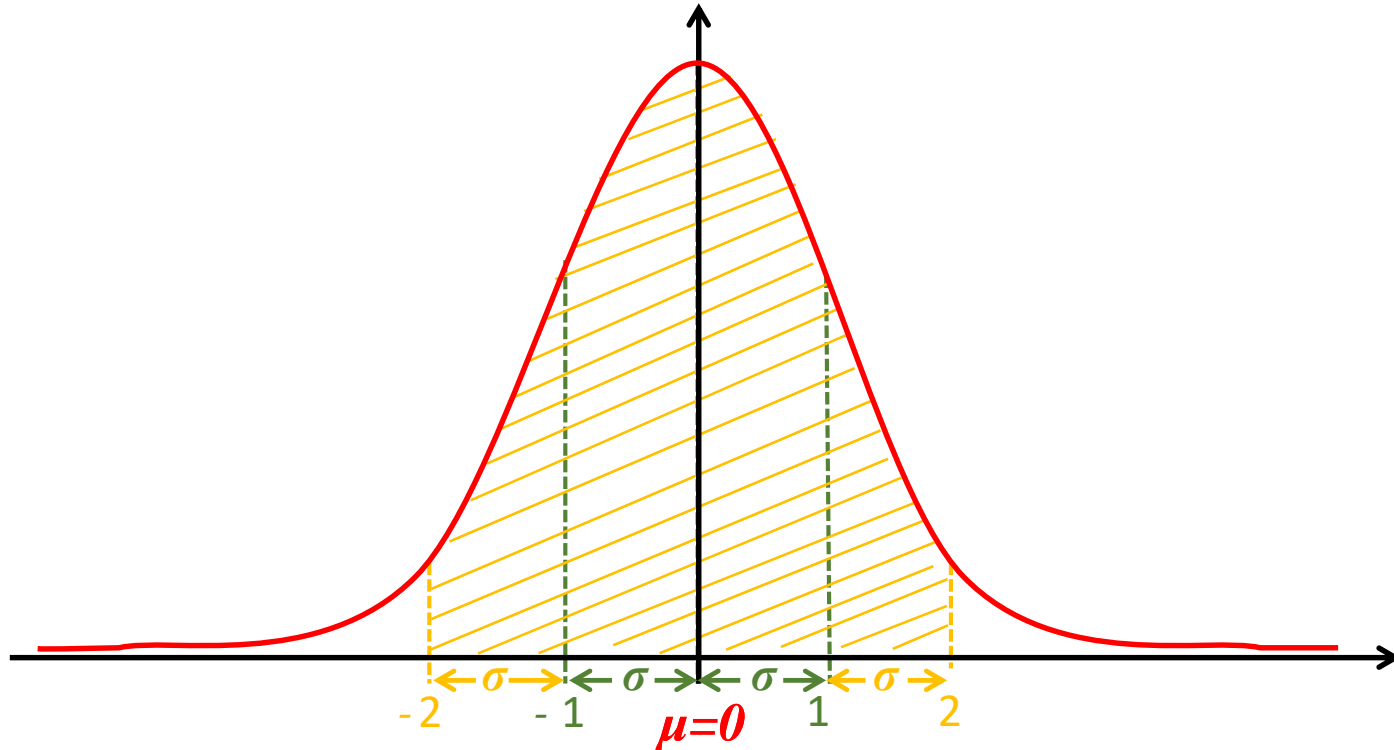
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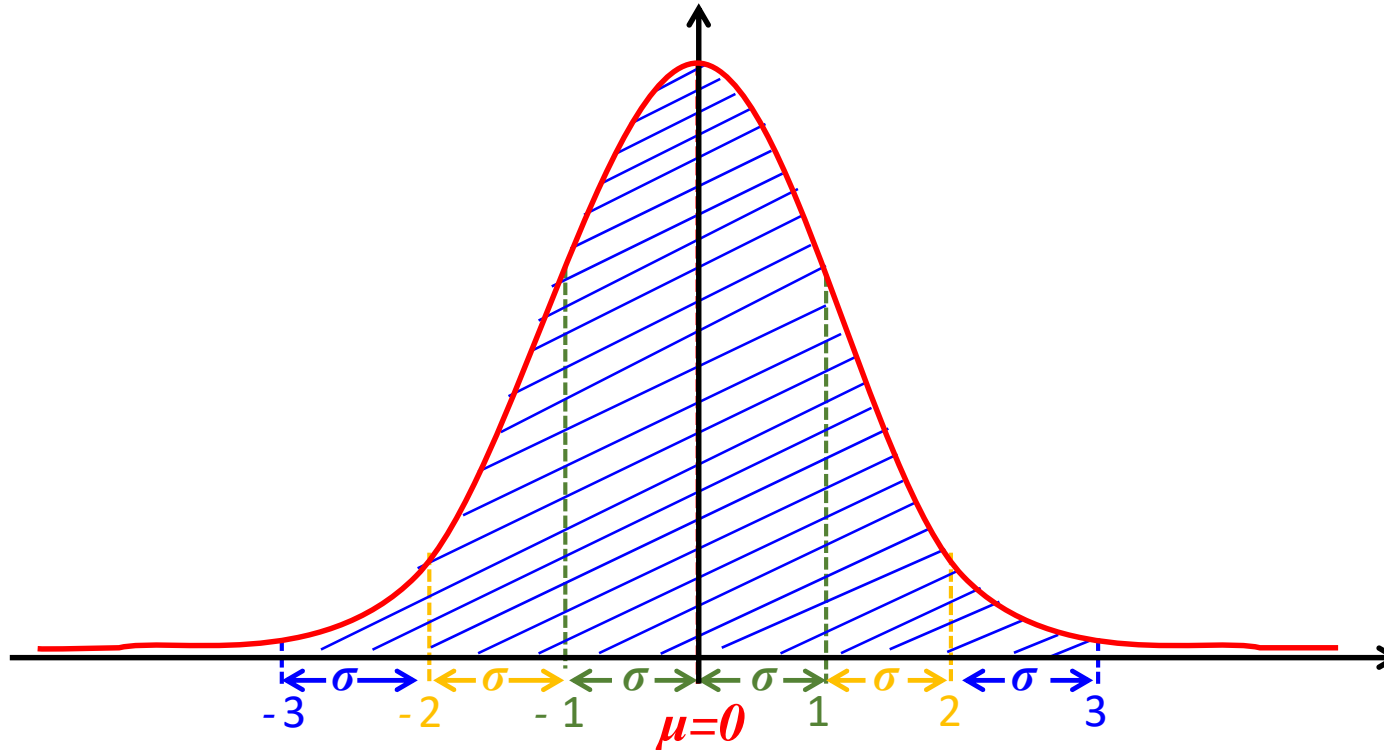
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- Approximately **95.4%** of the area is between $[\mu - 2\sigma = -2; \mu - 2\sigma = 2]$

