A PREMISE:

Misbehaviour in the Classroom

Each class starts 15 minutes after the posted starting time.

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NO ENTRANCE will be allowed after this time.

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Talking is forbidden during classes.
For any question or doubt raise your hand and ask the teacher, not your colleagues!

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Those who disturb the lesson will be invited to leave the classroom immediately.

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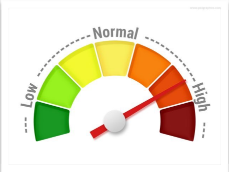

If the noise in the classroom exceeds a certain threshold, the lesson will be ended.
The undiscussed topics will still be subject of examination.

Structure of the course and of the exam

https://economia.uniroma2.it/ba/business-administration-economics/faculty/374/pirino-davide-erminio

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BACHELOR DEGREE IN
BUSINESS ADMINISTRATION AND ECONOMICS

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Davide Erminio Pirino

Associate Professor

Sector:	MATHEMATICS FOR ECONOMICS AND FINANCE [SECS-S/06]
Email:	davide.pirino@gmail.com
Office hour:	write me an e-mail to have an appointment
Room:	1B2-4 (first floor, building B)
Phone:	nd
Curriculum Vitae (EN):	download (pdf)

Courses A.Y. 2019-2020

MATHEMATICS MATHEMATICS I
Lecturer - Bachelor degree

TIME SERIES AND ECONOMETRICS
Lecturer - Master of Science

Structure of the course and of the exam

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MATHEMATICS I

Davide Erminio Pirino

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Program

A.Y. 2019-2020

Updated A.Y. 2019-2020

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Set theory. Set Operations: union, intersection, difference, complement, Cartesian product. Number sets. Intervals. Interior, exterior and accumulation points. Maximum, minimum, inferior and supremum. Powers with rational and real exponents. Funcions: domain, range, image. Injective and surjective functions.

Structure of the course and of the exam

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Calendar

The course is scheduled for the **Fall semester**.
Course lectures follow the current schema:

Day	Starting at	to	Typology	Room
Tuesday	15:00	17:00	Lecture	Aula T3
Tuesday *	14:00	16:00	Lecture	Aula T3
Thursday *	16:00	18:00	Practice	Aula I2
Thursday *	11:00	13:00	Lecture	Aula T2

Structure of the course and of the exam

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With me until October 15th

Structure of the course and of the exam

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With Prof. Colaneri from October 17th

Structure of the course and of the exam

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Structure of the course and of the exam

Detailed time schedule

Day		Typology	Starting at	to	Room
Tuesday	Sep 17, 2019	Lecture	15:00	17:00	Aula T3
Thursday	Sep 19, 2019	Lecture	11:00	13:00	Aula T2
Tuesday	Sep 24, 2019	Lecture	15:00	17:00	Aula T3
Thursday	Sep 26, 2019	Lecture	11:00	13:00	Aula T2
Thursday	Sep 26, 2019	Practice	16:00	18:00	Aula I2
Tuesday	Oct 01, 2019	Lecture	15:00	17:00	Aula T3
Thursday	Oct 03, 2019	Lecture	11:00	13:00	Aula T2
Thursday	Oct 03, 2019	Practice	16:00	18:00	Aula I2
Tuesday	Oct 08, 2019	Lecture	15:00	17:00	Aula T3
Thursday	Oct 10, 2019	Lecture	11:00	13:00	Aula T2
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Structure of the course and of the exam

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Structure of the course and of the exam

Berkan
Acar

Structure of the course and of the exam

Berkan
Acar

All surnames
from
A to D

Structure of the course and of the exam

Berkan
Acar

All surnames
from
A to D

Alessio
Fiorentini

Structure of the course and of the exam

Berkan
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All surnames
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Alessio
Fiorentini

All surnames
from
E to M

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Fernando
Loaiza Erazo

Structure of the course and of the exam

Berkan
Acar

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from
A to D

Alessio
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All surnames
from
E to M

Fernando
Loaiza Erazo

All surnames
from
N to Z

Structure of the course and of the exam

Berkan Acar

All surnames from A to D

Alessio Fiorentini

All surnames from E to M

Fernando Loaiza Erazo

All surnames from N to Z

Teaching assistants will help you and provide further guidance.

