Mathematics 1 A Fall 2019
Second Practice

1. Consider the function equation $f(x) = x^2 - 2x - 1$.

   (a) Determine the maximal domain for this equation, that is, the maximal subset $D \subseteq \mathbb{R}$ such that

   $$f : D \to \mathbb{R}, \quad x \mapsto f(x) = x^2 - 2x - 1$$

   is a well-defined function.

   (b) Determine the image $I_f = f(D)$ of f.

   (c) Determine whether f is injective, surjective, bijective and/or invertible.

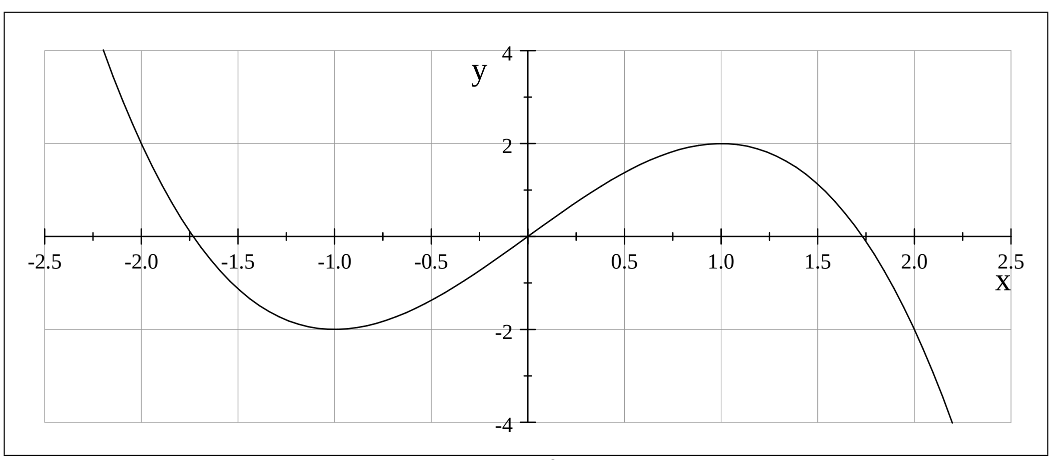
   (d) Let $A = \{0, 11, 101\}$. Determine the image $I_{f|A} = f(A)$ of the restriction $f|_A : A \to \mathbb{R}, \quad x \mapsto f(x) = x^2 - 2x - 1$.

   (e) If f is not injective, determine a suitable subset $E \subseteq D$ such that the *restriction* $f|_E : E \to \mathbb{R}, \quad x \mapsto f(x) = x^2 - 2x - 1$ is injective. Note that then $f|_E : E \to I_{f|E} = f(E)$ is invertible.

   (f) Determine the inverse function $(f|_E)^{(-1)}$ of $f|_E : E \to f(E)$ (including its domain).

2. Solve the problem 1 using the following functions instead of $f(x) = x^2 - 2x - 1$:

   - $f(x) = 2x - 8$
   - $f(x) = \sqrt[4]{x}$
   - $f(x) = \frac{1}{x^2 - 2x + 1}$
   - $f(x) = x^2 - 1$
   - $f(x) = -x^3 + 3x$ (you can use the graph and leave out point f)



$f(x) = -x^3 + 3x$

3. Line L_1 passes from the points A=(1,3) and B=(3,7), line L_2 passes from the points C=(2,1) and D=(-2,3).

   (a) Find the ratio of the distances:

   $$\frac{d(A,B)}{d(C,D)} = ?$$

   (b) Are L_1 and L_2 parallel?

   (c) Are L_1 and L_2 perpendicular?

   (d) Do they intersect? If yes, what is their intersection point

4. For the functions $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$, find the equations $fof(x), gog(x), gof(x)$,and $fog(x)$, and determine their corresponding domains.

5. Prove that the functions

   $$\begin{aligned} f &: \mathbb{R}\setminus\{2\} \to \mathbb{R}\setminus\{0\}, \; x \to f(x) = \frac{1}{x-2} \\ g &: \mathbb{R}\setminus\{0\} \to \mathbb{R}\setminus\{2\}, \; x \to g(x) = \frac{1}{x} + 2 \end{aligned}$$

   are each other's inverses.

   Note: This shows that f and g are invertible, hence we also know both are bijective. Finding an inverse function $[g(x) = f^{-1}(x)]$ and proving $fog(x) = gof(x) = x$ $= \mathcal{I}(x)$ (an identitiy function) is an alternative way of showing bijectivity.

6. Given the set $A = \{0, 1, 2\}$, find the power set $\mathcal{P}(A)$, the cardinality of A and $\mathcal{P}(A)$.