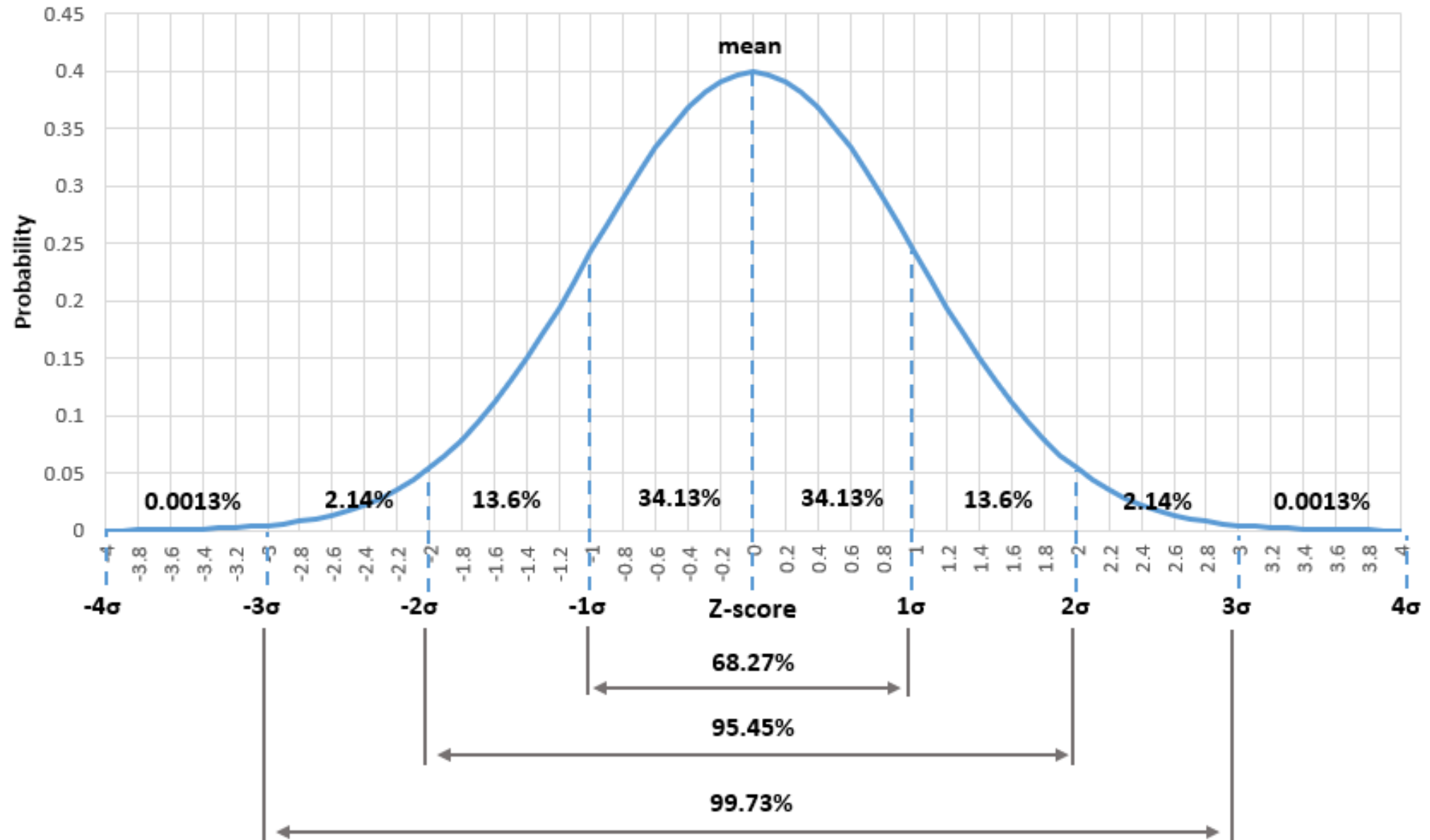


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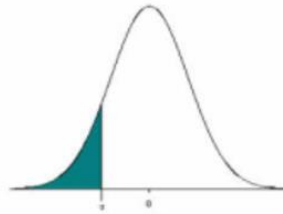
Z distribution with quantiles



Why it is so useful the Standard Normal Distribution $Z \sim N(0,1)$ and the standardization of a Normal distribution in a Standard Normal Distribution?

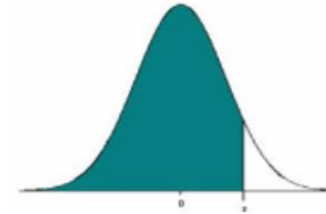
We have the tables of the Standard Normal Distribution $Z \sim N(0,1)$

Table of Standard Normal Probabilities for Negative Z-scores



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0015	0.0015	0.0014	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table of Standard Normal Probabilities for Positive Z-scores



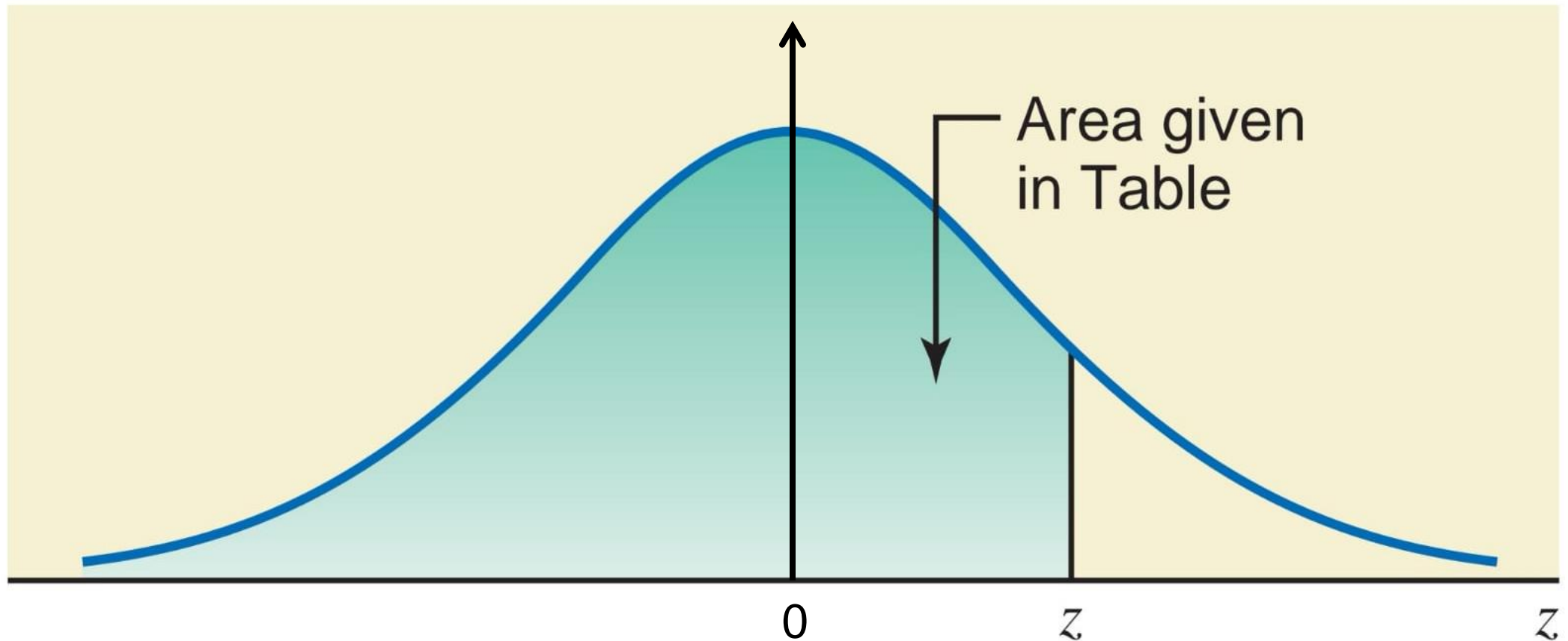
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9985	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Why it is so useful the Standard Normal Distribution $Z \sim N(0,1)$ and the standardization of a Normal distribution in a Standard Normal Distribution?

