Mathematics II

Third Practice

1. Compute (when possible) the inverse of the following matrices:

$$a)\ \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix} \qquad b)\ \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix} \qquad c)\ \begin{pmatrix} 1 & 3 \\ -2 & -1 \end{pmatrix}$$

$$d)\ \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \end{pmatrix} \quad e)\ \begin{pmatrix} 1 & 2 & 2 \\ 1 & -3 & -1 \\ 1 & 0 & -1 \end{pmatrix} \quad f)\ \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$g)\ \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & -3 & 0 & -5 \\ -4 & 3 & 4 & 1 \\ 4 & 3 & -4 & 0 \end{pmatrix}$$

2. Compute the rank of the following matrices:

$$a)\ \begin{pmatrix} 1 & 3 & 1 \\ -1 & 1 & 2 \\ 3 & 1 & -3 \end{pmatrix} \qquad b)\ \begin{pmatrix} 0 & 1 & 2 & -4 \\ 1 & -1 & 1 & 1 \\ -2 & 3 & 0 & -6 \end{pmatrix}$$

$$c)\ \begin{pmatrix} 3 & 1 & 0 & -1 & 2 \\ 1 & 5 & 3 & 2 & 1 \\ -1 & 0 & 0 & -1 & 4 \end{pmatrix} \qquad d)\ \begin{pmatrix} 0 & 2 & -4 & 3 \\ 0 & 1 & -2 & 1 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 1 & 0 & -2 \end{pmatrix}$$

3. Determine for which value of k the following vectors in $\mathbb{R}^4$ are linearly independent:

$$\vec{v} = (2, -1, 2, -1); \quad \vec{w} = (1, 1, 1, 1); \quad \vec{z} = (3, k, 3, -k).$$

4. Which of the following sets of vectors are linearly independent? Which of them span the space $\mathbb{R}^n$ to which they belong? Which of them form a basis?

$$
\begin{aligned}
&a)\ \mathbb{R}^2 :\ \{(0,0),(1,0),(0,1)\}\\
&b)\ \mathbb{R}^3 :\ \{(1,1,1),(1,0,0)\}\\
&c)\ \mathbb{R}^2 :\ \{(1,2),(3,-1)\}\\
&d)\ \mathbb{R}^4 :\ \{(2,0,1,1),(3,1,0,2),(2,0,1,4)\}
\end{aligned}
$$

5. Calculate (if any) all solutions for each of the following systems:

$$
(a)\ :\begin{cases} x+y+z=1 \\ 3x+3y-2z-2=0 \end{cases} \quad , \quad (b)\ :\begin{cases} x+y+z=1 \\ 3x+3y-2z-2=0 \\ z=1 \end{cases}
$$

$$
(c)\ :\begin{cases} x+y=1 \\ 3x+y=0 \\ 5x+3y=0 \end{cases} \quad , \quad (d)\ :\begin{cases} x+y=1 \\ 3x+y=0 \\ 5x+3y=2 \end{cases}
$$

6. Calculate the solutions of the following systems

$$
a)\ :\begin{cases} kx+y-z=0 \\ 2y+kz=k \\ x-z=0 \end{cases} \qquad b)\ :\begin{cases} x+y+z+w=0 \\ y-z=1 \\ x+2y+kw=2 \end{cases}
$$

$\forall k \in \mathbb{R}$.