Set Theory (continued), $\mathbb{N}$, $\mathbb{Q}$, max, min, sup and inf

Davide Pirino

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Sets: a summary of the quantifiers

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- $\exists! a \in A$ reads as "there exists one and only one a in A".

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The $\cup$ and $\cap$ symbols: some examples.

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The symbol $\emptyset$ indicates the set WITHOUT elements, also said the empty set.

Remark. For ANY set E

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- We say that $A = B$ if and only if
$$A \subseteq B \wedge B \subseteq A.$$

Example.

$$A = \{\text{All cities of Europe}\}\,, \quad B = \{x \in A \mid x \text{ is a capital city}\}\,.$$

then $B \subset A$.

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The complement

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then $B^c = \{x \in A \mid x \text{ is not a capital city}\}$.

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$$A \times B = \{(\triangle, \triangle), (\triangle, \diamond), (\bigcirc, \triangle), (\bigcirc, \diamond), (\bigstar, \triangle), (\bigstar, \diamond)\}$$

and

$$B \times A = \{(\triangle, \triangle), (\triangle, \bigcirc), (\triangle, \bigstar), (\diamond, \triangle), (\diamond, \bigcirc), (\diamond, \bigstar)\}$$

so typically $A \times B \neq B \times A$.

Definition

For any set A the power set is the set denoted with $\mathcal{P}(A)$ and it is defined as the set of all possible subsets of A, that is

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Examples.

$$A = \{\triangle, \bigcirc, \bigstar\} \Rightarrow \mathrm{Card}\,(A) = 3$$

Since

$$\mathcal{P}\,(A) = \{\emptyset, \{\triangle, \bigcirc, \bigstar\}, \{\triangle, \bigcirc\}, \{\triangle, \bigstar\}, \{\bigcirc, \bigstar\}, \{\triangle\}, \{\bigcirc\}, \{\bigstar\}\}$$

then $\mathrm{Card}\,(\mathcal{P}\,(A)) = 8 = 2^3$.

More generally if $\mathrm{Card}\,(A) = n$ then $\mathrm{Card}\,(\mathcal{P}\,(A)) = 2^n$.

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$$\mathrm{Card}\,(A) = +\infty.$$

Theorem

Given A, B and C sets, the following properties hold trivially

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$$\frac{n}{m}\cdot\frac{k}{q}=\frac{n\cdot k}{m\cdot q}. \quad \text{Example:} \quad \frac{1}{3}\cdot\frac{7}{4}=\frac{1\cdot 7}{3\cdot 4}=\frac{25}{12}.$$

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$$q = k_n \cdot 10^n + k_{n-1} \cdot 10^{n-1} + k_{n-2} \cdot 10^{n-2} + \dots + k_0 \cdot 10^0 + d_1 \cdot 10^{-1} + d_2 \cdot 10^{-2} + \dots$$

Example

$$\begin{aligned}
\frac{3}{10} &= 0,3 = 0 \cdot 10^0 + 3 \cdot 10^{-1} \\
\frac{173}{2} &= 86,5 = 8 \cdot 10^1 + 6 \cdot 10^0 + 5 \cdot 10^{-1} \\
\frac{1}{3} &= 0,3333333 \dots =
\end{aligned}$$

Decimal representation of $\mathbb{Q}$

Definition

For every $q \in \mathbb{Q}$ we use the notation

$$q = k_n k_{n-1} \ldots k_0, d_1 d_2 \ldots$$

to indicate the decimal representation of q, that is

$$q = k_n \cdot 10^n + k_{n-1} \cdot 10^{n-1} + k_{n-2} \cdot 10^{n-2} + \ldots + k_0 \cdot 10^0 + d_1 \cdot 10^{-1} + d_2 \cdot 10^{-2} + \ldots$$

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Decimal representation of $\mathbb{Q}$

Number	Decimal Representation	Length of the period
$\frac{9}{11}$	$0,8181818181\ldots = 0,\overline{81}$	2
$\frac{1}{7}$	$0,14285714285714\ldots = 0,\overline{142857}$	6
$\frac{1}{81}$	$0,01234567901234679\ldots = 0,\overline{012345679}$	9
$\frac{1}{29}$	$0,\overline{0344827586206896551724137931}$	28