

MATHEMATICS - THIRD PRACTICE

10-10-2019

October 11, 2019

1 Logarithm and exponential.

1. Find the domain of

$$f(x) = \log_{10}(x^4 - 4x^2 + 1).$$

Hint: $\sqrt{2 - \sqrt{3}} \approx 0.51 < \sqrt{2 + \sqrt{3}} \approx 1.93$.

2. Find the domain of the function

$$f(x) = \frac{1}{2^x - 1}.$$

3. Find the domain and study the sign of the function

$$f(x) = \log_{\frac{1}{3}}(1 - x^4).$$

4. Solve the inequality

$$\log_7(x) > 2.$$

2 Using the *Principle of Induction*, show that:

$$1 + 3 + 3^2 + \dots + 3^{(n-1)} = \frac{(3^n - 1)}{2} \quad (2.1)$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \quad (2.2)$$

$$1.2 + 2.3 + 3.4 + \dots + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right] \quad (2.3)$$

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+n)} = \frac{2n}{(n+1)} \quad (2.4)$$

3 Compute each of the following limits:

$$\lim_{n \rightarrow \infty} \frac{n^2(2n+1)(3n-2)}{2n^2(5n-8)(n+6)} \quad (3.1)$$

$$\lim_{n \rightarrow \infty} \left(\frac{n^3}{2n^2 - 1} - \frac{n^2}{2n + 1} \right) \quad (3.2)$$

$$\lim_{n \rightarrow \infty} \sqrt{n^2 + n} - n \quad (3.3)$$

$$\lim_{n \rightarrow \infty} \frac{4n^2 + n^6}{1 - 5n^3} \quad (3.4)$$

$$\lim_{n \rightarrow \infty} \frac{2n^4 - n^2 + 8n}{-5n^4 + 7} \quad (3.5)$$