

MATHEMATICS II
Thursday, March 15 2018
Third Exercise Class

1. Given in \mathbb{R}^3 the vectors $\vec{v} = (0, 3, 2)$, $\vec{w} = (1, 0, -2)$ and $\vec{z} = (0, 1, 1)$, calculate:
 - a) the combination $2\vec{v} + 3\vec{w} - \vec{z}$;
 - b) the inner product $\vec{v} \cdot \vec{w}$;
 - c) the combination $(\vec{v} + \vec{z}) \times \vec{w}$;
 - d) the length of each vector.
2. Given in \mathbb{R}^3 the vectors $\vec{v} = (1, 0, 1)$, $\vec{w} = (1, 1, 1)$ and $\vec{z} = (2, 1, 2)$, calculate the values of the parameters α , β , γ , such that : $\alpha\vec{v} + \beta\vec{w} + \gamma\vec{z} = 0$.
3. Given the vectors $\vec{v} = (3, 0, 2)$ and $\vec{w} = (k, 3, k + 2)$ determine values for k such that their inner product is zero.
4. For each of the following vectors, find the correspondent versor
 - a) $\vec{v} = (\alpha, 3\alpha)$;
 - b) $\vec{v} = (\alpha - 3, 2\alpha, 1)$;
 - c) $\vec{v} = (3, 2, -\alpha)$;
 - d) $\vec{v} = (1, \alpha, 4\alpha)$.
5. Determine the parametric equation and the Cartesian equation of the line on the space
 - a) passing through the points A(2,-1,2) and B(4,3,6);
 - b) passing through the point P(1,-1,0) and parallel to the vector $\vec{v} = (2, 0, 1)$;
 - c) of Cartesian equation $\begin{cases} y = 2x + 3 \\ x + 2y - z = 0 \end{cases}$.
6. Determine the reciprocal position of the lines r and s of Cartesian equations

$$r : \begin{cases} x + 3y = 4 \\ 2y + z = 0 \end{cases} \quad ; \quad s : \begin{cases} 3x + 2y = 5 \\ x - 2z = 5 \end{cases} .$$

7. Consider the lines of parametric equations

$$r : \begin{cases} x = 1 + 4t \\ y = -1 + 6t \\ z = 2 - 2t \end{cases} \quad ; \quad s : \begin{cases} x = 3 + 2t \\ y = 1 + 3t \\ z = -2 - t \end{cases} .$$

- a) Verify that the lines are parallel;
- b) Calculate the distance between r and s ;
- c) Find the plane π to which both the lines belong (Does the point $C(2,0,2)$ belong to the plane π ?).

8. Given the lines

$$r : \begin{cases} x + y - 2 = 0 \\ x + y - 2z = 0 \end{cases} \quad ; \quad s : \begin{cases} x - 2y + 1 = 0 \\ 2x - 2z - 2 = 0 \end{cases} ;$$

find the equation of the family of planes parallel to both r and s . Moreover find a plane to which the line r belongs and a plane to which both r and s belong.

9. Establish if the vectors $\vec{v} = (3, 0, 4)$ and $\vec{w} = (6, 0, 8)$ in \mathbb{R}^3 are linearly dependent.

10. Determine for which value of k the following vectors in \mathbb{R}^4 are linearly independent

$$\vec{v} = (1, 2, k, 3); \quad \vec{w} = (k, 2, 3, 1); \quad \vec{z} = (k, 6, 5, 5).$$

11. Calculate the rank of the following matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ k & 2k & k \\ 2 & k+2 & -2 \end{pmatrix}$$