Modeling Monetary Economies

Second Edition

Bruce Champ
Scott Freeman

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The approach of this text for upper-level undergraduates is to teach monetary economics using the classical paradigm of rational agents in a market setting. Too often monetary economics has been taught as a collection of facts about existing institutions for students to memorize. By teaching from first principles instead, the authors aim to instruct students not only in the monetary policies and institutions that exist today in the United States but also in what policies and institutions may or should exist tomorrow and elsewhere. The text builds on a simple, clear monetary model and applies this framework consistently to a wide variety of monetary questions. The authors have added in this second edition new material on speculative attacks on currencies, social security, currency boards, central banking alternatives, the payments system, and the Lucas model of price surprises. Discussions of many topics have been extended, presentations of data greatly expanded, and new exercises added.

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Second Edition

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Preface

WE OFFER THIS book as an undergraduate-level exposition of lessons about monetary economics gleaned from overlapping generations models of money. Assembling recent advances in monetary theory for the instruction of undergraduates is not a quixotic goal; these models are well within the reach of undergraduate students at the intermediate and advanced levels. These elegantly simple models strengthen our fundamental understanding of the most basic questions in monetary economics: How does money promote exchange? What should serve as money? What causes inflation? What is the cost of inflation?

This approach to teaching monetary economics follows the profession's general recoginition of the need to build the microeconomic foundations of monetary and other macroeconomic topics – that is, to explain aggregate economic phenomena as the implications of the choices of rational individuals who seek to improve their welfare within their limited means. The use of microeconomic foundations makes macroeconomics easier to understand because the performance of abstract economic processes such as gross national product and inflation is linked to something intuitively understood by all – rational individual behavior. It also brings powerful microeconomic tools familiar to undergraduates, such as indifference curves and budget lines, to bear on the questions of interest. Finally, the joining of micro- and macroeconomics introduces an element of consistency across undergraduate studies. Certainly, students will be puzzled if taught that people are rational and prices clear markets when studied by microeconomists but not when studied by macroeconomists.

Inertia and tradition, however, have mired the teaching of monetary economics in a swamp of institutional details, as if monetary economics were only an unchanging set of facts to be memorized. The rapid pace of change in the financial world belies this view. Undergraduates need a way to analyze a wide variety of monetary events and institutional arrangements because the events and institutions of the future will not be the same as those that the students studied in the classroom. The teaching of analysis, the heart of a liberal education, is best accomplished by having the students analyze clear, explicit, and internally consistent models. In this way, the students may uncover the links between the assumptions underlying the models and the performance of these model economies and thus may apply their lessons to new events or changes in government priorities and policies.

This book implements our goals by starting with the simplest model of money – the basic overlapping generations model – which we analyze for its insights into the most basic questions of monetary economics, including the puzzling demand for useless pieces of paper and the costs of inflation. Of course, such a simple model will not be able to discuss all issues of monetary economics. Therefore, we proceed in successive chapters by asking which features of actual economies the simple model does not address. We then introduce these neglected features into the model to enable us to discuss these more advanced topics. We believe this gradual approach allows us to build, step by step, an integrated model of the monetary economy without overwhelming the students.

The book is organized into three parts of increasing complexity. Part I examines money in isolation. Here we take up the questions of the demand for fiat money, a comparison of fiat and commodity monies, inflation, and exchange rates. In Part II, we add capital, to study money's interaction with other assets, banking, the intermediation of these assets into substitutes for fiat money, and alternative arrangements of central banking. In Part III, we look at the effects of money on saving, investment, and output through its effect on nonmonetary government debt.

This book is written for undergraduate students. Its mathematical requirements are no more advanced than the understanding of basic graphs and algebra; calculus is not required. (Those who want to use calculus can find a basic exposition of this approach in the appendix to Chapter 1.) Although we hope the book may also prove useful to graduate students as a primer in monetary theory, the main text is pitched at the undergraduate level. This pitch has held us back from a few demanding topics, such as nonstationary equilibria, but we hope the reader will be satisfied by the large number of topics we present in simple, clear models within a single basic framework. Material that is difficult but within the grasp of advanced undergraduates is set apart in appendices and thus easily skipped or inserted. The appendices also contain extensions, like the model of credit, that many instructors may want to use but that are not essential to the exposition of the main topics.

The references display the most tension between the needs of undergraduates and the technical base in which this approach originated. Whenever possible we reference related material written for undergraduates or general audiences; these references are marked by asterisks. We also reference the works from which our models and data have been drawn. Finally, where undergraduate-level references were not available, we have inserted references to a few academic articles and surveys to offer graduate and advanced undergraduate students some places to start with more advanced work. These are not intended to be a full survey of the advanced literature.

The choice of topics to be covered was also difficult. We make no claim to encyclopedic coverage of every topic or opinion related to monetary economics. We limited coverage to the topics most directly linked to money, covering banking (but not finance in general) and government debt (but not macroeconomics in general). We insisted on ideas consistent with fully rational people operating in explicitly specified environments to promote the unity and consistency of our approach across topics. We also selected topics tractably teachable in the basic framework of the overlapping generations model. Finally, we offer what we best know and understand. We hope individual instructors will build on our foundations to fill any gaps.

To reduce these gaps, in this second edition we have added new material on speculative attacks, the not-very-monetary topic of social security, currency boards, central banking alternatives, the payments system, and the Lucas model of price surprises. We have greatly expanded our presentations of data and have added new exercises.

Many have contributed to the development of this book. We owe Neil Wallace a tremendous intellectual debt for impressing upon us the importance of microeconomic theory in monetary economics. Many others have provided helpful suggestions, criticisms, encouragement, and other assistance during the writing of this book, including David Andolfatto, Leonardo Auernheimer, Robin Bade, Valerie Bencivenga, Mike Bryan, John Bryant, Douglas Dacy, Silverio Foresi, Christian Gilles, Paul Gomme, Joseph Haslag, Dennis Jansen, David Laidler, Kam Liu, Mike Loewy, Helen O'Keefe, John O'Keefe, Michael Parkin, Dan Peled, Tom Sargent, Pierre Siklos, Bruce Smith, Ken Stewart, Dick Tresch, François Velde, Paula Hernandez Verme, Warren Weber, and Steve Williamson. We thank the large number of students at Boston College, the University of California at Santa Barbara, the University of Western Ontario, Fordham University, and the University of Texas at Austin, who persevered through the various preliminary versions of this book. We are grateful for their patience and suggestions. The views stated in this book are those of the authors and not necessarily those of the Federal Reserve Bank of Cleveland or of the Board of Governors of the Federal Reserve System.

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Part I

Money

IN PART I we develop and learn to work with our most basic model of money, applying it to the study of fiat and commodity monies, inflation, and international monetary systems. In studying each of these topics, we will examine how money can facilitate trade and we will ask which form of money or system of exchange does so most efficiently.

Chapter 1 begins with a simple model of money designed to answer the most basic question in monetary economics: Why do people use money? Why do people value pieces of paper that cannot be consumed? With this model, we discover that intrinsically worthless pieces of paper can have value by providing a means by which individuals can acquire goods that they do not possess. Because of this, we also find that the introduction of money into an economy improves welfare. The model of Chapter 1 serves as the foundation on which we later build more complex models.

Chapter 2 considers two alternatives to the paper money of Chapter 1 – barter and commodity money. This chapter first presents a model that demonstrates, in a multiple good model, why barter, the direct trade of goods owned for goods desired, may be more costly than monetary exchange, the trade of goods owned for money and the subsequent trade of money for goods desired. Chapter 2 also demonstrates that commodities may be exchanged as monies. We will ask which commodities make the best monies and we will compare the efficiency of exchange with commodity money to exchange with fiat money.

Chapter 3 uses the basic model of money of Chapter 1 to analyze the effects of the expansion of the money supply on prices, the willingness of people to use money, and the welfare of individuals. We also see how the printing of fiat money can be used to raise revenue for government expenditures. We compare the efficiency of this revenue device with that of taxes.

Chapter 4 examines international monetary systems. We discover some of the important determinants of exchange rates, we discuss floating versus fixed exchange rate regimes and speculative attacks, and we compare alternative international monetary systems.

Chapter 5 looks at some effects of price surprises on labor and output. We introduce the concept of rational expectations and show that a positive correlation between price surprises and output may not imply that the monetary authority is able to increase output in any systematic way. More generally, the chapter is intended to demonstrate the pitfalls of giving policy advice based on purely statistical correlations without understanding the workings of the economy that generated those correlations.

Chapter 1

A Simple Model of Money

Building a Model of Money

IN THIS BOOK we will try to learn about monetary economies through construction of a series of model economies that replicate essential features of actual monetary economies. All such models are simplifications of the complex economic reality in which we live. They may be useful, however, if they are able to illustrate key elements of the behavior of people who choose to hold money and to predict the reactions of important economic variables such as output, prices, government revenue, and public welfare to changes in policies that involve money. We start our analysis with the simplest conceivable model of money. We will learn what we can from this simple model and then ask how the model fails to represent reality adequately. Throughout the book we will try to correct the model's oversights by adding, one by one, the features that it lacks.

To arrive at the simplest possible model of money, we must ask ourselves what features are essential to monetary economies. The demand for money is distinct from the demand for the goods studied elsewhere in economics. People want goods for the utility received from their consumption. In contrast, people do not want money in order to consume it; they want money because money helps them get the things they want to consume. In this way, money is a **medium of exchange** – something acquired to make it easier to trade for the goods whose consumption is desired.

A model of this distinction in the demand for money therefore requires two special features. First, there must be some "friction" to trade that inhibits people from directly acquiring the goods they desire in the absence of money. If people could costlessly trade what they have for what they want, there would be no role for money.

Second, someone must be willing to hold money from one period to the next. This is necessary because money is an asset held over some period of time, however short, before it is spent. Therefore, we will look for models in which there is always someone who will live into the next period.

Two possible frameworks meet this second requirement. People (or households) could live infinite lives or could live finite lives in generations that overlap (so that some but not all people will live into the next period). For many of the topics we study, life span does not matter. We identify where it does matter in Appendix B of Chapter 15, where infinitely lived households are studied in detail.

With the exception of that appendix, we concentrate on the second framework – the **overlapping generations model**. This model, introduced by Paul Samuelson (1958), has been applied to the study of a large number of topics in monetary theory and macroeconomic theory. Among its desirable features are the following:

- Overlapping generations models are highly tractable. Although they can be used to analyze quite complex issues, they are relatively easy to use. Many of their predictions may be described on a simple two-dimensional graph.
- Overlapping generations models provide an elegantly parsimonious framework in which to introduce the existence of money. Money in overlapping generations models dramatically facilitates exchange between people who otherwise would be unable to trade.
- Overlapping generations models are **dynamic**. They demonstrate how behavior in the present can be affected by anticipated future events. They stand in marked contrast to **static** models, which assume that only current events affect behavior.

We begin this chapter with a very simple version of an overlapping generations model. As we proceed through the book, we introduce extensions to this basic model. These extensions allow us to analyze a variety of interesting issues. For now, let us turn to the development of the basic overlapping generations model.

The Environment

In the basic overlapping generations model, individuals live for two periods. We call people in the first period of life **young** and those in the second period of life **old**.