Structure of the course and of the exam

Berkan
Acar

All surnames
from
A to D

Alessio
Fiorentini

All surnames
from
E to M

Fernando
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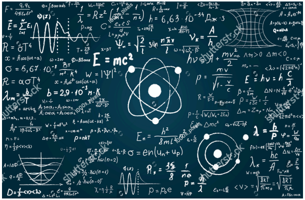
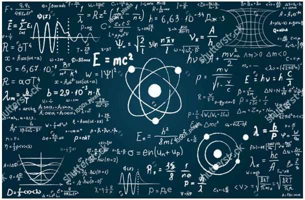
Be respectful with them!

LECTURES



Topics of the program are exposed through slides and the blackboard.

LECTURES



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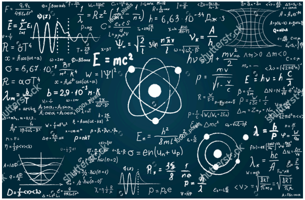
Focus

LECTURES



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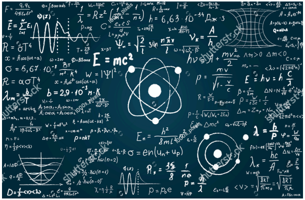
Focus

Takes notes

LECTURES



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Focus

Takes notes

Raise as many questions as you want

PRACTICES

At the beginning of each practice the TA will assign a “Classwork” to each student present.

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You will be assigned a grade form 0 to 3. The average of all the grades will be summed to the mid-term.

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You will be assigned a grade form 0 to 3. The average of all the grades will be summed to the mid-term.

If you are absent…this will be counted as a zero.

Structure of the course and of the exam

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MATHEMATICS
MATHEMATICS I

Davide Erminio Pirino

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A.Y. 2019-2020

Updated A.Y. 2019-2020

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Structure of the course and of the exam

https://economia.uniroma2.it/ba/business-administration-economics/corso/1206/

MATHEMATICS

M1 - I canale

Davide Erminio Pirino

Program | Notice | Teachers | Calendar | Exams | Teaching material | Info

My Courses | Exam rules | examination board

Teaching material

» **Additional material: update of 25 November 2016.**

data inserimento: 2016-11-25 11:27:26

» **Exercises, 5 October 2016**

data inserimento: 2016-10-20 10:16:32

» **Exercizes, 07 November 2016**

data inserimento: 2016-10-28 11:47:00

» **Exercizes, 10 October 2016**

data inserimento: 2016-10-20 10:16:50

» **Exercizes, 14 November 2016**

data inserimento: 2016-11-11 17:12:59

» **Exercizes, 17 October 2016**

data inserimento: 2016-10-20 10:17:02

» **Exercizes, 21 November 2016**

data inserimento: 2016-11-25 11:29:49

Structure of the course and of the exam

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MATHEMATICS

M1 - I canale

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The list of exercises of each practice will be **published the day before**.

Structure of the course and of the exam

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MATHEMATICS

M1 - I canale

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Structure of the course and of the exam

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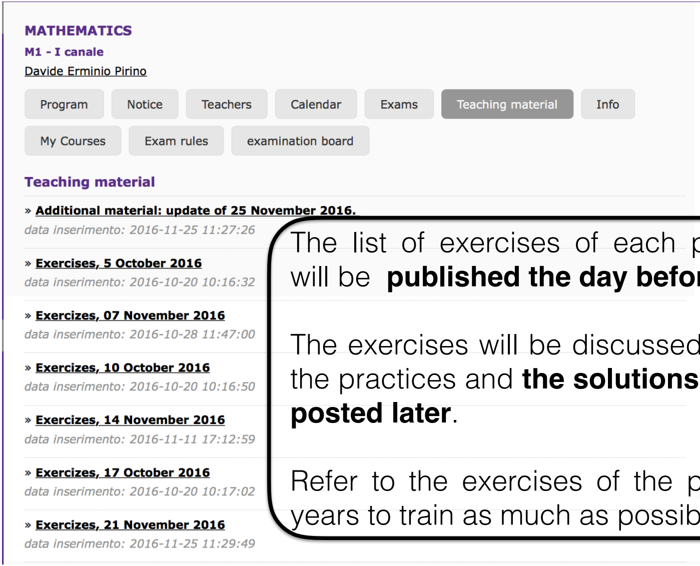

MATHEMATICS

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Refer to the exercises of the previous years to train as much as possible.