We have the tables of the Standard Normal Distribution $Z \sim N(0,1)$

For each z-score, the tables give to us the value of the area between $-\infty$ to z

The area in the tables is also the **probability of having a value lower than z**



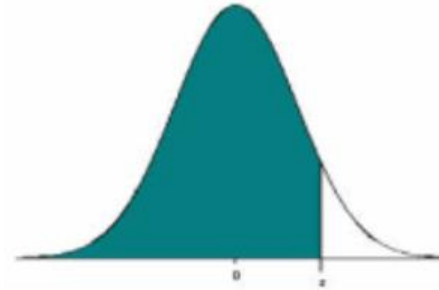
A normal distribution curve is shown with a horizontal axis. The mean is marked as 0. A vertical line is drawn at -1. The area under the curve to the left of -1 is shaded in teal.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
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-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

A normal distribution curve is shown. The area under the curve to the left of a point labeled 'b' on the horizontal axis is shaded in teal. The horizontal axis is labeled with '0' at the center and 'b' at a point to the right of the center.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Table of Standard Normal Probabilities for Positive Z-scores

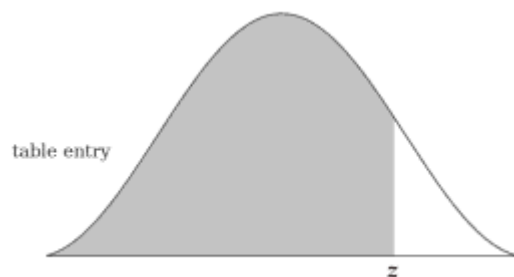


If we have only the table of Standard Normal Probabilities for Positive Z-scores we can find all the values (also for negative z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964

How to use the table?

Positive value of z



Find the value of z in the table.

This is the area between $[-\infty; z]$ or the **probability lower than z**

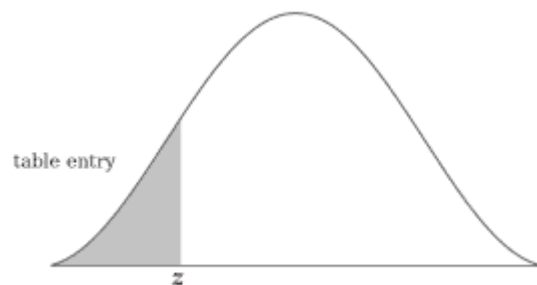
Ex. $P(z < 2.04) = ?$

$P(z < 2.04) = 0.9793$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

How to use the table?

Negative value of z



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884

From the condition of probability distribution:

$$\Sigma P(-\infty < Z < +\infty) = 1$$

Z is also symmetric around the mean

You can find the negative value of z taking the value of z in absolute value (or in modulus) doing:

$$1 - |z|$$

Ex. $P(z < -2.04) = ?$

$$P(z < -2.04) = 1 - P(z < 2.04) = 1 - 0.9793 = 0.0207$$

Exercise 1

Z is a normally standard distributed variable with mean $\mu = 0$ and standard deviation $\sigma = 1$.

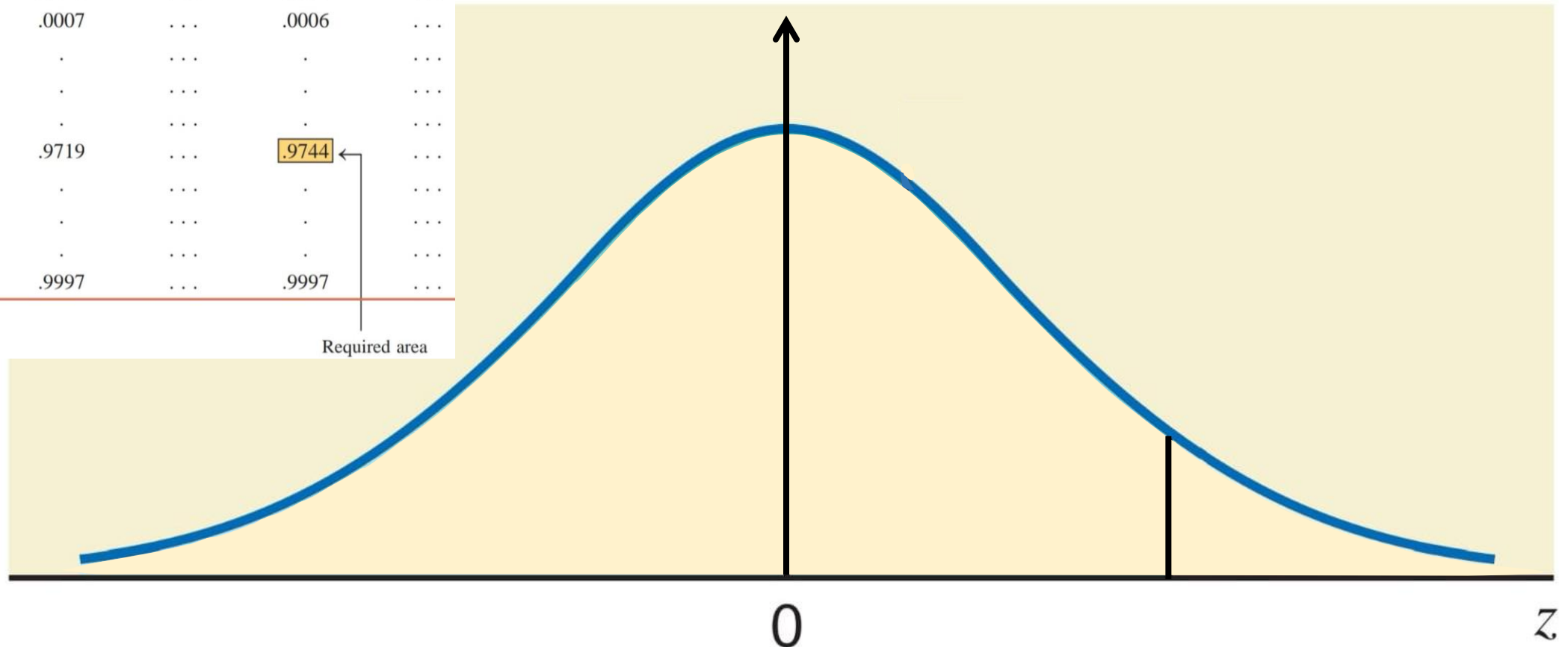
Find

- a. $P(z < 1.95)$
- b. $P(z > 2.32)$
- c. $P(1.19 < z < 2.12)$

Exercise with the 2 Tables of Standard Normal Probabilities for positive and negative Z-scores