The economy begins in period 1. In each period $t \geq 1$, N_t individuals are born. Note that we index time with a subscript. For example, N_2 is our notation for the number of individuals born in period 2. The individuals born in periods $1, 2, 3, \ldots$ are called the **future generations** of the economy. In addition, in period 1 there are N_0 members of the **initial old**.

Hence, in each period t, there are N_t young individuals and N_{t-1} old individuals alive in the economy. For example, in period 1, there are N_0 initial old individuals and N_1 young individuals who were born at the beginning of period 1.

For simplicity, there is only one good in this economy. The good cannot be stored from one period to the next. In this basic setup, each individual receives an **endowment** of the consumption good in the first period of life. The amount of this

Generation \ Period		1	2	3	4	5	6	7
Initial old	0	0						
Future generations	1	y	0					
	2		y	0				
	3			y	0			
	4				y	0		
	5					y	0	
							...	...

Figure 1.1. The pattern of endowments. In each period t, generation t is born. Each individual lives for two periods. Individuals are endowed with y units of the consumption good when young and 0 units when old. In any given period, one generation of young people and one generation of old people are alive. The name of this model, the overlapping generations model, follows from this generational structure.

endowment is denoted as y. Each individual receives no endowment in the second period of life. This pattern of endowments is illustrated in Figure 1.1.

We can also interpret the endowment as an endowment of labor – the ability to work. By using this labor endowment (by working), the individual is able to obtain a real income of y units of the consumption good.

Preferences

Individuals consume the economy's sole commodity and obtain satisfaction or, in the economist's jargon, utility from having done so.

Future Generations

Members of future generations in an overlapping generations model consume both when young and when old. An individual member's utility therefore depends on the combination of personal consumption when young and when old. We make the following assumptions about an individual's preferences about consumption:

1. For a given amount of consumption in one of the periods, an individual's utility increases with the consumption obtained in the other period.

2. Individuals like to consume some of this good in both periods of life. An individual prefers the consumption of positive amounts of the good in both periods of life over the consumption of any quantity of the good in only one period of life.
3. To receive another unit of consumption tomorrow, an individual is willing to give up more consumption today if the good is currently abundant than if it is scarce relative to consumption tomorrow.

With these assumptions, we are assuming that individuals are capable of ranking combinations (or bundles) of the consumption good over time in order of preference. We denote the amount of the good that is consumed in the first period of life by an individual born in period t with the notation $c_{1,t}$. Similarly, $c_{2,t+1}$ denotes the amount the same individual consumes in the second period of life. It is important to note that $c_{2,t+1}$ is consumption that actually occurs in period $t+1$, when the person born at time t is old. When the time period is not crucial to the discussion, we denote first- and second-period consumption as c_1 and c_2.

Suppose we offer an individual the following consumption choices:

- Bundle A, which consists of 3 units of the consumption good when young and 6 units of the consumption good when old. We denote this bundle as $c_1 = 3$ and $c_2 = 6$.
- Bundle B, which consists of 5 units of the consumption good when young and 4 units of the consumption good when old ($c_1 = 5, c_2 = 4$).

By assuming that an individual can rank these bundles, we are saying that this individual can state a preference for bundle A over bundle B, for bundle B over bundle A, or equal happiness with either bundle. The individual can rank any number of bundles of the consumption good that we might offer in this manner.

It will be extremely useful to portray an individual's preferences graphically. We do this with indifference curves. An **indifference curve** connects all consumption bundles that yield equal utility to the individual. In other words, if offered any two bundles on a given indifference curve, the individual would say, "I do not care which I receive; they are equally satisfying to me." In the preceding example, if the individual were indifferent to bundles A and B, then those two bundles would lie on the same indifference curve. Figure 1.2 displays a typical indifference curve.

On this indifference curve we show the two points A and B from our earlier example. We also illustrate a third point, C, representing the bundle $c_1 = 11$ and $c_2 = 2$. Because C lies on the same indifference curve as points A and B, point C yields the same level of utility as points A and B for the individual. In fact, any point along the illustrated indifference curve represents a bundle that yields the same utility level.

Note some features of the indifference curve. The first feature is that the curve gets flatter as we move from left to right. This is how indifference curves represent assumption 3. This property of indifference curves is called the assumption of **diminishing marginal rate of substitution**. To illustrate this assumption, start

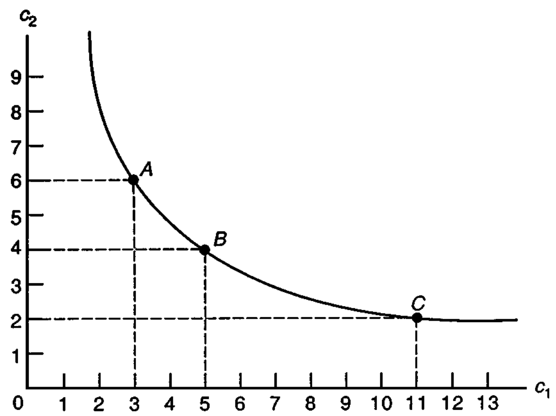


Figure 1.2. An indifference curve. Individual preferences are represented by indifference curves. The figure portrays an indifference curve for a typical individual. Along any particular indifference curve, utility is constant. Here, the individual is indifferent between points A, B, and C.

at point A, where $c_1 = 3$ and $c_2 = 6$. Suppose we reduce the individual's second period consumption by 2 units. The indifference curve tells us that, to keep the individual's utility constant, we must compensate the individual by providing 2 more units of first-period consumption. This places the individual at point B on the indifference curve. Now suppose we reduce second-period consumption by another 2 units. To remain indifferent, 6 more units of first-period consumption must be given to the individual. In other words, we must compensate the individual with ever-increasing amounts of first-period consumption as we successively cut second-period consumption. This should make intuitive sense; individuals are more reluctant to give up something they do not have much of to begin with.

Take food and clothing as an example. A person who has a large amount of clothing and very little food would be willing to give up a fairly large amount of clothing for another unit of food. Conversely, this person would be willing to give up only a small amount of food to obtain another unit of clothing.

This assumption of diminishing marginal rate of substitution is captured by drawing an indifference curve so that it becomes flatter as we move downward and to the right along the curve.

We also assume that the indifference curves become infinitely steep as we approach the vertical axis and perfectly flat as we approach the horizontal axis. The curves never cross either axis. This might be justified by saying that consuming nothing in any one period would mean horrible starvation, to which consuming even a small amount is preferable. This is assumption 2.

It is also important to keep in mind that the indifference curves are dense in the (c_1, c_2) space. This means that if you pick a combination of first- and second-period

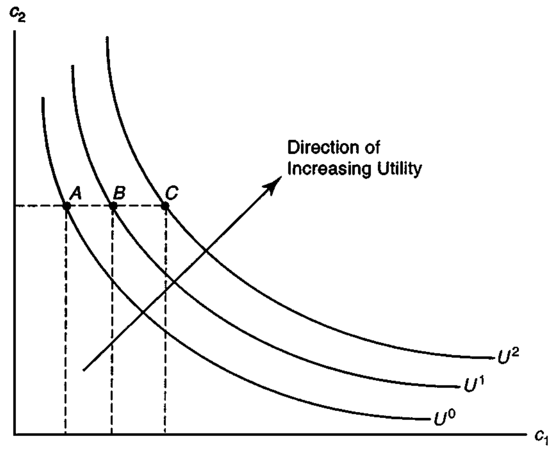


Figure 1.3. An indifference map. An indifference map consists of a collection of indifference curves. For a constant amount of consumption in one period, individuals prefer a greater amount of consumption in the other period. For this reason, individuals prefer point C to point B and point B to point A. Utility increases in the general direction of the arrow.

consumption, there is an indifference curve running through that point. However, to avoid clutter, we normally show only a few of these indifference curves. A group of indifference curves shown on one graph is often called an **indifference map**. Figure 1.3 illustrates an indifference map that obeys our assumptions.

Note that utility is increasing in the direction of the arrow. How do we know this? Compare points A, B, and C. Each of these bundles gives the individual the same amount of second-period consumption. However, moving from point A to B to C, the individual receives more and more first-period consumption. Hence, the individual will prefer point B to point A. Likewise, point C will be preferable to both points A and B. This is assumption 1.

It is often useful to draw an analogy between an indifference map and a contour map that shows elevation. On a contour map, the curves represent points of constant elevation; on an indifference map, the curves represent points of constant utility. Extending the analogy, if we think of traversing the indifference map in a northeasterly direction, we would be going uphill. In other words, utility would be increasing. In fact, an indifference map, like a contour map, is merely a handy way to illustrate a three-dimensional concept on a two-dimensional drawing. The three dimensions here are first-period consumption, second-period consumption, and utility.

One other important concept about our individual's ranking of preferences is that they are transitive. If an individual prefers bundle B to bundle A and bundle C to bundle B, then that individual must also prefer bundle C to bundle A. Graphically, this implies that indifference curves cannot cross. To do so would violate this property of transitivity and assumption 1. To see this, refer to Figure 1.4. In this figure, we have

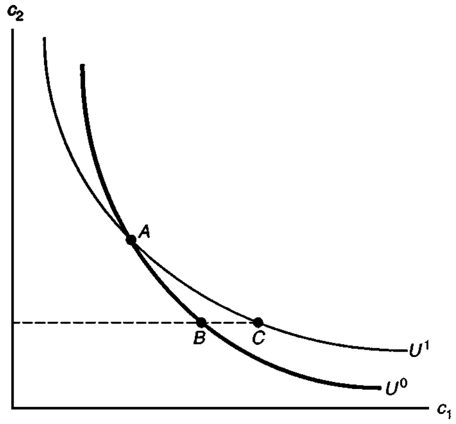


Figure 1.4. Indifference curves cannot cross. By our first assumption about preferences, the individual whose preferences are represented by these indifference curves prefers bundle C over bundle B because bundle C consists of more first-period consumption and the same amount of second-period consumption compared with bundle B. However, because the individual must be indifferent between all three bundles, A, B, and C, a contradiction arises. Our assumptions rule out the possibility of indifference curves that cross.

portrayed two indifference curves that cross at point A. We know that indifference curves represent bundles that give an individual the same level of utility. In other words, the individual whose preferences are represented by Figure 1.4 is indifferent between bundles A and B, because they lie on the same indifference curve U^0. Similarly, the individual must be indifferent between bundles A and C on indifference curve U^1. We see, then, that the individual is indifferent between all three bundles. However, if we compare bundles B and C, we also observe that they consist of the same amount of second-period consumption, but C contains more first-period consumption than B. By assumption 1, the individual must prefer C to B. But this contradicts our earlier statement about indifference between the three bundles. For this reason, indifference curves that cross violate our assumptions about preferences.

The Initial Old

The preferences of the initial old are much easier to describe than those of future generations. The initial old live and consume only in the initial period and thus simply want to maximize their consumption in that period.

The Economic Problem

The problem facing future generations of this economy is very simple. They want to acquire goods they do not have. Each has access to the nonstorable consumption good only when young but wants to consume in both periods of life. They must therefore find a way to acquire consumption in the second period of life and then decide how much they will consume in each period of life.[1]

We will examine, in turn, two solutions to this economic problem. The first, a centralized solution, proposes that an all-knowing, benevolent planner will allocate the economy's resources between consumption by the young and by the old. In the second, decentralized solution, we allow individuals to use money to trade for what they want. We will then compare the two solutions and ask which is more likely to offer individuals the highest utility. The answer will help to provide a first illustration of the economic usefulness of money.