Structure of the course and of the exam

https://economia.uniroma2.it/ba/business-administration-economics/corso/1206/

Teaching material

Academic Year 2017-2018

» **Classwork 11 October 2017**

data inserimento: 2017-11-14 14:43:28

» **Classwork 14 November 2017**

data inserimento: 2017-11-17 14:48:33

» **Classwork 15 November 2017**

data inserimento: 2017-11-17 14:48:55

» **Classwork-I 25 October 2017**

data inserimento: 2017-11-06 18:38:10

» **Classwork-II, 25 October 2017**

data inserimento: 2017-11-06 18:38:29

The solutions of the classwork will be published some day after the corresponding practice.

Structure of the course and of the exam

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Academic Year 2017-2018

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Ask your TA to keep trace of your errors and mistakes in the classworks!

Structure of the course and of the exam

https://economia.uniroma2.it/ba/business-administration-economics/corso/1206/

Teaching material

Books for the course

» **Additional Material**

data inserimento: 2018-09-15 1

» **NOTES**

data inserimento: 2018-09-15 1

The main text are my NOTES.

Some additional material is also available.

Structure of the course and of the exam

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Some additional material is also available.

Radicals, algebraic fractions, set theory, intervals of real numbers, functions, equations, inequalities, geometry and trigonometry.

Structure of the course and of the exam

https://economia.uniroma2.it/ba/business-administration-economics/corso/1206/

Teaching material

Books for the course

» **Additional Material**

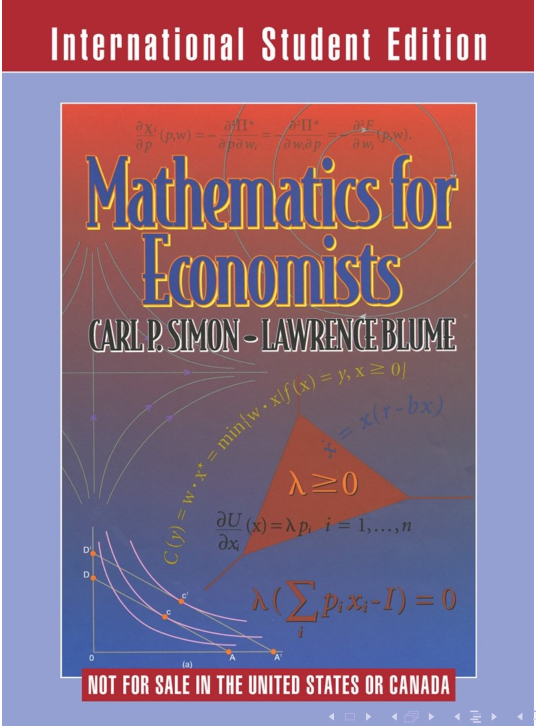
data inserimento: 2018-09-15 19:32:02

» **NOTES**

data inserimento: 2018-09-15 19:31:38

WARNING

SLIDES ARE NOT AVAILABLE!
Pay attention during
each class, ask questions and take notes.


International Student Edition
Mathematics for Economists
CARL P. SIMON – LAWRENCE BLUME
NOT FOR SALE IN THE UNITED STATES OR CANADA

Structure of the course and of the exam

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The mid-term is scheduled for the last week of November (to be defined).

Structure of the course and of the exam

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The mid-term is an examination on all the topics discussed during lectures and practices.

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The **arithmetic mean** of all the class-works is added to the grade of the mid-term.

Structure of the course and of the exam

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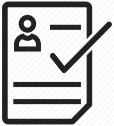
The **arithmetic mean** of all the class-works is added to the grade of the mid-term.



Registration to the mid-term is compulsory!
Registrations will be open from November, 1st, 2019 to five days before the exam date.
Info: Silvia Tabuani.

