# MATHEMATICS I-Part A

## September, 25, 2019



## Classwork #1

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1) *(1 point)* Complete the following table of truth.

| $\mathcal{F}$ | $\mathcal{G}$ | $\mathcal{F} \vee \mathcal{G}$ | $\mathcal{F} \wedge \mathcal{G}$ | $\neg(\mathcal{F} \wedge \mathcal{G})$ | $(\mathcal{F} \vee \mathcal{G}) \wedge \neg(\mathcal{F} \wedge \mathcal{G})$ |
|---|---|---|---|---|---|
| T | T | T | T | F | F |
| T | F | T | F | T | T |
| F | T | T | F | T | T |
| F | F | F | F | T | F |

2) *(1 point)* Given the set $A = \{1, 2\}$, find its power set $\mathcal{P}(A)$. Which is the cardinality of $\mathcal{P}(A)$ ? **Motivate your answers.**

The power set of $A$ is the set whose elements are all the subsets of $A$. Whence

$$\mathcal{P}(A) = \{\emptyset, \{1, 2\}, \{1\}, \{2\}\}$$

and so the cardinality is

$$\mathrm{card}(\mathcal{P}(A)) = 4 = 2^{\mathrm{card}(A)}.$$

3) *(1 point)* Consider the set $E = \left\{\frac{1}{n^2} \;\middle|\; n \in \mathbb{N} \wedge n > 0\right\}$. Compute, provided that they exist, max and min. Compute infimum and supremum. **Motivate your answers.**

Since $1 \in E$ and for all $q \in E$ it holds that $q \leq 1$ then 1 is the maximum element of $E$. We know that when the maximum exists it coincides with the supremum. Hence

$$\sup(E) = 1.$$

The set has no minimum because, no matter how close to zero is an element $q$ of $E$, there will be another element $u \in E$ which is smaller than $q$. To find out the infimum, we note that the set of lower bounds is

$$L_E = (-\infty, 0]$$

and since $\max(L_E) = 0$ we have, by definition of infimum,

$$\inf(E) = 0.$$