Exercise 1

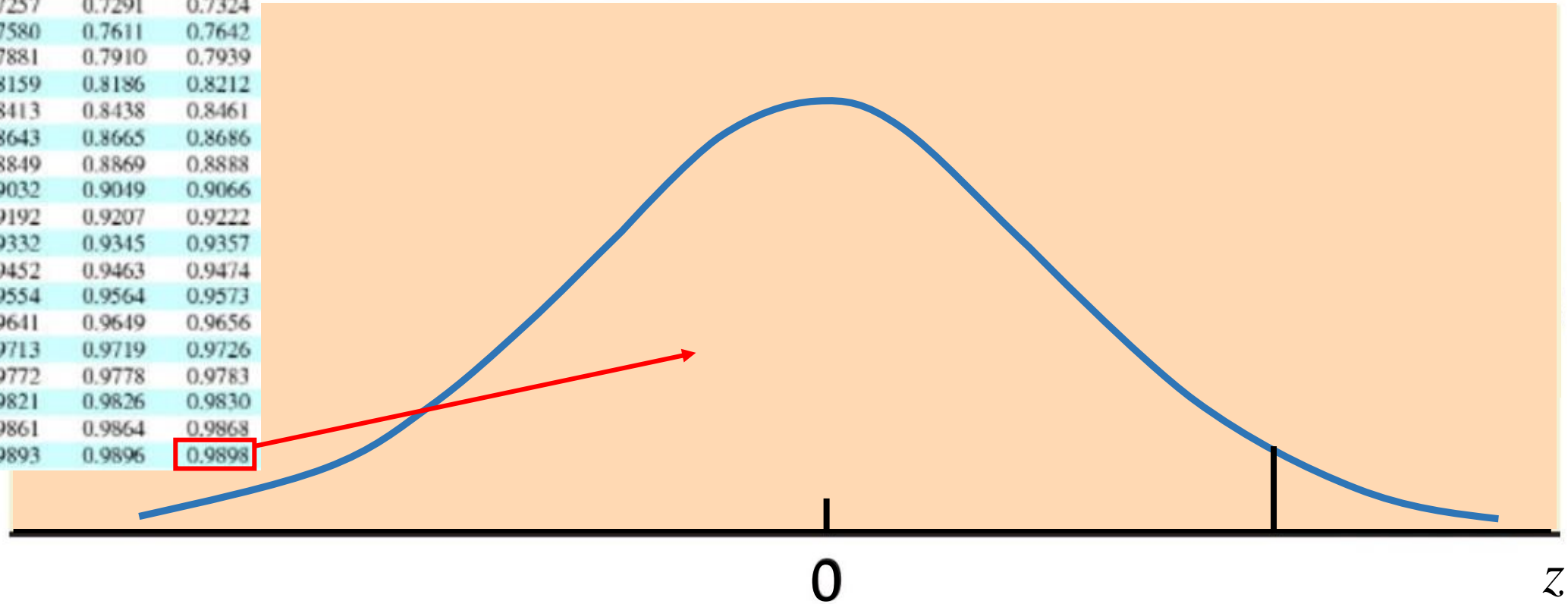
z	.00	.0105	...
-3.4	.0003	.00030003	...
-3.3	.0005	.00050004	...
-3.2	.0007	.00070006	...
.
.
.
1.9	.9713	.97199744	...
.
.
.
3.4	.9997	.99979997	...