Feasible Allocations

Imagine for a moment that we are central planners with complete knowledge of and total control over the economy. Our job is to allocate the available goods among the young and old people alive in the economy at each point in time.

As central planners, under what constraint would we operate? Put simply, it is that at any given time we cannot allocate more goods than are available in the economy. Recall that only the young people are endowed with the consumption good at time t. There are N_t of these young people at time t. We have

$$(\text{total amount of consumption good})_t = N_t y. \tag{1.1}$$

Suppose that every member of generation t is given the same lifetime allocation $(c_{1,t}, c_{2,t+1})$ of the consumption good (our society's view of equity). In this case, total consumption by the young people in period t is

$$(\text{total young consumption})_t = N_t c_{1,t}. \tag{1.2}$$

Furthermore, total old consumption in period t is

$$(\text{total old consumption})_t = N_{t-1} c_{2,t}. \tag{1.3}$$

Let us make sure the notation is clear. Recall that the old people at time t are those who were born at time $t-1$. There were N_{t-1} of these people born at time $t-1$. Furthermore, recall that $c_{2,t}$ denotes the second period (time t) consumption by someone who was born at time $t-1$. This implies that total consumption by the old at time t must be $N_{t-1} c_{2,t}$.

[1] We could make this model more complex by assuming that there are many types of goods and many periods in which to consume them. A model with many types of goods is introduced in Chapter 2. We will see, however, that this simple model is all that is needed to illustrate a demand for money.

Total consumption by young and old is the sum of the amounts in Equations 1.2 and 1.3. We are now ready to state the constraint facing us as central planners: total consumption by young and old cannot exceed the total amount of available goods (Equation 1.1) . In other words,

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y. \tag{1.4}$$

For simplicity, we assume for now that the population is constant ($N_t = N$ for all t). In this case, we rewrite Equation 1.4 as

$$N c_{1,t} + N c_{2,t} \leq N y.$$

Dividing through by N, we obtain the per capita form of the constraint facing us as central planners:

$$c_{1,t} + c_{2,t} \leq y. \tag{1.5}$$

For now, we are also concerned with a stationary allocation. A **stationary allocation** is one that gives the members of every generation the same lifetime consumption pattern. In other words, in a stationary allocation, $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for every period $t = 1, 2, 3, \ldots$. However, it is important to realize that a stationary allocation does not necessarily imply that $c_1 = c_2$. With a stationary allocation, the per capita constraint becomes

$$c_1 + c_2 \leq y. \tag{1.6}$$

This represents a very simple linear equation in c_1 and c_2, which is illustrated in Figure 1.5.

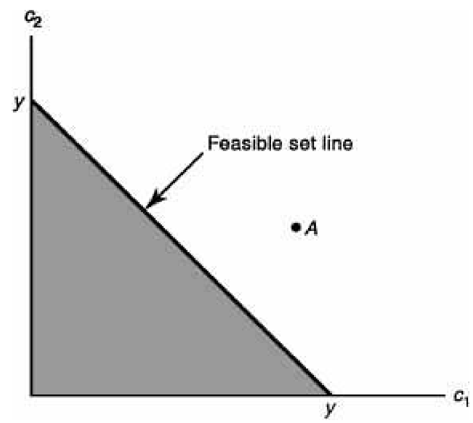


Figure 1.5. The feasible set. The feasible set, the gray triangle, represents the set of possible allocations that can be attained given the resources available in the economy. Points outside the feasible set, such as point A, are unattainable given the resources of the economy.

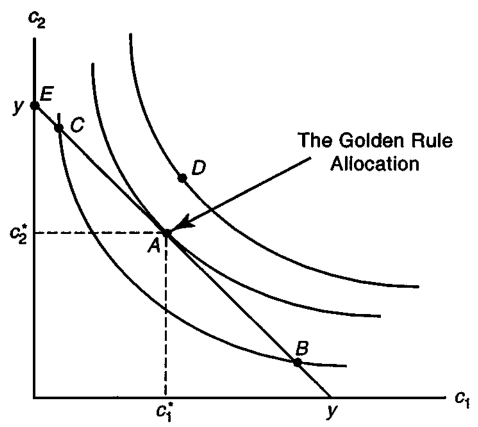


Figure 1.6. The golden rule allocation. The golden rule allocation is the stationary, feasible allocation of consumption that maximizes the welfare of future generations. It is located at a point of tangency between the feasible set line and an indifference curve (point A). This is the highest indifference curve in contact with the feasible set. As drawn, the golden rule allocation A allocates more goods to people when old than when young ($c_2^* > c_1^*$), but this is arbitrary. The tangency can just as easily have been drawn at a point where $c_2^* < c_1^*$.

The set of stationary, feasible, per capita allocations – the feasible set, for short – is bounded by the triangle in the diagram. We refer to the triangular region as the **feasible set**. The thick diagonal line on the boundary of the feasible set is called the **feasible set line**. The feasible set line represents Equation 1.6, evaluated at equality.

The Golden Rule Allocation

If we now superimpose a typical individual's indifference map on this diagram, we can identify the preferences of future generations among feasible stationary allocations. This is done in Figure 1.6.

The feasible allocation that a central planner selects depends on the objective. One reasonable and benevolent objective is the maximization of the utility of the future generations, an objective we call the **golden rule**. The golden rule in Figure 1.6 is represented by point A, which offers each individual the consumption bundle (c_1^*, c_2^*). This combination of c_1 and c_2 yields the highest feasible level of utility over an individual's entire lifetime. Note that the golden rule occurs at the unique point of tangency between the feasible set boundary and an indifference curve. Any other point that lies within the feasible set yields a lower level of utility. For example, points B and C are feasible because they lie on the boundary of the feasible set. However, they lie on an indifference curve that represents a lower level of utility than the one on which point A lies. Point D is preferable to point A, but it is unattainable. The endowments of the economy simply are not large enough to support the allocation implied by point D.

The Initial Old

It is important to consider the welfare of all participants in the economy – including the initial old – when considering the effects of any policy. Although the golden rule allocation maximizes the utility of future generations, it does not maximize the utility of the initial old. To see this, recall that the initial old's utility depends solely and directly on the amount of the good they consume in their second period of life. The goal of the initial old is to get as much consumption as possible in period 1, the only period in which they live. (You may want to imagine that the initial old also lived in period 0; however, because this period is in the past, it cannot be altered by the central planner, who assumes control of the economy in period 1.) If the central planner's goal were to maximize the welfare of the initial old, the planner would want to give as much of the consumption good as possible to the initial old. This would be accomplished among stationary feasible allocations at point E of Figure 1.6, which allocates y units of the good for consumption when old (including consumption by the initial old) and nothing for consumption when young.

This stationary allocation, which implies that people consume nothing when young, would not maximize the utility of the future generations. They prefer the more balanced combination of consumption when young and old represented by (c_1^*, c_2^*). Faced with this conflict in the interests of the initial old and the future generations, an economist cannot choose among them on purely objective grounds. Nevertheless, the reader will find that, on subjective grounds (influenced by the fact that there are an infinite number of future generations and only a single generation of initial old), we tend to pay particular attention to the golden rule in this book.

Decentralized Solutions

In the previous section, we found the feasible allocation that maximizes the utility of the future generations. However, to achieve this allocation, in each period the central planner would have to take away c_2^* from each young person and give this amount to each old person. Such a redistribution requires that the central planner have the ability to reallocate endowments costlessly between the generations. Furthermore, in order to determine c_1^* and c_2^*, this central planner also must know the exact utility function of the subjects.

These are strong assumptions about the power and wisdom of central planners. This leads us to ask if there is some way we can achieve this optimal allocation in a more decentralized manner, one in which the economy reaches the optimal allocation through mutually beneficial trades conducted by the individuals themselves. In other words, can we let a market do the work of the central planner?

Before we answer this question, we need to define some terms that are used throughout the book. First, we discuss the notion of a competitive equilibrium. A

competitive equilibrium has the following properties:

1. Each individual makes mutually beneficial trades with other individuals. Through these trades, the individual attempts to attain the highest level of utility that he can afford.
2. Individuals act as if their actions have no effect on prices (rates of exchange). There is no collusion between individuals to fix total quantities or prices.
3. Supply equals demand in all markets. In other words, markets clear.

Equilibrium Without Money

Let us consider the nature of the competitive equilibrium when there is no money in our economy of overlapping generations. Recall that agents are endowed with some of the consumption good when young. Their endowment is zero when old. Their utility can be increased if they give up some of their endowment when they are young in exchange for some of the goods when they are old. Without the presence of an all-powerful central planner, we must ask ourselves if there are trades between individuals in the economy that could achieve this result.

No such trades are possible. Refer to Figure 1.1, which outlines the pattern of endowments. A young person at period t has two types of people with whom to trade potentially in period t – other young people of the same generation or old people of the previous generation. However, trade with fellow young people would be of no benefit to the young person under consideration. They, like him, have none of the consumption good when they are old. Trade with the old would also be fruitless; the old want the good the young have, but they do not have what the young want (because they will not be alive in the next period). The source of the consumption good at time $t + 1$ is from the people who are born in that period. However, in period t, these people have not yet come into the world and so do not want what young people have to trade. This lack of possible trades is the manner in which the basic overlapping generations model captures the "absence of double coincidence of wants" [a term introduced by the nineteenth century economist W. S. Jevons (1875) to explain the need for money]. Each generation wants what the next generation has but does not have what the next generation wants.[2]

The resulting equilibrium is **autarkic** – individuals have no economic interaction with others. Unable to make mutually beneficial trades, each individual consumes his entire endowment when young and nothing when old. In this autarkic equilibrium, utility is low. Both the future generations and the initial old are worse off than they would be with almost any other feasible consumption bundle. A member of the future generations would gladly give up some of his endowment when young in order to consume something when old. A member of the initial old would also like to consume something when old.

[2] People cannot trade directly in this model because they are separated in time. The same absence of trade would result if they were separated in space, as in the models of Robert Townsend (1980).

Equilibrium with Money

To open up a trading opportunity that might permit an exit from this grim autarkic equilibrium, we now introduce fiat money into our simple economy. **Fiat money** is a nearly costlessly produced commodity that cannot itself be used in consumption or production and is not a promise to anything that can be used in consumption or production.

For the purposes of our model, we assume the government can produce fiat money costlessly but that it cannot be produced or counterfeited by anyone else. Fiat money can be costlessly stored (held) from one period to the next and it is costless to exchange. Pieces of paper distinctively marked by the government generally serve as fiat money.

Because individuals derive no direct utility from holding or consuming money, fiat money is valuable only if it enables individuals to trade for something they want to consume.

A **monetary equilibrium** is a competitive equilibrium in which there is a valued supply of fiat money. By valued, we mean that the fiat money can be traded for some of the consumption good. For fiat money to have value, its supply must be limited and it must be impossible (or very costly) to counterfeit. Obviously, if everyone has the ability to print money costlessly, its supply will rapidly approach infinity, driving the value of any one unit to zero.

We begin our analysis of monetary economies with an economy with a fixed stock of M perfectly divisible units of fiat money. We assume that each of the initial old begins with an equal number, M/N, of these units.[3]

The presence of fiat money opens up a trading possibility. A young person can sell some of his endowment of goods (to old persons) for fiat money, hold the money until the next period, and then trade the fiat money for goods (with the young of that period).

