

MATHEMATICS II
Thursday March 22 2018
Fourth Exercise Class

1. Determine the rank of the following sets of vectors.

a) $\vec{v}_1 = (1, 2, 0), \vec{v}_2 = (2, 0, 1), \vec{v}_3 = (1, 1, 0), \vec{v}_4 = (1, -1, 2)$.

SOLUTION: 3

b) $\vec{v}_1 = (2, 0, 1, 0), \vec{v}_2 = (-1, 1, 0, 1), \vec{v}_3 = (2, -2, 0, 0), \vec{v}_4 = (0, 1, 3, 0)$.

SOLUTION: 4

c) $\vec{v}_1 = (0, 0, 1, 1, 0), \vec{v}_2 = (1, 1, 0, 0, 1), \vec{v}_3 = (1, 0, 1, 0, 1)$.

SOLUTION: 3

2. Determine the rank of the following set of vectors on the varying parameter α .

$\vec{v}_1 = (\alpha, 1, 1), \vec{v}_2 = (1, \alpha, 1), \vec{v}_3 = (0, 1, \alpha)$.

SOLUTION: 3 if $\alpha \neq 1, \frac{\sqrt{5}-1}{2}, -\frac{\sqrt{5}-1}{2}$; 2 otherwise.

3. Determine the rank of the following matrices.

a)

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & -3 & 1 \\ 4 & 4 & 0 \end{pmatrix}$$

.

SOLUTION: 2

b)

$$\begin{pmatrix} 2 & 3 & -1 & 1 \\ 1 & -2 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ -1 & -3 & 3 & 1 \end{pmatrix}$$

.

SOLUTION: 4

4. Determine the rank of the following matrices on the varying parameter α .

a)

$$\begin{pmatrix} 0 & 1-\alpha & 2 \\ 1+\alpha & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

SOLUTION: 3, $\forall \alpha \in \mathbb{R}$.

b)

$$\begin{pmatrix} 0 & \alpha & 1 \\ \alpha + 2 & 0 & -1 \\ 1 & 2 & 0 \end{pmatrix}$$

SOLUTION: 3 if $\alpha = -4$, 2 otherwise.

5. Study the following systems of equations. For each of them determine the solutions if they exist.

a)

$$\begin{cases} x + y - z = 1 \\ 2x - y - 2z = -1 \\ -x + y - z = 2 \end{cases}$$

SOLUTION: $x = -\frac{1}{2}, y = 1, z = -\frac{1}{2}$

b)

$$\begin{cases} 2x - 5y + 2z = 1 \\ 3x - 2y + z = 4 \\ x + 3y - z = -3 \end{cases}$$

SOLUTION: inconsistent

c)

$$\begin{cases} 2x - 3z = 4 \\ -y + 4w = -3 \\ x - 2y = -2 \\ 2w + z = 2 \end{cases}$$

SOLUTION: $x = \frac{52}{11}, y = \frac{37}{11}, w = \frac{1}{11}, z = \frac{20}{11}$

d)

$$\begin{cases} 2x - 2y = 0 \\ x + 2z = 1 \\ 3x - w = 2 \\ w + z = -1 \end{cases}$$

SOLUTION: $x = \frac{1}{5}, y = \frac{1}{5}, w = -\frac{7}{5}, z = \frac{2}{5}$

6. Study the following systems of equations on the varying parameter λ and μ .

a)

$$\begin{cases} x - y - \lambda x = 0 \\ -x + y - \lambda y = 0 \end{cases}$$

SOLUTION: $\lambda = 0 \Rightarrow$ System underdetermined, $(x, y) = (t, t), t \in \mathbb{R}$;

$\lambda = 2 \Rightarrow$ System underdetermined, $(x, y) = (t, -t), t \in \mathbb{R}$;

$$\lambda \neq 0, 2 \Rightarrow x = 0, y = 0.$$

b)

$$\begin{cases} y + z = 1 \\ 2x + 2z = \lambda \\ x + z = 1 \end{cases}$$

SOLUTION: $\lambda \neq 2 \Rightarrow$ System inconsistent;

$$\lambda = 2 \Rightarrow (x, y, z) = (1, t, 1 - t)t \in \mathbb{R}.$$

c)

$$\begin{cases} y + z = 1 \\ 2x + 2z = \lambda \\ x + y + z = 1 \end{cases}$$

$$\text{SOLUTION: } x = 0, y = \frac{2-\lambda}{2}, z = \frac{\lambda}{2}, \lambda \in \mathbb{R}.$$

d)

$$\begin{cases} y + z = 1 \\ 2x + 2z = \lambda \\ x + \gamma y + z = 1 \end{cases}$$

$$\text{SOLUTION: } \gamma \neq 0 \Rightarrow x = 0, y = \frac{2-\lambda}{2\gamma}, z = \frac{\lambda}{2\gamma}, \lambda \in \mathbb{R};$$

$\gamma = 0, \Rightarrow$ see part b).

e)

$$\begin{cases} x + y + z = \lambda x \\ 2x - y + z = \lambda y \\ -x + 2y + 2z = \lambda z \end{cases}$$