a. $P(z < 1.95) = 0.9744 = 97.44\%$

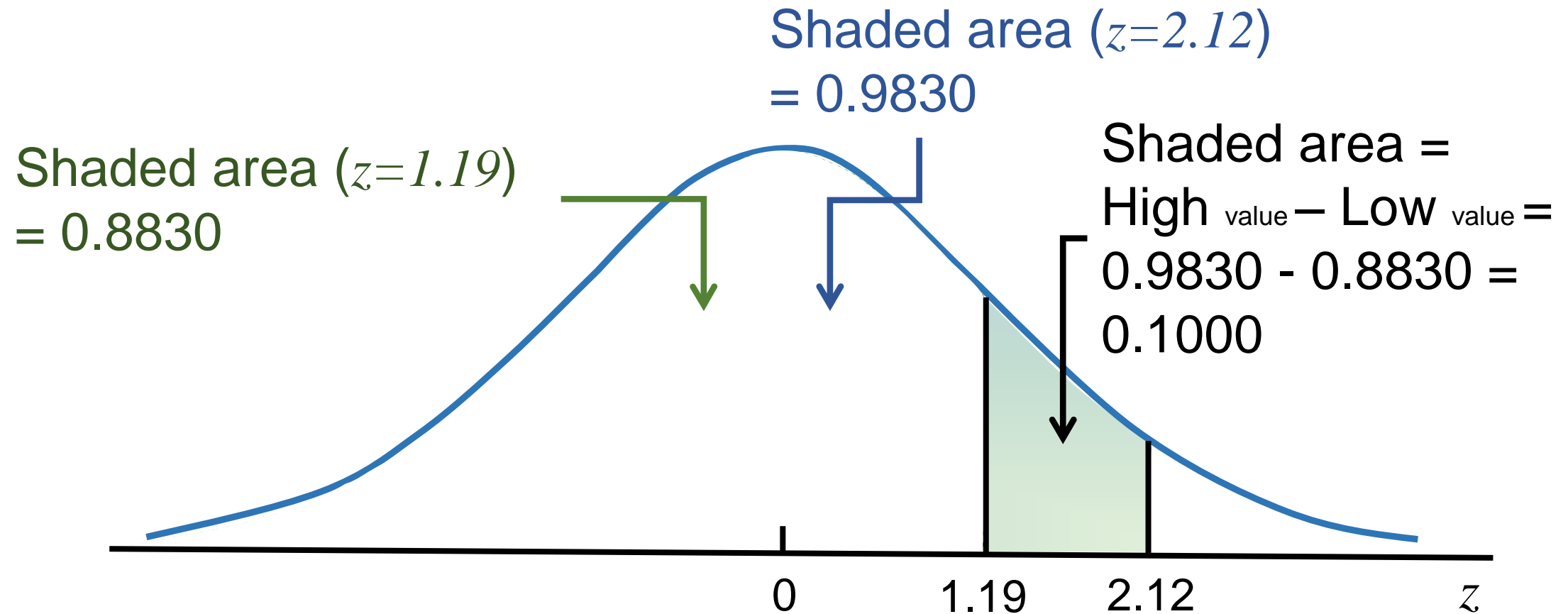
Exercise 1

z	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5478
0.2	0.5793	0.5832	0.5871
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628
0.5	0.6915	0.6950	0.6985
0.6	0.7257	0.7291	0.7324
0.7	0.7580	0.7611	0.7642
0.8	0.7881	0.7910	0.7939
0.9	0.8159	0.8186	0.8212
1.0	0.8413	0.8438	0.8461
1.1	0.8643	0.8665	0.8686
1.2	0.8849	0.8869	0.8888
1.3	0.9032	0.9049	0.9066
1.4	0.9192	0.9207	0.9222
1.5	0.9332	0.9345	0.9357
1.6	0.9452	0.9463	0.9474
1.7	0.9554	0.9564	0.9573
1.8	0.9641	0.9649	0.9656
1.9	0.9713	0.9719	0.9726
2.0	0.9772	0.9778	0.9783
2.1	0.9821	0.9826	0.9830
2.2	0.9861	0.9864	0.9868
2.3	0.9893	0.9896	0.9898



b. $P(z > 2.32) = 0.0102 = 1.02\%$

Exercise 1



c. $P(1.19 < z < 2.12) = 0.1000 = 10\%$

Exercise 2

X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

- a. $P(x < 40)$
- b. $P(x > 21)$
- c. $P(30 < x < 35)$

Exercise with the 2 Tables of Standard Normal Probabilities for positive and negative Z-scores

Solution

To solve the point a., b., c. we have to convert the Normal distribution, X in the Normal standard distribution, Z, by standardization:

$$z = \frac{x - \mu}{\sigma}$$

with mean $\mu = 30$ and standard deviation $\sigma = 4$.

- a. For $x = 40$, the z-value (or z-score) is: $z = (40 - 30) / 4 = 2.5$
Hence $P(x < 40) = P(z < 2.5) = [\text{area to the left of } 2.5] \text{ (from the Normal standard distribution table)} = 0.9938 = 99.38\%$
- b. For $x = 21$, the z-score is: $z = (21 - 30) / 4 = -2.25$
Hence $P(x > 21) = P(z > -2.25) = [\text{total area}] - [\text{area to the left of } -2.25] = 1 - P(z < -2.25) = 1 - 0.0122 = 0.9878 = 98.78\%$
- c. For $x = 30$: $z = (30 - 30) / 4 = 0$ and for $x = 35$: $z = (35 - 30) / 4 = 1.25$
Hence $P(30 < x < 35) = P(0 < z < 1.25) = \text{High value} - \text{Low value} = P(z < 1.25) - P(z < 0) = [\text{area to the left of } z = 1.25] - [\text{area to the left of } 0] = 0.8944 - 0.5 = 0.3944 = 39.44\%$

Exercise 2bis

X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$. Find

- a. $P(x < 40)$
- b. $P(x > 21)$
- c. $P(30 < x < 35)$

Exercise with only the Tables of Standard Normal Probabilities for positive Z-scores

Solution

We have to convert the Normal distribution, X in the Normal standard distribution, Z, by standardization:

$$z = \frac{x - \mu}{\sigma}, \text{ with mean } \mu = 30 \text{ and standard deviation } \sigma = 4.$$

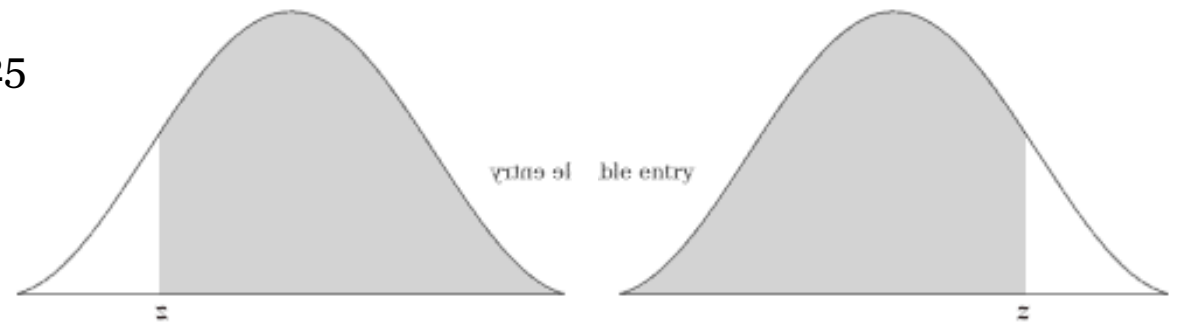
- a. $P(x < 40) = P(z < 2.5) = 0.9938 = 99.38\%$
- b. For $x = 21$, the z-score is: $z = (21 - 30) / 4 = -2.25$
Hence $P(x > 21) = P(z > -2.25)$.

The Z function is Symmetric

$$\text{Hence } P(z > -2.25) = P(z < 2.25)$$

$P(z < 2.25)$ from the table is 0.9878 so, $P(z > -2.25) = P(z < 2.25) = 98.78\%$

- c. $P(30 < x < 35) = P(0 < z < 1.25) = P(z < 1.25) - P(z < 0) = 0.8944 - 0.5 = 0.3944 = 39.44\%$



Exercise 3

A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of 90 km/hr and a standard deviation of 10 km/hr. What is the probability that a car picked at random is travelling at more than 100 km/hr?

Solution

Let X be the random variable that represents the speed of cars.

X has $\mu = 90$ and $\sigma = 10$.

We have to find the probability that x is higher than 100 or $P(x > 100)$

For $x = 100$, the z-score is: $z = (100 - 90) / 10 = 1$

$$P(x > 90) = P(z > 1) = [\text{total area}] - [\text{area to the left of } z = 1] = 1 - P(z < 1) = 1 - 0.8413 = 0.1587$$

The probability that a car selected at a random has a speed greater than 100 km/hr is the 15.87%

Exercise 4

For a certain type of computers, the length of time between charges of the battery is normally distributed with a mean of 50 hours and a standard deviation of 15 hours. John owns one of these computers and wants to know the probability that the length of time will be between 50 and 70 hours.

Solution

Let x be the random variable that represents the length of time.
It has a mean of 50 and a standard deviation of 15.

We have to find the probability that x is between 50 and 70 or $P(50 < x < 70)$

For $x = 50$ the z-score is: $z = (50 - 50) / 15 = 0$

For $x = 70$ the z-score is: $z = (70 - 50) / 15 = 1.33$ (rounded to 2 decimal places)

$P(50 < x < 70) = P(0 < z < 1.33) = [\text{area to the left of } z = 1.33] - [\text{area to the left of } z = 0] = P(z < 1.33) - P(z < 0) = 0.9082 - 0.5 = 0.4082$.

