

Mathematics II

First Practice: Solutions

1. Solve each of the following indefinite integrals (i.e. find all the antiderivatives of each given function):

$$a) \quad \int x^3 \cos x^4 dx = \frac{1}{4} \sin x^4 + c$$

$$b) \quad \int \cos x \cdot e^{\sin x} dx = e^{\sin x} + c$$

$$c) \quad \int e^x \tan(e^x) dx = -\log |\cos(e^x)| + c$$

$$d) \quad \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = -2 \cos \sqrt{x} + c$$

$$e) \quad \int x \arctan x dx = \frac{1}{2}(x^2 + 1) \arctan x - \frac{1}{2}x + c$$

$$f) \quad \int x \sin x dx = -x \cos x + \sin x + c$$

$$g) \quad \int (3 + x^5) \log x dx = \left(3x + \frac{x^6}{6}\right) \log x - 3x - \frac{x^6}{36} + c$$

$$h) \quad \int \frac{1}{x(\log x + \log^2 x)} dx = \log |\log x| - \log |\log x + 1| + c$$

$$i) \quad \int \frac{x-3}{(x-1)(x-2)} dx = \log(x-1)^2 - \log|x-2| + c$$

2. Evaluate the following definite integrals:

$$a) \quad \int_3^4 x\sqrt{x-3} dx = \frac{12}{5}$$

$$b) \quad \int_2^3 \frac{x+1}{\sqrt{x^2+2x+3}} dx = \sqrt{18} - \sqrt{11}$$

$$c) \quad \int_0^1 \sqrt{1+\sqrt{x}} dx = \frac{8}{15}(\sqrt{2}+1)$$

$$d) \quad \int_0^1 \arctan x dx = \frac{\pi}{4} - \frac{1}{2} \log 2$$

$$\begin{aligned}
e) \quad & \int_1^2 x^3 e^{x^2} dx = \frac{3e^4}{2} \\
f) \quad & \int_0^1 \frac{\ln(x+1)}{\sqrt{x+1}} dx = 2\sqrt{2} \log 2 - 4\sqrt{2} + 4 \\
g) \quad & \int_0^{\pi/2} \sin^2 x dx = \frac{\pi}{4} \\
h) \quad & \int_0^1 \frac{1}{e^x + 1} dx = 1 + \log 2 - \log(1+e) = \log \frac{2e}{1+e} \\
i) \quad & \int_1^2 \frac{1}{(1+x)(2+x)} dx = 2 \log 3 - 3 \log 2
\end{aligned}$$