Finding the Demand for Fiat Money

Of course, this new trading possibility exists only if fiat money is valued – in other words, if people are willing to give up some of the consumption good in trade for fiat money and vice versa. Because fiat money is intrinsically useless, its value depends on one's view of its value in the future, when it will be exchanged for the goods that do increase an individual's utility.

If it is believed that fiat money will not be valued in the next period, then fiat money will have no value in this period. No one would be willing to give up some of the consumption good in exchange for it. That would be tantamount to trading something for nothing.

[3] Because the government is the creator of fiat money, we are implicitly assuming that the government has made a gift of the initial money stock to the initial old.

Extending this logic, we can predict that fiat money will have no value today if it is known with complete certainty that fiat money will be valueless at any future date T. To see this, first ask what the value of fiat money will be at time $T - 1$; in other words, ask how many goods you would be willing to give for money at $T - 1$ if it is known that it will be worthless at time T. The answer, of course, is that you would not be willing to give up any goods at time $T - 1$ for money. In other words, fiat money would have no value at time $T - 1$. Then what must its value be at time $T - 2$? By similar reasoning, we see that it will also be valueless at time $T - 2$. Working backward in this manner, we can see that fiat money will have no value today if it will be valueless at some point in the future.

Now let us consider a more interesting equilibrium where money has a positive value in all future periods. We define v_t as the value of 1 unit of fiat money (let us call the unit a dollar) in terms of goods; that is, it is the number of goods that one must give up to obtain one dollar. It is the inverse of the dollar price of the consumption good, which we write as p_t. For example, if a banana costs 20 cents, $p_t = 1/5$ dollars and the value of a dollar, v_t, is five bananas. Note also that because our economy has only one good, the price of that good p_t can be viewed as the price level in this economy.

An Individual's Budget

Let us now examine how individuals will decide how much money to acquire (assuming that fiat money will have a positive value in the future). To answer, we must first establish the constraints on the choices of the individual – why he cannot simply enjoy infinite consumption both when young and when old. As it was for the entire society, the constraints on an individual are that he cannot give up more goods than he has. We will refer to the limitations on an individual's consumption as his **budget constraints**.

In the first period life, an individual has an endowment of y goods. The individual can do two things with these goods – consume them and/or sell them for money. Notice that no one in the future generations is born with fiat money. To acquire fiat money, an individual must trade. If the number of dollars acquired by an individual (by giving up some of the consumption good) at time t is denoted by m_t, then the total number of goods sold for money is $v_t m_t$. We can therefore write the budget constraint facing the individual in the first period of life as

$$c_{1,t} + v_t m_t \leq y. \tag{1.7}$$

The left-hand side of Equation 1.7 is the individual's total uses of goods (consumption and acquisition of money). The right-hand side of Equation 1.7 represents the total sources of goods (the individual's endowment).

In the second period of life, the individual receives no endowment. Hence, when old, an individual can acquire goods for consumption only by spending the money acquired in the previous period. In the second period of life (period $t + 1$), this money will purchase $v_{t+1}m_t$ units of the consumption good. The only use for these goods is second-period consumption. This means that the constraint facing the individual in the second period of life is

$$c_{2,t+1} \leq v_{t+1}m_t. \tag{1.8}$$

In a monetary equilibrium where, by definition, $v_t > 0$ for all t, we can rewrite this constraint as $m_t \geq (c_{2,t+1})/(v_{t+1})$ and substitute it into the first-period constraint (Equation 1.7) to obtain

$$c_{1,t} + \frac{v_t c_{2,t+1}}{v_{t+1}} \leq y, \tag{1.9}$$

or

$$c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq y. \tag{1.10}$$

Equation 1.10 expresses the various combinations of first- and second-period consumption that an individual can afford over a lifetime. In other words, it is the individual's **lifetime budget constraint**.

We can graph this budget constraint as shown in Figure 1.7. We can easily verify that the intercepts of the budget line are as illustrated. The budget line represents

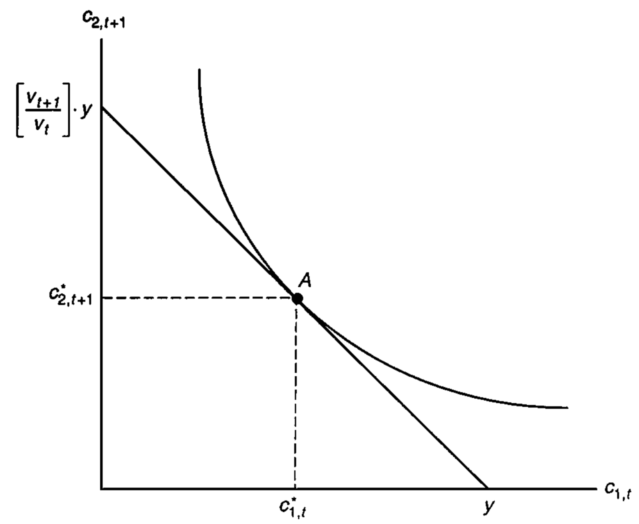


Figure 1.7. The choice of consumption with fiat money. At point A individuals maximize utility given their lifetime budget set in the monetary equilibrium. Point A is found by locating a point of tangency between an indifference curve and the individual's lifetime budget set line. The rate of return on fiat money determines the slope of the budget set line.

Equation 1.10 at equality. If nothing is consumed in the second period of life ($c_{2,t+1} = 0$), then the constraint implies that $c_{1,t} = y$. This is the horizontal intercept of the budget line. On the other hand, if nothing is consumed in the first period of life ($c_{1,t} = 0$), so that the entire endowment of y is used to purchase money, the constraint implies that $[(v_t)/(v_{t+1})]c_{2,t+1} = y$ or $c_{2,t+1} = [(v_{t+1})/(v_t)]y$. This represents the vertical intercept of the budget line.

Note that $(v_{t+1})/(v_t)$ can be considered as the (real) **rate of return of fiat money** because it expresses how many goods can be obtained in period $t + 1$ if one unit of the good is sold for money in period t.[4]

For a given rate of return of money, $(v_{t+1})/(v_t)$, we can find the $(c_1^*, c_{2,t+1}^*)$ combination that will be chosen by individuals who are seeking to maximize their utility. This point is shown in Figure 1.7. It is the point along the budget line that touches the highest indifference curve. This must occur at a point where the budget line is tangent to an indifference curve.

Finding Fiat Money's Rate of Return

But how can we determine the rate of return on intrinsically useless fiat money? The value that individuals place on a unit of fiat money at time t, v_t, depends on what people believe will be the value of one unit of money at $t + 1$, v_{t+1}. By similar logic, the value of a unit of fiat money at time $t + 1$ depends on people's beliefs about the value of money in period $t + 2$, v_{t+2}. And so on. We see that the value of fiat money at any point in time depends on an infinite chain of expectations about its future values. This indefiniteness is not due to any peculiarity in our model but rather to the nature of fiat money, which, because it has no intrinsic value, has a value that is determined by views about the future.

Whatever the views of the future value of money, a reasonable benchmark is the case in which these views are the same for every generation. This is plausible because in our basic model every generation faces the same problem; endowments, preferences, and population are the same for every generation. If views about the future are also the same across generations, then individuals will react in the same manner in each period, choosing $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for each period t. We call such equilibria **stationary equilibria**. Notice that because individuals face different circumstances, depending on whether they are young or old, c_1 will not in general be equal to c_2 in a stationary equilibrium. People may choose to consume more when young or more when old. It turns out that the relative mix of first- and second-period consumption depends on preferences and on the rate of return on fiat money.

[4] Note that v_{t+1}/v_t is the gross rate of return of fiat money. The *net* rate of return of fiat money is equal to $[(v_{t+1})/(v_t)] - 1$.

We also assume that individuals in our economy form their expectations of the future rationally. In this nonrandom economy, where there are no surprises, **rational expectations** means that individuals' expectations of future variables equal the actual values of these future variables. In this special case, we say that people have **perfect foresight**. With perfect foresight, there are no errors in individuals' forecasts of the important economic variables that affect their decisions. In the context of our model, this assumption means that an individual born in period t will perfectly forecast the value of money in the next period, v_{t+1}. The individual's expectation of this value will be exactly realized. This assumption would be less credible in an economy buffeted by random shocks than in our model economy, where preferences and the environment are unchanging and therefore are perfectly predictable.[5]

To see the importance of perfect foresight, consider the alternative in a nonrandom economy – that individuals always expect a value of money greater or less than the value of money that actually occurs. Individuals with wrong beliefs about the future value of money will not choose the money balances that maximize their utility. They therefore have an incentive to figure out the value of money that will actually occur.

Let us now employ the assumptions of stationarity and perfect foresight to find an equilibrium time path of the value of money. In perfectly competitive markets, the price (or value) of an object is determined as the price at which the supply of the object equals its demand. This applies to the determination of the price (value) of money as well as the price of any good.

The demand for fiat money of each individual is the number of goods each chooses to sell for fiat money, which equals the goods of the endowment that the individual does not consume when young, $y - c_{1,t}$. The total money demand by all individuals in the economy at time t is therefore $N_t(y - c_{1,t})$.

The total supply of fiat money measured in dollars is M_t, implying that the total supply of fiat money measured in goods is the number of dollars multiplied by the value of each dollar, or $v_t M_t$. Equality of supply and demand therefore requires that

$$v_t M_t = N_t(y - c_{1,t}). \tag{1.11}$$

This in turn implies that

$$v_t = \frac{N_t(y - c_{1,t})}{M_t}, \tag{1.12}$$

[5] We examine individuals' formations of expectations in a random economy in Chapter 5.

which states that the value of a unit of fiat money is given by the ratio of the real demand for fiat money to the total number of dollars. Similarly, at time $t+1$,

$$v_{t+1} = \frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}}. \tag{1.13}$$

Using Equations 1.12 and 1.13 together, we have

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}}}{\frac{N_t(y - c_{1,t})}{M_t}}. \tag{1.14}$$

To simplify this, we look for a stationary solution, where $c_{1,t} = c_1$ and $c_{2,t} = c_2$ for all t. Because all generations have the same endowments and preferences and anticipate the same future pattern of endowments and preferences, it seems quite reasonable to look for a stationary equilibrium. Then, after some cancelation, Equation 1.14 becomes

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y - c_1)}{M_{t+1}}}{\frac{N_t(y - c_1)}{M_t}} = \frac{\frac{N_{t+1}}{M_{t+1}}}{\frac{N_t}{M_t}}. \tag{1.15}$$

Because we are assuming a constant population ($N_{t+1} = N_t$) and a constant supply of money ($M_{t+1} = M_t$), the terms in Equation 1.15 cancel out and we find that

$$\frac{v_{t+1}}{v_t} = 1 \quad \text{or} \quad v_{t+1} = v_t, \tag{1.16}$$

implying a constant value of money. Because the price of the consumption good p_t is the inverse of the value of money, it too is constant over time.

Notice that the rate of return on fiat money is also a constant (1) in the stationary equilibrium. Identical people who face the same rate of return will choose the same consumption and money balances over time, a stationary equilibrium. Therefore, the stationary equilibrium is internally consistent.

