

Mathematics I A

First Practice

1. Determine the truth values of \mathcal{P} , \mathcal{Q} , $\mathcal{P} \wedge \mathcal{Q}$ and $\mathcal{P} \vee \mathcal{Q}$ for each of the following pairs of logical propositions:

$$\begin{array}{l} a) \quad \mathcal{P} : \text{A whale is an animal.} \\ \quad \mathcal{Q} : \text{A whale is a plant.} \end{array}$$

$$\begin{array}{l} b) \quad \mathcal{P} : \text{Dublin is the capital of Ireland.} \\ \quad \mathcal{Q} : 2 > 3. \end{array}$$

$$\begin{array}{l} c) \quad \mathcal{P} : 3 \text{ is an even number.} \\ \quad \mathcal{Q} : 4 \in (-\infty, 5) \cup [6, +\infty]. \end{array}$$

2. Determine the truth values of \mathcal{P} , $\neg \mathcal{Q}$, $\mathcal{P} \wedge (\neg \mathcal{Q})$ and $\mathcal{P} \vee (\neg \mathcal{Q})$ for each of the following pairs of propositions:

$$\begin{array}{l} a) \quad \mathcal{P} : 3 \text{ is a rational number.} \\ \quad \mathcal{Q} : \sqrt{2} \text{ is a rational number.} \end{array}$$

$$\begin{array}{l} b) \quad \mathcal{P} : 0 \notin (-3, 0) \cap [0, 3]. \\ \quad \mathcal{Q} : (-\infty, 2] \cap [1, +\infty] = [1, 2]. \end{array}$$

3. Determine if the following subsets of real numbers are open, closed or neither; for each of them describe the set of interior, exterior, and boundary points:

$$A := [0, 1]; \quad B := [-1, 1]; \quad C := (-1, 1);$$

$$D := [-1, 2); \quad E := (-\infty, 1]; \quad F := [-1, 3] \cup [4, 5];$$

4. For each of the following subsets in \mathbb{R} find the infimum (*inf*) and the supremum (*sup*); describe the minimum (*min*) and the maximum (*max*) too, if they exist:

$$A := [1, 2]; \quad B := [1, 2); \quad C := (0, +\infty);$$

$$D := \left\{ \frac{1}{n} \mid n \in \mathbb{N} \right\}; \quad E := \left\{ \frac{(-1)^n}{n} \mid n \in \mathbb{N} \right\}; \quad F := \{x \in \mathbb{R} \mid x^2 < 9\};$$

5. Given the following subsets of \mathbb{R} :

$$A := (-\infty, 1) \quad B := (-\infty, 1]$$

$$C := (1, +\infty) \quad D := [1, +\infty)$$

$$E := [0, 3] \quad F := (-6, 6)$$

compute the set operations:

$$\begin{array}{llll} a) & A \cup B; & b) & A \cap B; & c) & B \cup F; & d) & B \cup C; \\ e) & A \cup C; & f) & B \setminus A; & g) & \mathbb{R} \setminus A; & h) & D \cap E; \end{array}$$