Structure of the course and of the exam

Mid-term

Teaching material

Previous exams and mid-terms

» **Exam, Part I, September 2018**

data inserimento: 2018-09-13 17:19:28

» **Mid-term of 2017/2018 (February) with solutions**

data inserimento: 2018-02-14 13:56:15

» **Exam, Part I, January 2018**

data inserimento: 2018-01-24 14:20:21

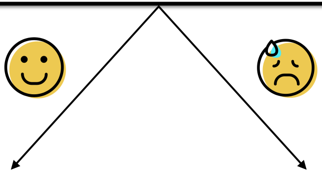
» **Exam, Part I, July 2017**

data inserimento: 2018-01-11 11:34:03

» **Exam, Part I, July 2018**

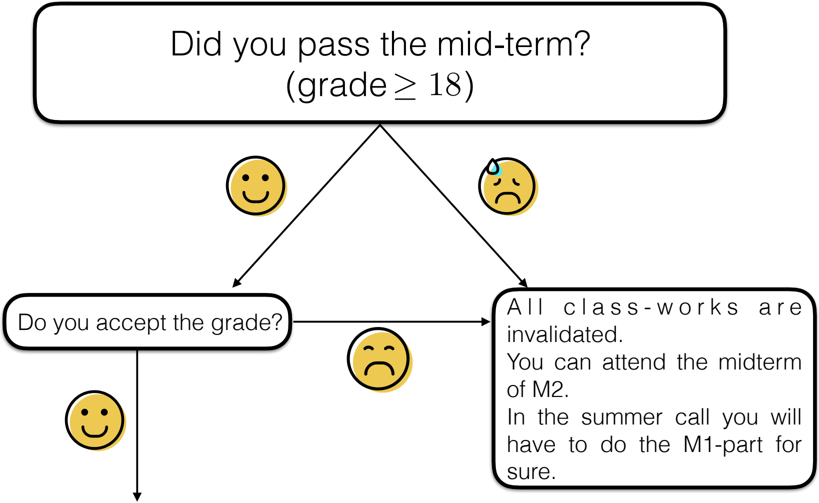
data inserimento: 2018-07-03 19:00:06

Did you pass the mid-term?
(grade ≥ 18)



Did you pass the mid-term?
(grade ≥ 18)

All class-works are invalidated.
You can attend the midterm of M2.
In the summer call you will have to do the M1-part for sure.


Did you pass the mid-term?
(grade ≥ 18)
Do you accept the grade?
All class-works are invalidated.
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Did you pass the mid-term?
(grade ≥ 18)

Do you accept the grade?

All class-works are invalidated.
You can attend the midterm of M2.
In the summer call you will have to do the M1-part for sure.

You must choose **one and only one call of the summer session** and you will have to do only the M2 part.

Should you be successful also in the M2 mid-term, the exam is approved.

Propositional Calculus and Set Theory

Davide Pirino

September 17, 2019

Definition

*A **proposition** is any claim/assertion that can be either TRUE or FALSE.*

We indicate propositions with the calligraphic letter $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, ...

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Example

Definition

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We indicate propositions with the calligraphic letter $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, ...

Example

- $\mathcal{R} =$ *"Rome is a nice city".*

Definition

*A **proposition** is any claim/assertion that can be either TRUE or FALSE.*

We indicate propositions with the calligraphic letter $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, ...

Example

- $\mathcal{R} =$*"Rome is a nice city". It attributes a subjective quality* $\Rightarrow$ *it is not possible to establish CERTAINLY whether it is TRUE or FALSE (for someone will be true for someone else will be false).*

Definition

*A **proposition** is any claim/assertion that can be either TRUE or FALSE.*
We indicate propositions with the calligraphic letter $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, ...

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We indicate propositions with the calligraphic letter $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, ...

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- $\mathcal{P}$ =*"Rome is the capital of France",* $\mathcal{P}$ *is FALSE.*
- $\mathcal{Q}$ =*"Paris is the capital of France",* $\mathcal{Q}$ *is TRUE.*

Definition

Given a proposition $\mathcal{P}$ *we indicate with* $\neg\mathcal{P}$ *a new proposition which is FALSE if* $\mathcal{P}$ *is TRUE and vice versa. The proposition* $\neg\mathcal{P}$ *is called the negation of* $\mathcal{P}$*.*

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$\mathcal{P}$	$\neg\mathcal{P}$
T	

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$\mathcal{P}$	$\neg\mathcal{P}$
T	F
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$$\left\{ \right.$$