The probability that John's computer has a length of time between 50 and 70 hours is the 40,82%.

Exercise 5

Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Tom wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Tom takes the test and scores 585. Will he be admitted to this university?

Solution

Let X be the random variable that represents the scores.

X is normally distributed with a mean of 500 and a standard deviation of 100.

The total area under the normal curve represents the total number of students who took the test.

If we multiply the values of the areas under the curve by 100, we obtain percentages.

For $x = 585$ the z-score is: $z = (585 - 500) / 100 = 0.85$

The proportion P of students who scored below 585 is given by:

$$P(z < 0.85) = [\text{area to the left of } z = 0.85] = 0.8023 = 80.23\%$$

Tom scored better than 80.23% of the students who took the test and he will be admitted to this University.

Exercise 6

The time taken to assemble a car in a certain plant is a random variable having a normal distribution of 20 hours and a standard deviation of 2 hours. What is the probability that a car can be assembled at this plant at this plant in a period of time

- a) less than 19.5 hours?
- b) between 20 and 22 hours?

Exercise with the 2 Tables of Standard Normal Probabilities for positive and negative Z-scores

Solution

- a) We have to calculate the probability that a car can be assembled in less than 19.8 hours or $P(x < 19.5)$

The z-score for $x=19.9$ is: $z = (19.5-20)/2$, so we have to calculate from the table of Z the probability:

$$P(z < -0.25) = 0.4013 = 40.13\%$$

- b) We have to calculate the probability that a car can be assembled between 20 and 22 hours or $P(20 < x < 22)$

The z-score for $x=20$ is $z=0$ and for $x=22$ is $z=1$. So,

$$P(0 < z < 1) = P(z < 1) - P(z < 0) = 0.8413 - 0.5 = 0.3413 = 34.13\%$$

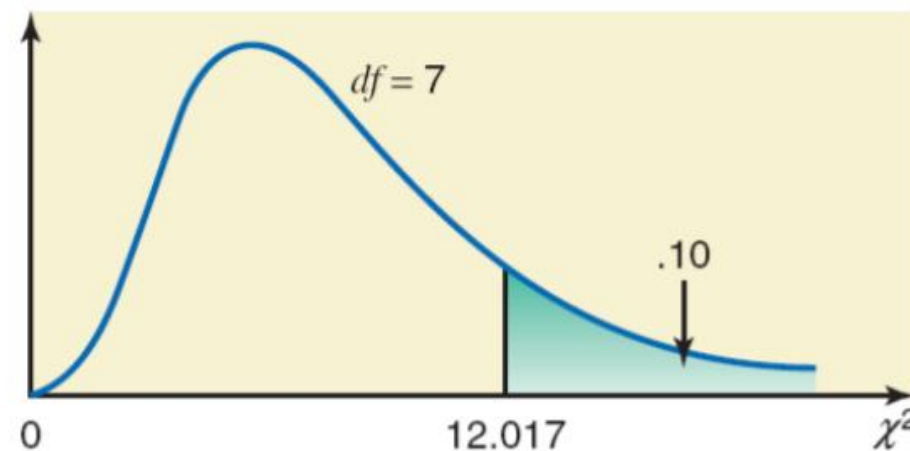
Chi square distribution, χ^2

Let $Z_1, Z_2, Z_3, \dots, Z_n$, be n i.i.d. random variables $Z_i \sim N(0,1)$

$$X = \sum_{i=1}^n Z_i^2 \sim \chi_n^2$$

where n =degrees of freedom (only parameter)

$$E(X) = n \quad \text{and} \quad V(X) = 2n$$



t-Student distribution, t_n

Let $Z \sim N(0,1)$ and $Y \sim \chi_n^2$,

$$X = \frac{Z}{\sqrt{Y/n}} \sim t_n$$

where n = degrees of freedom

- The total area under a t-student distribution curve is 1
- Distribution is bell-shaped and symmetric around $E(X) = 0$
- $V(X) = n/(n-2)$, for $n > 2$
- For $n > 60$, X converges to $Z \sim N(0,1)$

F distribution, $F_{n,m}$

Let $X \sim \chi_m^2$ and $Y \sim \chi_n^2$,

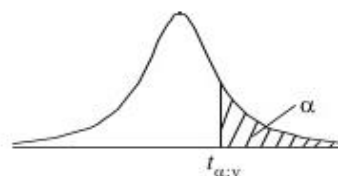
$$F = \frac{X/m}{Y/n} \sim F_{m,n}$$

where m = numerator's degree of freedom, n = denominator's degree of freedom (2 parameters)

The total area under F distribution curve is 1

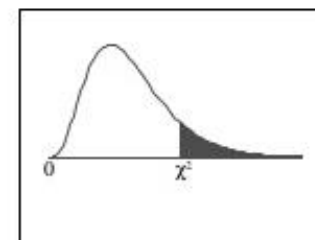
Table of the Student's t -distribution

The table gives the values of $t_{\alpha;v}$ where
 $\Pr(T_v > t_{\alpha;v}) = \alpha$, with v degrees of freedom



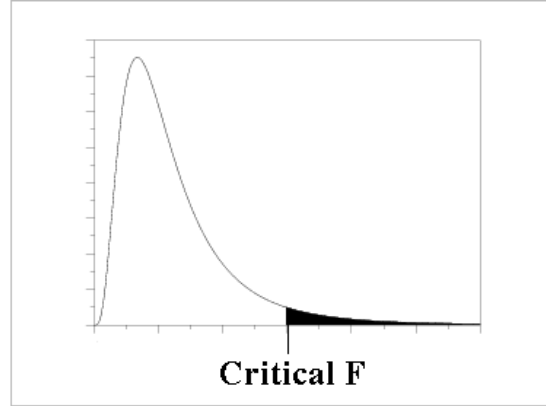
α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
v							
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Chi-Square Distribution Table



The shaded area is equal to α for $\chi^2 = \chi^2_{\alpha}$.

