

Mathematics II

Fifth Practice: Solutions

1. Determine for which values of the parameter $k \in \mathbb{R}$ the following matrix is diagonalizable:

$$\begin{pmatrix} k & 1 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & k \end{pmatrix}$$

2. Describe the domain $D_f \subset \mathbb{R}^2$ of each of the following functions:

$$\begin{aligned} a) \quad f(x, y) &= \sqrt{4 - x^2 - y^2} \\ b) \quad f(x, y) &= \sqrt{\sin^2(xy) + 2e^{x+y}} \\ c) \quad f(x, y) &= \frac{\cos(x+y) + e^x}{x-y} \\ d) \quad f(x, y) &= \frac{x+y}{\sqrt{x^2+y^2}} \\ e) \quad f(x, y) &= \frac{\log y}{e^x} \\ f) \quad f(x, y) &= \frac{\log(x-y) + \log(x+y)}{\sqrt{x}} \end{aligned}$$

3. Describe the contour curves of the following functions:

$$\begin{aligned} a) \quad f(x, y) &= \sqrt{x^2 + y^2} \\ b) \quad f(x, y) &= \sqrt{x-y} \\ c) \quad f(x, y) &= \frac{y}{x} \end{aligned}$$

4. Find the following multivariable limits:

$$\begin{aligned} a) \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sin xy} \\ b) \quad \lim_{(x,y) \rightarrow (2,0)} \frac{1 - \cos y}{xy^2} \\ c) \quad \lim_{(x,y) \rightarrow (0,0)} xy e^{\frac{xy}{x^2+y^2}} \end{aligned}$$

5. Consider the function

$$f(x, y) = \frac{2xy^2 \sin^2(y)}{(x^2 + y^2)^2}.$$

Does the limit exist when (x, y) tends to $(0, 0)$?

6. Calculate the partial derivatives, $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$, of the following functions:

$$a) f(x, y) = x^2 + y - e^{xy}$$

$$b) f(x, y) = \frac{1}{1 + xy}$$

$$c) f(x, y) = \sqrt{x^2 + y^2}$$

7. Study the continuity and the differentiability of the following function

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

8. Study the differentiability of

$$f(x, y) = 2xy + \frac{x}{y}$$

at $(1, 1)$.

9. Study the continuity and the differentiability of the following function

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

10. Find all the stationary points of the following functions and classify them:

$$a) \quad x^6 + y^2$$

$$b) \quad 3x^2 y - xy$$

$$c) \quad x^2 + 2y^2 - 3y^3$$

$$d) \quad y^3 + y + 7xy + 7x^2 + 4$$

$$e) \quad x - x^3 - 4xy^2$$