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$\mathcal{P}$	$\neg\mathcal{P}$
T	F
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Example

$$\begin{cases} \mathcal{P} = \text{“Rome is the capital of France”} \\ \neg\mathcal{P} = \end{cases}$$

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$\mathcal{P}$	$\neg\mathcal{P}$
T	F
F	T

Example

$$\begin{cases} \mathcal{P} = \text{“}\textit{Rome is the capital of France}\text{”} \\ \neg\mathcal{P} = \text{“}\textit{Rome is not the capital of France}\text{”} \end{cases}$$

Definition

Given two propositions $\mathcal{P}$ and $\mathcal{Q}$, we define a third proposition denoted with

$$\mathcal{P} \wedge \mathcal{Q},$$

which is read "$\mathcal{P}$ and $\mathcal{Q}$" and that is TRUE if and only if both $\mathcal{P}$ and $\mathcal{Q}$ are TRUE, and it is FALSE in all the other cases.

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \wedge \mathcal{Q}$
T	T	

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T	T	T
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T	T	T
T	F	F
F	T	F
F	F	F

Example

$$\begin{cases} \mathcal{P} = \text{“Rome is the capital of France”} \\ \mathcal{Q} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{P} \wedge \mathcal{Q} \ \textit{is} \qquad .$$

Example

$$\begin{cases} \mathcal{R} = \text{“Rome is the capital of Italy”} \\ \mathcal{S} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{R} \wedge \mathcal{S} \ \textit{is} \qquad .$$

Example

$$\begin{cases} \mathcal{P} = \text{“Rome is the capital of France”} \\ \mathcal{Q} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{P} \wedge \mathcal{Q} \textit{ is FALSE.}$$

Example

$$\begin{cases} \mathcal{R} = \text{“Rome is the capital of Italy”} \\ \mathcal{S} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{R} \wedge \mathcal{S} \textit{ is } \qquad .$$

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$\mathcal{P} \wedge \mathcal{Q}$ *is FALSE.*

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$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \vee \mathcal{Q}$
T	T	

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T	T	T
T	F	

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T	T	T
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T	T	T
T	F	T
F	T	T
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$\mathcal{P} \vee \mathcal{Q}$ *is TRUE.*

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$\mathcal{R} \vee \mathcal{S}$ *is* .

Example

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Propositional Calculus: some identities

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \wedge \mathcal{Q}$	$\neg(\mathcal{P} \wedge \mathcal{Q})$
T	T	T	F

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T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \vee \neg\mathcal{Q}$
T	T	F	F	F

Propositional Calculus: some identities

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \wedge \mathcal{Q}$	$\neg(\mathcal{P} \wedge \mathcal{Q})$
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F	T	F	T
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$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \vee \neg\mathcal{Q}$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T

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T	F	F	T	T
F	T	T	F	T
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Propositional Calculus: some identities

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T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \vee \neg\mathcal{Q}$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

That is $\neg(\mathcal{P} \wedge \mathcal{Q}) = \neg\mathcal{P} \vee \neg\mathcal{Q}$

Propositional Calculus: some identities

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \vee \mathcal{Q}$	$\neg(\mathcal{P} \vee \mathcal{Q})$
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T	F	T	F
F	T	T	F
F	F	F	T

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \wedge \neg\mathcal{Q}$
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$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \vee \mathcal{Q}$	$\neg(\mathcal{P} \vee \mathcal{Q})$
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F	F	F	T

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \wedge \neg\mathcal{Q}$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F

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T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \wedge \neg\mathcal{Q}$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Propositional Calculus: some identities

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \vee \mathcal{Q}$	$\neg(\mathcal{P} \vee \mathcal{Q})$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{P} \wedge \neg\mathcal{Q}$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

That is $\neg(\mathcal{P} \vee \mathcal{Q}) = \neg\mathcal{P} \wedge \neg\mathcal{Q}$

Definition

Given two propositions $\mathcal{P}$ and $\mathcal{Q}$, we define a third proposition denoted with

$$\mathcal{P} \Rightarrow \mathcal{Q},$$

read as "If $\mathcal{P}$ then $\mathcal{Q}$" or, also,"$\mathcal{P}$ implies $\mathcal{Q}$" and such that

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T

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$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F

Propositional Calculus: logical operators

Definition

Given two propositions $\mathcal{P}$ and $\mathcal{Q}$, we define a third proposition denoted with

$$\mathcal{P} \Rightarrow \mathcal{Q},$$

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$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T

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Given two propositions $\mathcal{P}$ and $\mathcal{Q}$, we define a third proposition denoted with

$$\mathcal{P} \Rightarrow \mathcal{Q},$$

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$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
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$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

If we assume a TRUE hypothesis the implication is TRUE if and only if the thesis is also TRUE, while from a FALSE hypothesis we can derive anything (both a TRUE and a FALSE thesis).

