MATHEMATICS II
Thursday April 19 2018
Seventh Exercise Class

1. Decide if the following function are differentiable at the specified point.

   a) $f(x,y) = xe^{x,y}$, $(x,y) = (1,0)$.

   SOLUTION: $f_x(x,y) = e^{xy} + xye^{xy}, f_y(x,y) = x^2e^{xy}$ are continuous in $(1,0)$, then $f(x,y)$ is differentiable at $(1,0)$.

   b) $f(x,y) = \sqrt{x^2+y^2}$, $(x,y) = (0,0)$.

   SOLUTION: no, since $f_x(x,y), f_y(x,y)$ are not defined for $(x,y) = (0,0)$.

   c) $f(x,y) = \begin{cases} \frac{xy^2}{\sqrt{x^4+y^4}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

   SOLUTION: no, since $f_x(0,0) = 0$, $f_y(0,0) = 0$, $\displaystyle\lim_{\sqrt{h^2+k^2}\to 0} \frac{f(h,k)}{\sqrt{h^2+k^2}}$ is not defined.

2. Find local maxima, local minima and saddle point of the following functions.

   a) $f(x,y) = e^{x^2+y^2}$.

   SOLUTION: $(x,y) = (0,0)$ minimum.

   b) $f(x,y) = e^{x^2-y^2}$.

   SOLUTION: $(x,y) = (0,0)$ saddle point.

   c) $f(x,y) = \log(xy) - x - y$.

   SOLUTION: $(x,y) = (1,1)$ maximum.

   d) $f(x,y) = \log(xy) - x^2 - y^2$.

   SOLUTION: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ local maxima.

   e) $f(x,y,z) = 2x^2 - y^2 - z^2 + 2xz - 2x - 2z$.

   SOLUTION: $P = (0,0,-1)$ maximum.

   f) $f(x,y,z) = e^{x^2+y^2-z^2}$.

   SOLUTION: $P = (0,0,0)$ saddle point.

3. Find the maximum and the minimum values of f subject to the given constraint.

   a) $f(x,y) = x^2 + 2xy - y^2$ on $2x + 3y = 0$.

   SOLUTION: $(0,0)$ global max.

b) $f(x,y) = x - y$ on $x^2 + y = 2$

SOLUTION: $(x,y) = \left(-\frac{1}{1}, \frac{7}{4}\right)$ local min.

c) $f(x,y) = x^2 + 2y$ on $y^2 = x$.

SOLUTION: $(x,y) = \left(\frac{1}{\sqrt[3]{2}}, \sqrt[3]{2}\right)$ local max.

d) $f(x,y) = y^3 - 3x$ on $x - y = 1$.

SOLUTION: $(x,y) = (2,1)$ max, $(x,y) = (0,-1)$ min.

e) $f(x,y,x) = x + y + z$ on $xyz = 1$.

SOLUTION: $P = (1,1,1)$ mimimum.

f) $f(x,y,x) = x^2 + y^2 + z^2$ on $\begin{cases} x + y = z \\ y + z = 1 \end{cases}$.

SOLUTION : $P = \left(0, \frac{1}{2}, \frac{1}{2}\right)$ global minimum.