Using the information that $(v_{t+1})/(v_t) = 1$ and recalling that the budget line in a stationary monetary equilibrium is represented by $c_1 + [(v_t)/(v_{t+1})]c_2 = y$, we determine that $c_1 + c_2 = y$. Our graph of the budget line therefore becomes the one depicted in Figure 1.8.

Be aware that the stationary equilibrium may not be a unique monetary equilibrium. There also may exist more complicated nonstationary equilibria. In this text, however, we confine our attention to stationary equilibria because there is much that can be learned from these easy-to-study cases.[6]

[6] Nonstationary equilibria have been studied by Azariadas (1981) and by Cass and Shell (1983).

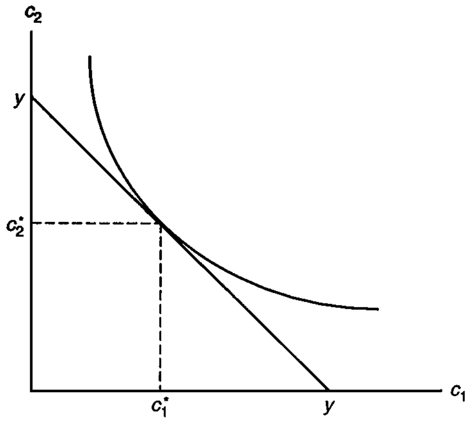


Figure 1.8. An individual's choice of consumption when the money supply and population are constant. With a constant money supply and population, the rate of return on fiat money is 1, implying the lifetime budget constraint of the diagram.

The Quantity Theory of Money

The simplest version of the **quantity theory of money** predicts that the price level is exactly proportional to the quantity of money in the economy. We would like to investigate whether this theory holds in our basic overlapping generations model.

Recall that, in Equation 1.12, we found that the value of money is determined by

$$v_t = \frac{N_t(y - c_{1,t})}{M_t}.$$

In a stationary equilibrium with a fixed population and a fixed stock of fiat money, this equation simplifies to

$$v_t = \frac{N(y - c_1)}{M}. \tag{1.17}$$

As we have seen, the value of money is constant in this simple economy. This is evident from the lack of time subscripts on the right-hand side of Equation 1.17.

Because the price level is the inverse of the value of money ($p_t = 1/v_t$), we can write an expression for the price level as

$$p_t = \frac{1}{v_t} = \frac{M}{N(y - c_1)}. \tag{1.18}$$

This illustrates that the price level in our model is, in fact, proportional to the stock of fiat money, M. As an example, suppose that the initial stock of fiat money in the economy M is doubled but remains constant from then on. (This is referred to as a *once-and-for-all increase* in the fiat money stock.) Equation 1.18 tells us that the price level in every period will also be twice as high. This demonstrates that our model is indeed consistent with the quantity theory of money.

The Neutrality of the Fiat Money Stock

The nominal (measured in dollars) size of the stock of fiat money M has no effect on the real (measured in goods) values of consumption or money demand $(y - c_1)$ of this monetary equilibrium. We see from Figures 1.7 and 1.8 that an individual's choices of consumption and real money balances do not depend on the total number of dollars but do depend on the rate of return of money. The rate of return of money is unaffected by the size of the constant stock of fiat money (notice in Equation 1.15 that the money stock terms canceled each other out). This property of the monetary equilibrium is referred to as the **neutrality of money**.[7]

The Role of Fiat Money

The introduction of valued fiat money into the basic overlapping generations model improves the welfare of the individuals of the economy. Why is this the case? All we have done is to introduce intrinsically worthless pieces of paper into an economy. How can this improve welfare? We hinted at the answer earlier. Without fiat money, people are unable to trade for the goods they desire (c_2) because they do not own anything that the owners of these goods, the next generation, desire. With fiat money, however, people are able to trade for the goods they desire despite this absence of a double coincidence of wants. People sell some of the goods they have for fiat money and then use the money to buy the goods they want. In this model economy, therefore, fiat money serves as a medium of exchange. It is not consumed nor does it produce anything that can be consumed. It is valued nevertheless because it helps people acquire goods they otherwise could not have acquired.

Second-period consumption is a market good in the sense that an individual must trade to obtain more of it. In contrast, first-period consumption is a nonmarket good; individuals already possess first-period consumption without needing to trade for it. We can say then that fiat money provides a means for individuals to purchase market goods.[8]

Is This Monetary Equilibrium the Golden Rule?

We have seen that fiat money can provide for second-period consumption, improving the welfare of individuals otherwise unable to trade. We would like to make the

[7] Keep in mind that we are discussing here the size of a constant stock of money. We will allow the stock of money to change over time in Chapter 3.

[8] The interpretation of the model as one in which people save for old age is not especially helpful here but is taken more seriously in later chapters.

individuals in our economy not just better off but as well off as possible. It remains to ask, therefore, whether the monetary equilibrium results in the best possible allocation of goods. In particular, we would like to see whether the stationary monetary equilibrium we have just found maximizes the welfare of future generations. In other words, does the monetary equilibrium reach the golden rule?

Compare the budget line of Figure 1.8 with the feasible set line of Figure 1.6. They are identical. The choice of consumption in this monetary equilibrium will be identical to the one we found when we were looking at the stationary allocation that was dictated by a central planner who wanted to maximize the utility of the future generations. This implies that the stationary monetary equilibrium obeys the golden rule. The introduction of fiat money allows the future generations not only to increase their utility through trade but, in this case, actually allows them to reach their maximum feasible utility. This will not always be the case. The budget set and the feasible set answer different economic questions. The budget set depicts the constraint on an individual, whereas the feasible set describes the constraint on the society as a whole. We will later find cases in which these two constraints differ and the monetary equilibrium does not obey the golden rule.

The initial old are also better off in the monetary equilibrium than they were with the autarkic equilibrium. In the monetary equilibrium, each person among the initial old will receive $v_1 m_0 = (v_1 M)/(N)$ units of the consumption good when they trade their initial holdings of money for goods with the young of period 1. This means their consumption will be positive. In the autarkic equilibrium, their consumption would be zero. They are certainly better off in the monetary equilibrium.

Because we concentrate on stationary monetary equilibria in this book, it may be useful to summarize the features of such equilibria. A stationary consumption bundle of a monetary equilibrium satisfies two basic properties:

- It provides the maximum level of utility given the individual's budget set. It is found where an indifference curve lies tangent to the individual's budget set.
- It lies on the feasible set line, with the boundary of the set representing all feasible per capita allocations.

A Monetary Equilibrium with a Growing Economy

In the example we just considered, we found that a constant value of money (constant prices) led to an equilibrium that maximized the welfare of future generations. Is this always the case? Are there cases in which a changing value of money maximizes the utility of future generations? To answer these questions, we now complicate our example by allowing the economy to grow over time. We accomplish this by assuming that the population is increasing over time. This implies that the total amount of the consumption good available in the economy will grow over time.

In a monetary equilibrium, the assumption of a growing population also implies a growing demand for fiat money.

Specifically, we will assume that the population of this economy is growing so that $N_t = nN_{t-1}$ for every period t, where n is a constant greater than 1. This says that the number of people born in any period is always n times the number born in the previous period. For example, if $n = 1.05$, then the number of people born in each period is growing by 5 percent from generation to generation. Five percent is the **net rate** of population growth; $n = 1.05$ is the **gross rate**. The gross rate is the net rate plus 1. To test your understanding of population growth rates, try Example 1.1.

Example 1.1 Suppose there are 100 initial old in an economy ($N_0 = 100$) and that the number of young born in the economy is changing according to $N_t = nN_{t-1}$ in each period t, where $n = 1.2$. Trace out the number of young and old people alive in periods 1 and 2. What is the growth rate of the total population?

The Feasible Set with a Growing Population

First, as before, consider the case of an all-powerful central planner who determines allocations of the available goods in each generation. We consider the case of a monetary equilibrium later. As we determined earlier, the total amount of goods available for allocation in period t is $N_t y$. Assuming that all persons within a generation will have identical consumption, total consumption in each period t consists of aggregate consumption by the young ($N_t c_{1,t}$) and aggregate consumption by the old ($N_{t-1}c_{2,t}$). We will consider the stationary case where $c_{1,t} = c_1$ and $c_{2,t} = c_2$. The constraint describing feasible allocations is the same as before

$$N_t c_1 + N_{t-1} c_2 \leq N_t y. \tag{1.19}$$

When we considered the case of a constant population ($N_t = N_{t-1}$), the N terms canceled out in the previous expression. Although this will not occur here, we can simplify Equation 1.19 by dividing through both sides of the inequality by N_t.

$$\left[\frac{N_t}{N_t}\right]c_1 + \left[\frac{N_{t-1}}{N_t}\right]c_2 \leq \left[\frac{N_t}{N_t}\right]y. \tag{1.20}$$

If we recall that $N_t = nN_{t-1}$, we can simplify this expression to

$$c_1 + \left[\frac{N_{t-1}}{nN_{t-1}}\right]c_2 \leq y,$$

or

$$c_1 + \left[\frac{1}{n}\right]c_2 \leq y. \tag{1.21}$$

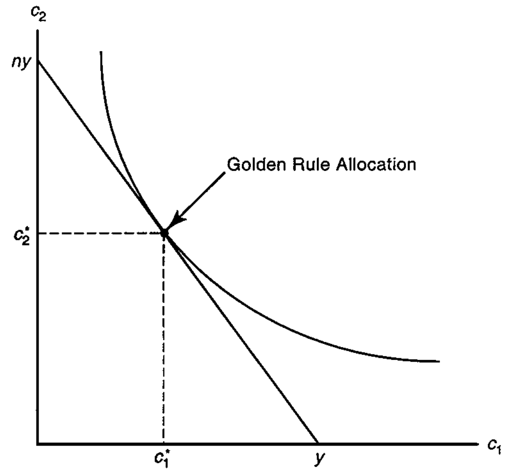


Figure 1.9. The golden rule allocation with a growing population. When the population grows at the rate n, the feasible set line has a horizontal intercept of y and a vertical intercept of ny. As before, the golden rule allocation is determined at a point of tangency between the feasible set line and an indifference curve.

We can easily graph this constraint, as is done in Figure 1.9. You should verify that the intercepts are as shown in the diagram.

Note that if the two axes are scaled the same, then because $n > 1$, the vertical intercept lies farther from the origin than does the horizontal intercept. Why is this vertical intercept greater than it was in the case of a constant population? With a growing population, there are n young people for each old person. Therefore, if we divide the entire endowment of the young equally among the old, there will be ny goods for each old person. It is easier for the planner to provide for consumption by the old because they are relatively few in number.

If we superimpose a typical individual's indifference curves on the graph with the feasible allocations line, we can find the stationary allocation that maximizes the utility of future generations. As always, this occurs at a point of tangency between the feasible allocations line and an indifference curve. This yields the point (c_1^*, c_2^*), which is illustrated in Figure 1.9. If the central planner were to give this combination of c_1 and c_2 to each member of future generations, his welfare would be maximized.

The Budget Set with a Growing Population

Now that we have determined the optimal allocation for future generations, let us turn to the case of a stationary monetary equilibrium. As before, we will eliminate the central planner and introduce fiat money into the economy. We again require that markets clear. In particular, the total demand for money must equal the aggregate supply. Earlier we found that this condition implies (see Equation 1.12) that

$$v_t = \frac{N_t(y - c_{1,t})}{M_t}. \tag{1.22}$$

Note that the numerator of Equation 1.22 is the total real demand for fiat money and the denominator is the total fiat money stock. The equation tells us that the value of fiat money in any period is determined by the relative demand for fiat money and its supply. A higher real demand for fiat money will raise its value, and a higher supply of fiat money will lower its value.