Example

$$\begin{cases} \mathcal{P} = \text{“}Rome\ is\ the\ capital\ of\ France\text{”} \\ \mathcal{Q} = \text{“}Paris\ is\ the\ capital\ of\ France\text{”} \end{cases} \qquad \mathcal{P} \Rightarrow \mathcal{Q}\ is \qquad .$$

Example

$$\begin{cases} \mathcal{R} = \text{“}Rome\ is\ the\ capital\ of\ France\text{”} \\ \mathcal{S} = \text{“}Paris\ is\ the\ capital\ of\ Italy\text{”} \end{cases} \qquad \mathcal{R} \Rightarrow \mathcal{S}\ is \qquad .$$

Example

$$\begin{cases} \mathcal{T} = \text{“}Rome\ is\ the\ capital\ of\ Italy\text{”} \\ \mathcal{U} = \text{“}Paris\ is\ the\ capital\ of\ Germany\text{”} \end{cases} \qquad \mathcal{T} \Rightarrow \mathcal{U}\ is \qquad .$$

Example

$$\begin{cases} \mathcal{P} = \text{“Rome is the capital of France”} \\ \mathcal{Q} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{P} \Rightarrow \mathcal{Q} \text{ is TRUE.}$$

Example

$$\begin{cases} \mathcal{R} = \text{“Rome is the capital of France”} \\ \mathcal{S} = \text{“Paris is the capital of Italy”} \end{cases} \qquad \mathcal{R} \Rightarrow \mathcal{S} \text{ is } \qquad .$$

Example

$$\begin{cases} \mathcal{T} = \text{“Rome is the capital of Italy”} \\ \mathcal{U} = \text{“Paris is the capital of Germany”} \end{cases} \qquad \mathcal{T} \Rightarrow \mathcal{U} \text{ is } \qquad .$$

Example

$$\begin{cases} \mathcal{P} = \text{“Rome is the capital of France”} \\ \mathcal{Q} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{P} \Rightarrow \mathcal{Q} \textit{ is TRUE.}$$

Example

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Example

$$\begin{cases} \mathcal{T} = \text{“Rome is the capital of Italy”} \\ \mathcal{U} = \text{“Paris is the capital of Germany”} \end{cases} \qquad \mathcal{T} \Rightarrow \mathcal{U} \textit{ is} \qquad .$$

Example

$$\begin{cases} \mathcal{P} = \text{“Rome is the capital of France”} \\ \mathcal{Q} = \text{“Paris is the capital of France”} \end{cases} \qquad \mathcal{P} \Rightarrow \mathcal{Q} \text{ is TRUE.}$$

Example

$$\begin{cases} \mathcal{R} = \text{“Rome is the capital of France”} \\ \mathcal{S} = \text{“Paris is the capital of Italy”} \end{cases} \qquad \mathcal{R} \Rightarrow \mathcal{S} \text{ is TRUE.}$$

Example

$$\begin{cases} \mathcal{T} = \text{“Rome is the capital of Italy”} \\ \mathcal{U} = \text{“Paris is the capital of Germany”} \end{cases} \qquad \mathcal{T} \Rightarrow \mathcal{U} \text{ is FALSE.}$$

Starting from

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

compute the table of truth of $\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$

Starting from

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

compute the table of truth of $\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$
T	T	F	F	T

Starting from

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

compute the table of truth of $\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$
T	T	F	F	T
T	F	F	T	F

Starting from

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

compute the table of truth of $\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T

Starting from

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

compute the table of truth of $\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Propositional Calculus: logical operators

Starting from

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \Rightarrow \mathcal{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

compute the table of truth of $\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$

$\mathcal{P}$	$\mathcal{Q}$	$\neg\mathcal{P}$	$\neg\mathcal{Q}$	$\neg\mathcal{Q} \Rightarrow \neg\mathcal{P}$
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that is $(\mathcal{P} \Rightarrow \mathcal{Q}) = (\neg\mathcal{Q} \Rightarrow \neg\mathcal{P})$.