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.800}$	$\chi^2_{.700}$	$\chi^2_{.600}$	$\chi^2_{.500}$	$\chi^2_{.400}$	$\chi^2_{.300}$	$\chi^2_{.200}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
1	0.000	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	10.597	12.838	14.860	16.750	18.548	20.278	21.955
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	11.345	13.442	15.488	17.535	19.591	21.666	23.685	25.788
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.143	13.277	15.152	17.045	18.925	20.793	22.660	24.433	26.191
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	15.152	17.045	18.925	20.793	22.660	24.433	26.191	27.989
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	18.548	20.278	21.955	23.685	25.422	27.081	28.758
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	20.278	21.955	23.685	25.422	27.081	28.758	30.578
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	21.955	23.685	25.422	27.081	28.758	30.578	32.358
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	23.685	25.422	27.081	28.758	30.578	32.358	34.154
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	25.188	26.893	28.599	30.341	32.191	33.924	35.718
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	26.757	28.460	30.191	31.900	33.566	35.307	37.566
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	28.300	30.191	31.900	33.566	35.307	37.566	39.597
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	30.191	31.900	33.566	35.307	37.566	39.597	41.401
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	31.599	33.409	35.172	36.932	38.932	40.781	42.796
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	33.159	34.805	36.191	37.566	39.597	41.401	43.666
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	34.267	35.718	37.156	38.582	40.154	42.000	44.566
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	35.718	37.156	38.582	40.154	42.000	44.566	46.566
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	37.156	38.582	40.154	42.000	44.566	46.566	48.566
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	38.582	40.154	42.000	44.566	46.566	48.566	50.566
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582	40.154	42.000	44.566	46.566	48.566	50.566	52.566
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	41.401	43.154	45.022	46.979	48.925	50.925	54.566
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	43.154	45.022	46.979	48.925	50.925	52.925	56.566
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796	44.566	46.566	48.566	50.566	52.566	54.566	58.566
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	45.979	47.979	49.979	51.979	53.979	55.979	60.566
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	47.559	49.559	51.559	53.559	55.559	57.559	62.566
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928	48.928	50.928	52.928	54.928	56.928	58.928	64.566
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	50.290	52.290	54.290	56.290	58.290	60.290	66.566
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	51.645	53.645	55.645	57.645	59.645	61.645	68.566
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993	52.993	54.993	56.993	58.993	60.993	62.993	70.566
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	54.336	56.336	58.336	60.336	62.336	64.336	72.566
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	55.672	57.672	59.672	61.672	63.672	65.672	74.566
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766	69.766	72.766	75.766	78.766	81.766	84.766	87.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	75.754	79.490	83.195	86.866	90.554	94.254	97.974	101.674	105.374
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	87.566	91.674	95.904	100.154	104.424	108.704	112.994	117.294	121.594
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	108.629	113.004	117.444	121.874	126.324	130.794	135.274
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	111.929	116.329	120.749	125.179	129.629	134.099	138.589	143.099	147.619
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	123.116	127.606	132.606	137.606	142.606	147.606	152.606	157.606	162.606
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.001	140.451	145.901	151.351	156.801	162.251	167.701	173.151	178.601

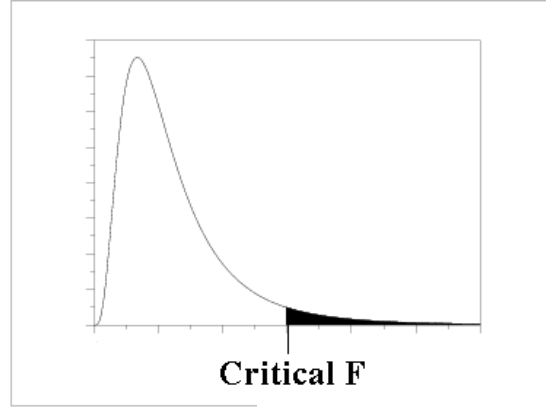


F - Distribution ($\alpha = 0.05$ in the Right Tail)

Denominator Degrees of Freedom df_2	Numerator Degrees of Freedom df_1	Numerator Degrees of Freedom								
		1	2	3	4	5	6	7	8	9
1		161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2		18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3		10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123
4		7.7086	9.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	6.9988
5		6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6		5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990
7		5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8		5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881
9		5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10		4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11		4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12		4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13		4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14		4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15		4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16		4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17		4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
18		4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19		4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20		4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21		4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660
22		4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23		4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24		4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25		4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26		4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27		4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28		4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29		4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229
30		4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40		4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240
60		4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401
120		3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588
∞		3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799

F - Distribution ($\alpha = 0.05$ in the Right Tail)

Denominator Degrees of Freedom df_2	Numerator Degrees of Freedom df_1	10	12	15	20	24	30	40	60	120	∞
1		241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2		19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
3		8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5264
4		5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.6281
5		4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.3650
6		4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7		3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8		3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9		3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10		2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11		2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12		2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13		2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14		2.6022	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15		2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16		2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17		2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18		2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19		2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780
20		2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21		2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22		2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23		2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24		2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25		2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26		2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27		2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28		2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29		2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30		2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40		2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60		1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120		1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
∞		1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000



F - Distribution ($\alpha = 0.01$ in the Right Tail)

F - Distribution ($\alpha = 0.01$ in the Right Tail)

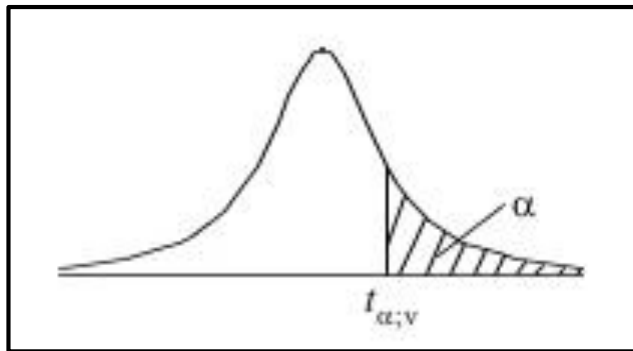
Denominator Degrees of Freedom df_2	Numerator Degrees of Freedom df_1	Numerator Degrees of Freedom								
		1	2	3	4	5	6	7	8	9
1		4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
2		98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388
3		34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345
4		21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659
5		16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158
6		13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1017	7.9761
7		12.246	9.5466	8.4513	7.8466	7.4604	7.1914	6.9928	6.8400	6.7188
8		11.259	8.6491	7.5910	7.0061	6.6318	6.3707	6.1776	6.0289	5.9106
9		10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10		10.044	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11		9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12		9.3302	6.9266	5.9525	5.4120	5.0643	4.8206	4.6395	4.4994	4.3875
13		9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14		8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15		8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16		8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17		8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18		8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19		8.1849	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20		8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21		8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22		7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23		7.8811	5.6637	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986
24		7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25		7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
26		7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818
27		7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494
28		7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195
29		7.5977	5.4204	4.5378	4.0449	3.7254	3.4995	3.3303	3.1982	3.0920
30		7.5625	5.3903	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40		7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60		7.0771	4.9774	4.1259	3.6490	3.3389	3.1187	2.9530	2.8233	2.7185
120		6.8509	4.7865	3.9491	3.4795	3.1735	2.9559	2.7918	2.6629	2.5586
∞		6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073