If we update the time subscripts in Equation 1.22 one period, we find that an expression for the value of fiat money in period $t + 1$ is

$$v_{t+1} = \frac{N_{t+1}(y - c_1)}{M_{t+1}}. \tag{1.23}$$

If we now look at the rate of return on money $(v_{t+1})/(v_t)$ we have

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{\frac{N_{t+1}}{M_{t+1}}}{\frac{N_t}{M_t}}. \tag{1.24}$$

If we assume a constant money supply, the M terms cancel. Previously, with a constant population, the N terms also canceled. However, with a growing population, we know that $N_{t+1} = nN_t$, so that Equation 1.24 becomes

$$\frac{v_{t+1}}{v_t} = \frac{N_{t+1}}{N_t} = \frac{nN_t}{N_t} = n. \tag{1.25}$$

The rate of return on money is merely equal to the rate of population growth n. Because $n > 1$, the value of money is increasing over time. This implies that the price of the consumption good is falling over time. Note that our earlier constant-population example is merely a special case of the one just considered. With a constant population, n is equal to 1. We therefore conclude that the rate of return on money is also equal to 1 in that case.

Now if we recall the individual's lifetime budget constraint (Equation 1.10), we find that

$$c_1 + \left[\frac{v_t}{v_{t+1}}\right]c_2 \leq y \Rightarrow c_1 + \left[\frac{1}{n}\right]c_2 \leq y. \tag{1.26}$$

This turns out to be the same constraint that faced our central planner (Equation 1.21) . Therefore, the best allocation in the budget sets of future generations must also be the golden rule, the best allocation in the feasible set for future generations. This implies that an omnipotent, omniscient, and benevolent central planner could do no better than individuals acting within their budget sets.

You should note that our analysis also applies to a shrinking economy, where $n < 1$. In such a case, the value of money falls over time, implying a rising price level. However, much of the previous analysis would still apply. The monetary equilibrium with a constant fiat money stock would still attain the golden rule.

Summary

In this chapter, we introduced the basic overlapping generations model. We found that fiat money, intrinsically worthless pieces of paper, can have value by providing a means for individuals to acquire goods that they do not possess. In addition, we saw that the introduction of a fixed stock of fiat money into an economy enables future generations to attain the maximum possible level of utility given the resources available.

So far, we have concentrated on factors that affect the demand for money. We found that, in a growing economy where the demand for money increases over time, a constant fiat money stock enables individuals to attain the golden rule. We might also be interested in knowing what effects a growing supply of fiat money has on an economy. We turn our attention to the case of an increasing fiat money stock in Chapter 3. Before doing so, in Chapter 2 we consider two alternative trading arrangements to using fiat money – the use of barter and the use of commodity money.

Exercises

1.1 Consider an economy with a constant population of $N = 100$. Individuals are endowed with $y = 20$ units of the consumption good when young and nothing when old.

- **a.** What is the equation for the feasible set of this economy? Portray the feasible set on a graph. With arbitrarily drawn indifference curves, illustrate the stationary combination of c_1 and c_2 that maximizes the utility of future generations.
- **b.** Now look at a monetary equilibrium. Write down equations that represent the constraints on first- and second-period consumption for a typical individual. Combine these constraints into a lifetime budget constraint.
- **c.** Suppose the initial old are endowed with a total of $M = 400$ units of fiat money. What condition represents the clearing of the money market in an arbitrary period t? Use this condition to find the real rate of return of fiat money.

For the remaining parts of this exercise, suppose preferences are such that individuals wish to hold real balances of money worth

$$\frac{y}{1 + \frac{v_t}{v_{t+1}}} \text{ goods.}$$

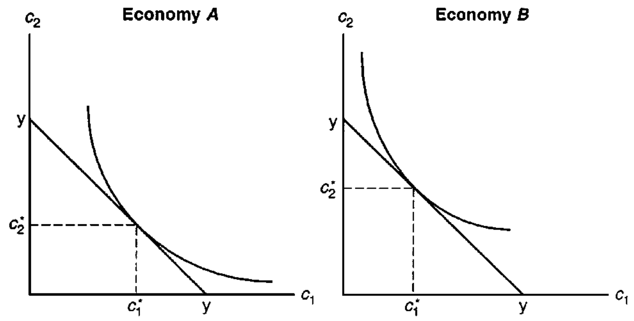
[In the appendix to this chapter, it is verified that this demand for fiat money comes from the utility function $(c_{1,t})^{1/2} + (c_{2,t+1})^{1/2}$.]

d. What is the value of money in period t, v_t? Use the assumption about preferences and your answer in part c to find an exact numerical value. What is the price of the consumption good p_t?

e. If the rate of population growth increased, what would happen to the rate of return of fiat money, the real demand for fiat money, the value of a unit of fiat money in the initial period, and the utility of the initial old? Explain your answers. *Hint:* Answer these questions in the order asked.

f. Suppose instead that the initial old were endowed with a total of 800 units of fiat money. How do your answers to part d change? Are the initial old better off with more units of fiat money?

1.2 Consider two economies, A and B. Both economies have the same population, supply of fiat money, and endowments. In each economy, the number of young people born in each period is constant at N and the supply of fiat money is constant at M. Furthermore, each individual is endowed with y units of the consumption good when young and zero when old. The only difference between the economies is with regard to preferences. Other things being equal, individuals in economy A have preferences that lean toward first-period consumption; individual preferences in economy B lean toward second-period consumption. We will also assume stationarity. More specifically, the lifetime budget constraints and typical indifference curves for individuals in the two economies are represented in the following diagram.



a. Will there be a difference in the rates of return of fiat money in the two economies? If so, which economy will have the higher rate of return of fiat money? Give an intuitive interpretation of your answer.

b. Will there be a difference in the value of money in the two economies? If so, which economy will have the higher value of money? Give an intuitive interpretation of your answer.

1.3 Consider an economy with a growing population in which each person is endowed with y_1 when young and y_2 when old. Assume that y_2 is sufficiently small that everyone wants to consume more than y_2 in the second period of life. Bear in mind that under the new assumptions the equations and graphs you find may differ from the ones found previously.

a. Apply the steps taken in Equations 1.1 to 1.6 to find the feasible set.
b. Assume that all individuals within a generation will be treated alike and graph the set of stationary per capita feasible allocations. Draw arbitrarily located, but correctly shaped, indifference curves on your graph and point out the allocation that maximizes the utility of the future generations.
c. Turning now to the monetary equilibrium, find the equation representing the equality of supply and demand in the market for money.
d. Assume a stationary solution and a constant money supply. Use the equation in (c) to find v_{t+1}/v_t.
e. Draw the budget set for an individual in this monetary equilibrium. Does this monetary equilibrium maximize the utility of future generations? Explain.

1.4 In this chapter, we modeled growth in an economy by a growing population. We could also achieve a growing economy by having an endowment that increases over time. To see this, consider the following economy. Let the number of young people born in each period be constant at N. There is a constant stock of fiat money, M. Each young person born in period t is endowed with y_t units of the consumption good when young and nothing when old. The individual endowment grows over time so that $y_t = \alpha y_{t-1}$ where $\alpha > 1$. For simplicity, assume that in each period t individuals desire to hold real money balances equal to one-half of their endowment, so that $v_t m_t = y_t/2$.
a. Write down equations that represent the constraints on first- and second-period consumption for a typical individual. Combine these constraints into a lifetime budget constraint.
b. Write down the condition that represents the clearing of the money market in an arbitrary period, t. Use this condition to find the real rate of return of fiat money in a monetary equilibrium. Explain the path over time of the value of fiat money.

Appendix: Using Calculus

With the use of simple calculus we can derive mathematical representations of the demand for fiat money from specific utility functions. In the main body of the text we have simply assumed certain demand-for-money functions to illustrate monetary equilibria. In this appendix, we demonstrate that these functions can be derived from utility functions that satisfy our basic assumptions about preferences. The appendix also serves as an illustration of a way to solve explicit examples of monetary equilibria. Following similar steps, advanced students may be able to solve examples of their own creation based on the simple model of this chapter or on the more complex economies of succeeding chapters.

If you do not know calculus, simply skip this appendix. It is not a prerequisite for any material in the succeeding chapters.

The problem facing a young person born at t is to maximize utility, which is a function, $U(c_{1,t}, c_{2,t+1})$, of consumption in each period of life. We assume that the function is continuous in each argument. The individual is constrained by his budget constraints

$$c_{1,t} + v_t m_t \leq y, \tag{1.27}$$

$$c_{2,t+1} \leq v_{t+1} m_t. \tag{1.28}$$

We want to solve for a young person's real demand for fiat money $v_t m_t$, which we write as q_t. We can now write the person's budget constraints (solved at equality) as

$$c_{1,t} + q_t \leq y, \tag{1.29}$$

$$c_{2,t+1} \leq v_{t+1} m_t = \frac{v_{t+1}}{v_t}[v_t m_t] = \frac{v_{t+1}}{v_t}[q_t]. \tag{1.30}$$

If we use the budget constraints to substitute for $c_{1,t}$ and $c_{2,t+1}$ in the utility function, we can write utility as the following function of q_t:

$$U\left(y - q_t, \frac{v_{t+1}}{v_t}[q_t]\right). \tag{1.31}$$

If we graph utility as a function of q_t, we find a function, like that in Figure 1.10, with a single peak. (That there is a single peak is ensured by our assumption of a diminishing marginal rate of substitution.) Maximum utility is reached at q_t^*, where the slope of the utility function is zero.

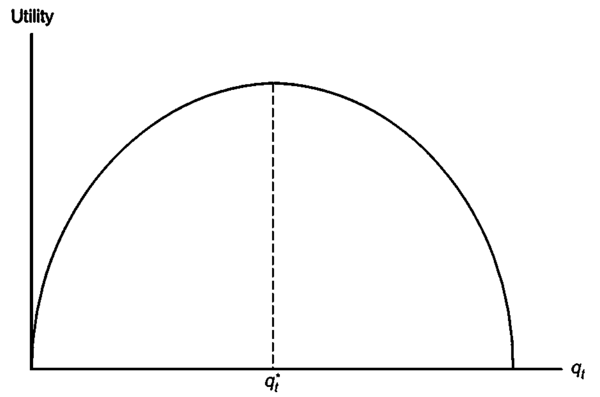


Figure 1.10. Utility as a function of an individual's real demand for fiat money. An individual's utility can be expressed as a function of real fiat money holdings. Utility is maximized by holding real fiat money balances of q_t^*.