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Proving a theorem: direct proof.

Direct proof is typically used for $\mathcal{H} \Rightarrow \mathcal{T}$.

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Proof. Since m is even, then $m = 2\,k$ for some k. Since n is even, then $n = 2\,q$ for some q. Then

$$m + n = 2\,k + 2\,q = 2\,(k + q) = 2\,h$$

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with $h = k + q$, so also $n + m$ is even, whence the thesis. □

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(the proof continues in the next slide).

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Example

We will prove that....

$$\underbrace{\text{If } f(x) \text{ is differentiable in } x_0}_{\mathcal{A}} \Rightarrow \underbrace{f(x) \text{ is continuous in } x_0 \quad (\bigstar)}_{\mathcal{B}}.$$

Necessary and sufficient conditions.

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Remark. The $\in$ symbol is frequently used in the intensive notation.

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- If $A = \{\text{All numbers between zero and one}\}$ then $\frac{1}{2} \in A$.

Remark. The $\in$ symbol is frequently used in the intensive notation.

$$A = \{\text{All cities of Europe}\}, \quad B = \{x \in A \mid x \text{ is a capital city}\}.$$

The $\in$ symbol

Write $a \in A$ to say that a is an element of A and $a \notin A$ to say the opposite.

- If $A = \{\triangle, \bigcirc, \bigstar\}$ then $\triangle \in A$, $\bigcirc \in A$ and $\bigstar \in A$.
- If $A = \{-20, 5, 13, 56.7\}$ then $-20 \in A$, $5 \in A$ and so on...
- If $A = \{\text{All cities of Europe}\}$ then $\text{Paris} \in A$
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Remark. The $\in$ symbol is frequently used in the intensive notation.

$$A = \{\text{All cities of Europe}\}, \quad B = \{x \in A \mid x \text{ is a capital city}\}.$$

So that $\text{Milan} \in A$ but $\text{Milan} \notin B$.

The $\forall$ symbol

Write $\forall a \in A$

The ∀ symbol

Write $\forall a \in A$ as a shortcut to declare a property that holds

"for **ALL** the elements of the set A".

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Examples:

- If $A = \{0,$

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Examples:

- If $A = \{0, 0.2, 0.5$

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Examples:

- If $A = \{0, 0.2, 0.5, 0.6$

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- If $A = \{0, 0.2, 0.5, 0.6, 0.7, 0.9, 1\}$ then we can say that $\forall a \in A$ it holds that $a \geq 0$.

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- If $A = \{0, 0.2, 0.5, 0.6, 0.7, 0.9, 1\}$ then we can say that $\nexists a \in A$ such that $a < 0$.

Sets: the $\exists!$ symbol

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Sets: a summary of the quantifiers

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- $\nexists a \in A$ reads as "it does not exist an a in A".
- $\exists! a \in A$ reads as "there exists one and only one a in A".

Negation of the logical symbol

Statement	Negation
$\forall x \in A \Rightarrow \mathcal{P}(x)$	

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Example

$$\neg\left(\forall x \in A \Rightarrow x^2 < 1\right) \quad =$$

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$$\neg\left(\forall x \in A \Rightarrow x^2 < 1\right) \quad = \quad \left(\exists x \in A : x^2 \geq 1\right)$$

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\end{aligned}$$

The $\cup$ and $\cap$ symbols

Let A and B be two sets

The $\cup$ and $\cap$ symbols

Let A and B be two sets

- The set $A \cup B$ is the set that contains all the elements of A and all the elements of B

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\} = \{x \mid x \in A \vee x \in B\}$$

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$$A \cup B = \{x \mid x \in A \text{ or } x \in B\} = \{x \mid x \in A \vee x \in B\}$$

- The set $A \cap B$ is the set that contains all the elements in common between A and B

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\} = \{x \mid x \in A \wedge x \in B\}$$