Denominator Degrees of Freedom df_2	Numerator Degrees of Freedom df_1	10	12	15	20	24	30	40	60	120	∞
1		6055.8	6106.3	6157.3	6208.7	6234.6	6260.6	6286.8	6313.0	6339.4	6365.9
2		99.399	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.491	99.499
3		27.229	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
4		14.546	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
5		10.051	9.8883	9.7222	9.5526	9.4665	9.3793	9.2912	9.2020	9.1118	9.0204
6		7.8741	7.7183	7.5590	7.3958	7.3127	7.2285	7.1432	7.0567	6.9690	6.8800
7		6.6201	6.4691	6.3143	6.1554	6.0743	5.9920	5.9084	5.8236	5.7373	5.6495
8		5.8143	5.6667	5.5151	5.3591	5.2793	5.1981	5.1156	5.0316	4.9461	4.8588
9		5.2565	5.1114	4.9621	4.8080	4.7290	4.6486	4.5666	4.4831	4.3978	4.3105
10		4.8491	4.7059	4.5581	4.4054	4.3269	4.2469	4.1653	4.0819	3.9965	3.9090
11		4.5393	4.3974	4.2509	4.0990	4.0209	3.9411	3.8596	3.7761	3.6904	3.6024
12		4.2961	4.1553	4.0096	3.8584	3.7805	3.7008	3.6192	3.5355	3.4494	3.3608
13		4.1003	3.9603	3.8154	3.6646	3.5868	3.5070	3.4253	3.3413	3.2548	3.1654
14		3.9394	3.8001	3.6557	3.5052	3.4274	3.3476	3.2656	3.1813	3.0942	3.0040
15		3.8049	3.6662	3.5222	3.3719	3.2940	3.2141	3.1319	3.0471	2.9595	2.8684
16		3.6909	3.5527	3.4089	3.2587	3.1808	3.1007	3.0182	2.9330	2.8447	2.7528
17		3.5931	3.4552	3.3117	3.1615	3.0835	3.0032	2.9205	2.8348	2.7459	2.6530
18		3.5082	3.3706	3.2273	3.0771	2.9990	2.9185	2.8354	2.7493	2.6597	2.5660
19		3.4338	3.2965	3.1533	3.0031	2.9249	2.8442	2.7608	2.6742	2.5839	2.4893
20		3.3682	3.2311	3.0880	2.9377	2.8594	2.7785	2.6947	2.6077	2.5168	2.4212
21		3.3098	3.1730	3.0300	2.8796	2.8010	2.7200	2.6359	2.5484	2.4568	2.3603
22		3.2576	3.1209	2.9779	2.8274	2.7488	2.6675	2.5831	2.4951	2.4029	2.3055
23		3.2106	3.0740	2.9311	2.7805	2.7017	2.6202	2.5355	2.4471	2.3542	2.2558
24		3.1681	3.0316	2.8887	2.7380	2.6591	2.5773	2.4923	2.4035	2.3100	2.2107
25		3.1294	2.9931	2.8502	2.6993	2.6203	2.5383	2.4530	2.3637	2.2696	2.1694
26		3.0941	2.9578	2.8150	2.6640	2.5848	2.5026	2.4170	2.3273	2.2325	2.1315
27		3.0618	2.9256	2.7827	2.6316	2.5522	2.4699	2.3840	2.2938	2.1985	2.0965
28		3.0320	2.8959	2.7530	2.6017	2.5223	2.4397	2.3535	2.2629	2.1670	2.0642
29		3.0045	2.8685	2.7256	2.5742	2.4946	2.4118	2.3253	2.2344	2.1379	2.0342
30		2.9791	2.8431	2.7002	2.5487	2.4689	2.3860	2.2992	2.2079	2.1108	2.0062
40		2.8005	2.6648	2.5216	2.3689	2.2880	2.2034	2.1142	2.0194	1.9172	1.8047
60		2.6318	2.4961	2.3523	2.1978	2.1154	2.0285	1.9360	1.8363	1.7263	1.6006
120		2.4721	2.3363	2.1915	2.0346	1.9500	1.8600	1.7628	1.6557	1.5330	1.3805
∞		2.3209	2.1847	2.0385	1.8783	1.7908	1.6964	1.5923	1.4730	1.3246	1.0000

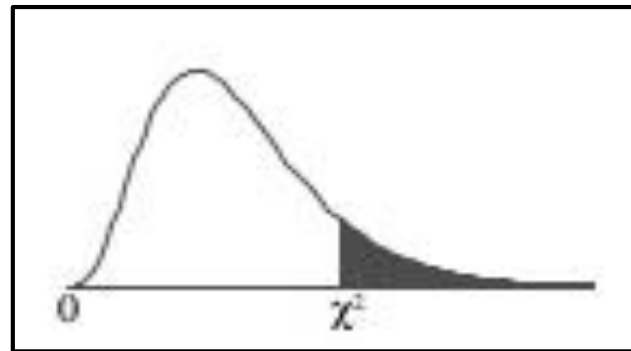
Exercise 7

- Find the value of t for 24 df and 0.05 area in the right tail (*one-sided*);
- Find the value of t for 24 df and 0.05 in the *two-sided* area ;
- Find the probability that a r.v. t for 24 df assumes a value greater than 2.5;
- Find the probability that a r.v. t for 24 df assumes a value greater than 1.5;
- Find the probability that a r.v. χ^2 for 3 df assumes a value greater than or equal to 7;
- Find the probability that a r.v. χ^2 for 5 df leaves in the right tail a probability of 0.1;
- Find the probability that a r.v. $F_{10,10}$ leaves in the right tail a probability of 0.05.

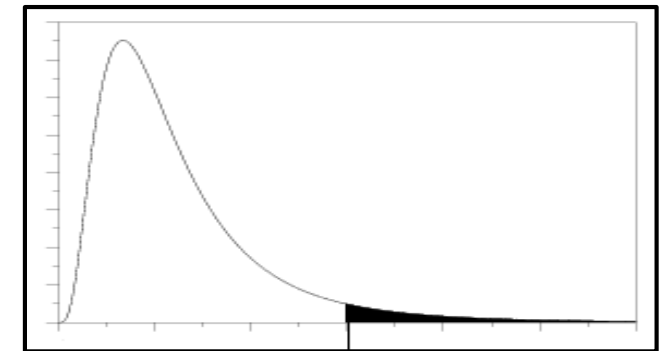
Student's T



χ^2



F



Exercise 7

Solution

- a. We have to find the 95% percentile of the t di Student with 24 df : $t_{24,0.05}$. From the table the value is: 1.711
- b. We have to find the 2.5% e 97.5% percentile that border the two-sided area that leaves in the tail a probability of 5%. From the table $t_{24,0.025} = 2.064$ and $t_{24,0.975} = -2.064$
- c. If $X \sim t_{24}$ then, $P(X > 2.5) = 0.01$
- d. If $X \sim t_{24}$ then, $0.05 \leq P(X > 1.5) \leq 0.1$
- e. If $X \sim \chi^2_3$, then $0.05 \leq P(X > 7) \leq 0.1$
- f. From the table the 90% percentile of the distribution of the r.v. $\chi^2_{(5, 0.1)}$ with 5 df and a probability of 0.1 on the right tail is 9.236
- g. From the table the 95% percentile of the distribution of the r.v. $F_{10,10}$ and a probability on the right tail of 0.05 is 2.9782