The derivative of a function is its slope. Therefore, we find maximum utility at the value of q_t where the derivative of $U(y - q_t, [v_{t+1}/v_t]q_t)$ with respect to q_t equals zero. Let U_i denote the derivative of utility with respect to c_i. Then the utility-maximizing demand for money, q_t^*, is defined by

$$\left.\frac{\partial U\big(y - q_t, \big[\frac{v_{t+1}}{v_t}\big][q_t]\big)}{\partial q_t}\right|_{q_t = q_t^*} = 0$$

$$\Rightarrow -U_1\left(y - q_t^*, \frac{v_{t+1}}{v_t}[q_t^*]\right) + \left[\frac{v_{t+1}}{v_t}\right]U_2\left(y - q_t^*, \frac{v_{t+1}}{v_t}[q_t^*]\right) = 0 \qquad (1.32)$$

$$\Rightarrow \frac{U_1\big(y - q_t^*, \frac{v_{t+1}}{v_t}[q_t^*]\big)}{U_2\big(y - q_t^*, \frac{v_{t+1}}{v_t}[q_t^*]\big)} = \frac{v_{t+1}}{v_t}.$$

Equation 1.32 states that the utility-maximizing demand for money occurs where the marginal rate of substitution between first- and second-period consumption equals the rate of return on money.

The marginal rate of substitution U_1/U_2, which is the ratio of the marginal utilities in the two periods of life, represents -1 times the slope of the indifference curve at the combination of $c_{1,t}$ and $c_{2,t+1}$ that corresponds to a given value of q_t. Because the slope of the budget set is -1 times the rate of return of fiat money, Equation 1.32 is simply a mathematical expression of the statement that utility is maximized where an indifference curve is tangent to the budget line.

An Example

Suppose that utility is given by $(c_{1,t})^{1/2} + (c_{2,t+1})^{1/2}$. If we use the budget constraints to substitute for $c_{1,t}$ and $c_{2,t+1}$, we can find utility as the following function of q_t:

$$(y - q_t)^{1/2} + \left(\frac{v_{t+1}}{v_t}[q_t]\right)^{1/2}. \qquad (1.33)$$

Now differentiate this function with respect to q_t and set the derivative equal to zero:

$$-\frac{1}{2}(y - q_t^*)^{-1/2} + \frac{1}{2}\left[\frac{v_{t+1}}{v_t}\right]^{1/2}(q_t^*)^{-1/2} = 0. \qquad (1.34)$$

Now solve this for q_t^*. (To start, take the first term over to the right-hand side and square both sides.) You should find the money demand function that we used in Exercise 1.1.

$$q_t^* = \frac{y}{1 + \frac{v_t}{v_{t+1}}}. \tag{1.35}$$

Appendix Exercise

1.1 Suppose utility equals $\ln(c_{1,t}) + \beta \ln(c_{2,t+1})$ where $\ln(c)$ represents the natural logarithm of c, whose derivative equals $1/c$. The parameter β is a positive number.

a. Prove that real money balances are

$$q_t^* = \frac{\beta y}{1 + \beta}.$$

b. Derive expressions for the lifetime consumption pattern $c_{1,t}^*$ and $c_{2,t+1}^*$.

c. What effect does an increase in β have on real money balances and the lifetime consumption pattern? Give an intuitive interpretation of the parameter β.

Chapter 2

Barter and Commodity Money

THE NEED FOR exchange is derived from the problem that the goods a person produces may not be the goods that person wants to consume. In Chapter 1 we modeled this problem by assuming that people had goods when young but also wanted to consume when old. Because of the model's simplicity we use it as the foundation on which we build more complicated models.

The simple model, however, allows no alternatives to fiat money – fiat money is used in exchange because there is no other way to trade what one has for what one wants. The model has only a single type of good in every period, so trading goods for goods is ruled out. In this chapter we consider models of two historically important alternative trading possibilities – direct barter and commodity money. In a fiat monetary system, goods trade for fiat money, but goods trade directly for goods in an economy with barter or commodity money. We distinguish between the two in the following way. In a direct barter economy, the goods one owns are exchanged for the goods one desires. In a commodity money economy, the goods one owns may be traded for a good that is not consumed but is traded, in turn, for the good one desires.

In each case, we compare the performance of the model economy using fiat money with the alternative trading device. The first model illustrates how direct barter may be more costly than monetary exchange, the trading of goods for money and, subsequently, money for goods. In the second model, real commodities (not just pieces of paper) serve as money; people trade for commodities they do not want to consume in order to trade later for the goods they do want to consume. We then compare economies using commodity monies to those using fiat money to determine whether one is preferred to the other.

A Model of Barter

If we look at primitive economies, we find that they were typically barter economies. A **barter economy** is one in which the goods one owns are traded directly for the goods one wants to consume. In a barter economy, no particular good is used as a medium of exchange. For small economies with few goods, barter does not present many problems for the typical trader. However, once an economy begins to produce a greater variety of goods and specialization in production develops, barter becomes increasingly inefficient. This is because trade in barter economies requires a **double coincidence of wants**. For a successful trade in a barter economy, the person with whom you wish to trade must not only have what you want but also want what you have. The inefficiency is apparent; a great deal of time is spent merely finding someone with whom to trade.

We turn now to a model that will illustrate the advantages of using fiat money to facilitate trades when there exist many types of goods.[1] Consider a model economy like the overlapping generations model of the Chapter 1, but in which there are J different types of goods. Each person is endowed with y units of one type of good when young and with nothing when old. Equal numbers of the young are endowed with each type of good. When young, individuals wish to consume the type of good with which they are endowed. When old, they will wish to consume one of the other types of goods. However, young people do not know what type of good they will want to consume when old.

There exists a fixed stock of M units of fiat money, which is also costlessly stored. In the first period, the stock of fiat money is owned by the initial old. To allow an alternative to fiat money, we assume that goods can be stored costlessly over time.

People live on a large number of spatially separated islands. Everyone on a given island has the same endowment and tastes. Hence, all young people on a given island will be endowed with the same type of good. For example, a large number of islands will have young people endowed with good 1 when young and similarly for the remaining goods. When old, all the people on a given island will again desire to consume the same type of good, a good with which they were not endowed.

People who want to trade must travel in a group to a trading area, where a group from one island is matched at random with a group from another island seeking to trade. When the people from a pair of islands meet, they can reveal to each other the type of good they are carrying and the type of good they want. If the groups agree to trade, they do so and go home. If they do not both wish to trade, they split and each is matched again with some other island. We assume that islands searching for trading partners can choose to search among the young or the old.

[1] The model is taken from Freeman (1989). Kiyotaki and Wright (1989) and Maeda (1991) offer other recent, interesting models of the use of money when there are many different goods.

Exchange is costly in the following way. Each time a group from one island is matched with a group from another island, each person in the group loses α units of utility. This represents the bother of searching for a suitable trading partner.

Let us now identify patterns of trade through which people in this economy may acquire the goods they desire.

Direct Barter

The most direct way for these people to get what they want is to store some of their endowment until they are old and then trade what they have for what they want to consume. Recall that, until they are old, they do not know what they want to consume.[2] When they know what they want, they can go out and seek a trade.

Let us now determine the probability on any given attempt that they will meet someone who has what they want and wants what they have. Figure 2.1 presents every possible combination of the good with which a person is endowed and the good that person wants to consume for $J = 3$, labeling the goods a, b, and c. The asterisks in Figure 2.1 represent the possible combinations of endowments and desires. If it were possible to desire when old the good with which one is endowed, there would be $J^2 = 9$ possible combinations. Because those three combinations are ruled out, there are $J^2 - J = 6$ possible combinations. Assuming that each group is equally likely to meet any of the possible combinations at any given meeting, the probability of finding a match in which your trading partner has what you want and wants what you have is only $1/(J^2 - J)$ on any given attempt. If there are many types of goods (if J is large), $1/(J^2 - J)$ is a small number. For example, if there are 100 goods, the probability of a successful trade for a given encounter is only $1/(10{,}000 - 100) = 1/(9{,}900)$.

Endowment / Good desired	*a*	*b*	*c*
a		*	*
b	*		*
c	*	*	

Figure 2.1. Endowments and desired goods. When old, individuals do not wish to consume the same good with which they are endowed. Only those combinations of endowments and goods desired marked by asterisks are possible.

[2] These people will not want to barter when young because they do not yet know what they will want to consume.

The small probability of finding someone who has what you want and wants what you have is a good illustration of Jevons' absence of a double coincidence of wants. The average (mean) number of attempts before finding a double coincidence of wants is $J^2 - J$, the inverse of the probability of success on any single try.[3] Given that each search costs α units of utility, the average of search costs under barter is therefore $\alpha(J^2 - J)$.

Monetary Exchange

An alternative pattern of trade uses fiat money as a medium of exchange. Suppose young people seek to trade their goods to the old for fiat money and then, when old, use the fiat money to buy the goods they want.

In this pattern of trade, people undertake two searches and exchanges over their lifetimes. Nevertheless, average lifetime search costs may be less with monetary exchange. In a single try, a young person's probability of finding an old person who wants what he is selling is $1/J$. The young person wants fiat money and does not care which type of old person is encountered, because all old people carry what the young person wants, fiat money. Therefore, the probability of a match on any given attempt is only $1/J$, which is greater than the probability of a match under barter, $1/(J^2 - J)$, where each side of the transaction cared about the type of good carried by the other side. With fiat money, it takes J searches on average for a successful trade.[4] Because each person undertakes two such searches, one when young and one when old, lifetime search costs will average $2\alpha J$ when people use money.

We would like to compare the search costs associated with using barter $[\alpha(J^2 - J)]$ with those when money is used $(2\alpha J)$. We find that the search costs when using

[3] Students of statistics know that the number of attempts before a success follows a geometric distribution. The mean of the geometric distribution is the probability of failure on any single trial [here, $1 - 1/(J^2 - J)$] divided by the probability of success on any one try, $1/(J^2 - J)$. Hence, in this problem, the average number of failures before a success is

$$\frac{1 - \frac{1}{J^2 - J}}{\frac{1}{J^2 - J}} = \left[1 - \frac{1}{J^2 - J}\right](J^2 - J) = (J^2 - J) - 1.$$

Because this number represents the average number of failures before a success, success will occur in the next search. Hence, the average number of searches (including the last successful one) is $J^2 - J$. In this problem the number of attempts before a success approximates a geometric distribution because the probability of finding a match will rise once almost everyone else has found a match. However, the difference is small in a large population of people seeking matches.

[4] As in the previous footnote, we compute the mean of the geometric distribution (mean number of failures before a success) as

$$\frac{\text{probability of failure}}{\text{probability of success}} = \frac{1 - \frac{1}{J}}{\frac{1}{J}} = \left(1 - \frac{1}{J}\right) J = J - 1.$$

Hence, on average, the first successful search occurs on the Jth attempt.

Figure 2.2. Search costs for barter and money. When there are fewer than three goods in an economy, the search costs associated with barter are less than those associated with using fiat money. With three goods present, the search costs are identical for both methods of exchange. When more than three goods exist, fiat money has a clear advantage relative to barter in terms of search costs. The search costs associated with barter rise exponentially with the number of goods.

barter are greater than those when using money if

$$\alpha(J^2 - J) > 2\alpha J \quad \Leftrightarrow \quad J > 3.$$

If there are more than three types of goods (if $J > 3$), average lifetime search costs are lower using money than barter. Although people must trade twice when using money, average search costs are lower (if $J > 3$) for monetary exchange because people do not have to search until they find a double coincidence of wants. It is easier to find someone who wants to buy the endowment good with money, and then, when old, find someone who has the desired good and will accept money. The key to money's usefulness is that everyone accepts money in trade, whereas people who barter accept only the goods they desire.

Notice that the search cost advantage of money grows with the complexity of the economy. Figure 2.2 graphs the search costs associated with barter and money for different numbers of goods. As the number of types of goods J increases, search costs increase faster for barter $[\alpha(J^2 - J)]$ than for money $(2\alpha J)$; with barter, it becomes more and more difficult to find someone who has what you want and wants what you have. If goats and spears are the only two tradable commodities in a primitive village economy, it does not take very long for a goatherd to find a hungry spear maker with whom to trade. In contrast, in a complex modern economy, it may take some time for a hungry economist to find a restaurant owner who wants a lesson in monetary economics.

What Should be Used as Money?

Nothing in our model of money and barter requires that the medium of exchange be fiat money. A commodity also can be used as a medium of exchange. Note that, as economies develop and a greater variety of goods are produced, the search costs associated with barter rise exponentially. As the number of wants and goods expands, an individual might come to accept one particular good in exchange for others even if the person did not wish to consume that good. This can occur if the individual believes that it will then be possible to trade that good for one the person wants to consume. Once most people in the economy come to accept this special good, these barter economies essentially become monetary economies – more specifically, commodity money economies. An often cited modern example of a commodity money is the cigarettes that circulated in prisoner-of-war camps in World War II.[5] Lacking any government currency, even nonsmoking prisoners of war came to accept cigarettes in trade, aware that the cigarettes could be used later to bribe guards or to trade for desired goods. The example demonstrates that money is a natural economic phenomenon not dependent on government for its existence.

A good that everyone accepts in payment for goods is called a commodity money. More precisely, a **commodity money** is a good with intrinsic value (at least some people derive utility from consuming this good directly) that is used as a medium of exchange. A commodity money stands in contrast to fiat money, which has no intrinsic value.

In humankind's long history, the use of fiat monies is a rarity. Most economies either have used some valuable commodity as their medium of exchange or have backed their paper currency with a promise that it can be exchanged for some specified amount of a valuable commodity.

Which commodities will surface as media of exchange? The usefulness of a commodity or fiat money as a medium of exchange depends on its exchange costs.

Exchange Costs

Monetary exchange involves two trades – goods for money, then money for goods – whereas barter requires only one trade. If a money is costly to exchange, its advantage in reducing search costs may be offset by the costs of the second trade.

To be more precise, assume that there is an exchange cost of λ units of utility per person each time goods are accepted. This represents the bother of verifying the quantity and quality of goods exchanged or some other cost of transferring the goods from one island to another. Let λ denote the exchange cost of goods

[5] This example was introduced to economists by Radford's (1945) "The Economic Organization of a P.O.W. Camp." This nontechnical article still makes interesting reading.

Table 2.1. *Search and Exchange Costs for Barter and Money*

	Search cost	Exchange cost	Total cost
Barter	$\alpha(J^2 - J)$	λ	$\alpha(J^2 - J) + \lambda$
Money	$2\alpha J$	$\lambda + \lambda_m$	$2\alpha J + \lambda + \lambda_m$

per person, and let λ_m denote the exchange cost associated with using money. An exchange cost is incurred whenever goods or money are accepted.

The lifetime exchange costs of barter equal λ because each person accepts delivery of goods once in a lifetime. The exchange costs of monetary exchange equal $\lambda_m + \lambda$ because each person accepts money when young and goods when old. The average costs associated with money and barter are summarized in Table 2.1.

When the exchange cost of money λ_m is zero, barter and monetary exchange have the same lifetime exchange costs. Monetary exchange is then superior to barter because of money's lower search costs (if $J > 3$). If, however, money has an exchange cost, its advantage over barter in search costs may be offset by the extra cost of exchange incurred by making two trades instead of one.

It follows that people will want to use something easy to exchange as money. What makes something easy to exchange? It must be easy to recognize and measure. Fiat money tends to possess these properties. Hence, exchange costs for fiat money λ_m are approximately zero.

However, exchange costs with a commodity money system are typically not equal to zero. In fact, exchange costs with commodity money systems may be quite high. For example, early examples of commodity money took the form of chunks of precious metals called *bullion*.[6] Individuals typically accepted these chunks of metal in payment for goods or services. A merchant who accepted bullion in exchange for goods had to assay the quality of the metal. Furthermore, accurate scales were needed to determine the weight of the metal. This process of verifying the quality of the money was costly. In the context of our model, λ_m, the exchange costs associated with using bullion as money were quite high relative to those that would be associated with using fiat money. This, as stated before, at least partially offset the lower search costs associated with the use of metals as money.

In an attempt to lower the exchange costs associated with commodity money, governments soon entered the picture by assaying metals and stamping them with their own insignia. This led to the minting of the metals into regular shapes (coins) stamped with their value.[7] The value that was stamped on the face of the coin was appropriately called the *face value* of the coin.

[6] For example, the Babylonians began using silver bullion as money around 2000 B.C.

[7] Herodotus attributes the origin of coinage to the kings of Lydia in the eighth century B.C., although evidence exists that suggests coinage may have existed in India prior to this time.

A Model of Commodity Money

We have seen that exchange costs may be higher in a commodity money system than in a fiat money system. Are there other advantages or disadvantages of using commodity money versus fiat money?

To answer this question, we introduce into the basic overlapping generations model of Chapter 1 a commodity money, which we will call *gold*.[8] We assume that the consumption good is not storable, but gold can be stored costlessly without physically changing. In other words, there is no depreciation or appreciation associated with gold storage in that 1 unit of gold stored at time t is still 1 unit of gold at time $t+1$. At any point in time, gold can be consumed. When consumed, each unit of gold gives an individual as much utility as the consumption of $\tilde{v}$ more units of the consumption good. We say that the **intrinsic value** of gold is $\tilde{v}$ in this economy, because an individual is indifferent between consuming 1 unit of gold or $\tilde{v}$ units of the consumption good. In contrast, the fiat money of Chapter 1 had an intrinsic value of zero.

Each member of the initial old is endowed with m_0^g units of gold, so that the total initial gold stock is $M^g = Nm_0^g$. There is no source of gold other than this initial stock. Furthermore, no fiat money is present in the economy at any point in time.

As before, each member of the future generations is endowed with y units of the (nongold) consumption good when young and with nothing when old. These people are not endowed with any gold. We assume a constant population in which N individuals are born in each period. We define m_t^g to be the number of units of gold purchased by an individual at time t and v_t^g as the value of a unit of gold in units of the consumption good.

A Commodity Money Equilibrium

Gold has two possible uses in this economy – consumption and trade. It follows that there are two possible equilibria – one in which gold is traded and not consumed and another in which gold is consumed. Let us look first at an equilibrium in which gold is traded but never consumed.

In each period, the young individuals consume a portion of their endowment and use the remainder to purchase gold. In this way, gold will be used as money to trade for second-period consumption. Given our notation, the number of units of the consumption good that will be used to purchase gold will be $v_t^g m_t^g$. This implies that the constraint facing each individual in the first period of life is

$$c_{1,t} + v_t^g m_t^g \leq y. \tag{2.1}$$

[8] The model is taken from Sargent and Wallace (1983). Commodity money could also be studied in the multiple good model of this chapter. We return to the model of Chapter 1, however, because it is simpler.

When old, each individual will trade holdings of gold for some of the consumption good. Therefore, the constraint facing each individual in the second period of life is

$$c_{2,t+1} \leq v^g_{t+1} m^g_t. \tag{2.2}$$

Substituting Equation 2.2 into Equation 2.1, we find the combined budget constraint for an individual born at time t:

$$c_{1,t} + \left[\frac{v^g_t}{v^g_{t+1}}\right] c_{2,t+1} \leq y. \tag{2.3}$$

We know that the market for gold must clear in each period. Recall that the supply of gold in each period is fixed at M^g. From Equation 2.1, we can see that each young individual's demand for gold in period t is

$$m^g_t = \frac{y - c_{1,t}}{v^g_t}, \tag{2.4}$$

so that the total demand for gold is $[N(y - c_{1,t})]/v^g_t$. Equating the total supply of gold to the total demand for gold, we see that

$$M^g = \frac{N(y - c_{1,t})}{v^g_t} \quad \Rightarrow \quad v^g_t = \frac{N(y - c_{1,t})}{M^g}. \tag{2.5}$$

As usual, we restrict our attention to the stationary case where $c_{1,t} = c_1$ and $c_{2,t+1} = c_2$ for all t. In this case, we find that the value of gold in each period is

$$v^g_t = \frac{N(y - c_1)}{M^g}. \tag{2.6}$$

Note that in this stationary equilibrium the value of gold is constant over time. This means that the rate of return of gold is 1 in every period ($v^g_{t+1}/v^g_t = 1$ for all t).

We have assumed in this equilibrium that gold is not consumed; the entire initial stock of gold is used as a medium of exchange. For this to represent the behavior of rational people, there must be no incentive for any individual to consume gold. What condition ensures that this gold consumption does not take place? If individuals can obtain greater utility by trading gold for the consumption good, then they will not choose to consume their gold. In this case, the trading value of a unit of gold exceeds $\tilde{v}$, which is its intrinsic value. In other words, we must have that

$$v^g_t = \frac{N(y - c_1)}{M^g} > \tilde{v}. \tag{2.7}$$

In this case, trading 1 unit of the gold will give an individual v_t^g units of the consumption good, which will generate a certain amount of utility. If people consumed the gold, they would obtain the amount of utility associated with consuming $\tilde{v}$ units of the consumption good. Clearly, then, if $v_t^g > \tilde{v}$, the amount of utility obtained by trading gold for the consumption good is higher than that obtained by consuming the gold. This, in turn, implies that individuals will choose to trade their gold, utilizing it as a medium of exchange.

The Consumption of Gold

The other possibility is worth noting. Suppose that the trading value of gold is less than $\tilde{v}$. This would occur if

$$v_t^g = \frac{N(y - c_1)}{M^g} < \tilde{v}. \tag{2.8}$$

In this case, the initial old will choose to consume gold rather than trade it. If they sell their gold for some of the consumption good, their utility will be less than if they consume gold for its intrinsic value.

Will they consume all the gold? As they consume gold, the total stock of gold in the economy begins to fall. From Equation 2.6, we see that the price of gold will begin to rise. As long as the price of gold is less than $\tilde{v}$, this process will continue and the price of gold will increase. Eventually, the price of gold must rise to its intrinsic value. At this point, the consumption of gold will stop and we will be in the situation described in the first scenario. The remaining gold will then be used as a medium of exchange from that point forward in time. If we denote the amount of gold used for monetary purposes (not consumed) as M^{g^*}, this variable is determined by

$$v_t^g = \frac{N(y - c_1)}{M^{g^*}} = \tilde{v} \quad \Rightarrow \quad M^{g^*} = \frac{N(y - c_1)}{\tilde{v}}. \tag{2.9}$$

The amount of gold in monetary use will be equal to the initial stock of gold minus the amount demanded for personal use (the amount consumed). More precisely, the real value (in units of the consumption good) of gold used as a medium of exchange, $\tilde{v}M^{g^*}$, will be a quantity that will just equal $N(y - c_1)$. The amount of gold consumed by the initial old will be $M^g - M^{g^*}$.

Because commodity money may be consumed, the quantity theory of money may not hold in quite the same way for commodity money as it did for fiat money. Recall that the quantity theory predicts that if two economies are identical except that the fiat money stock in one is twice as large as in the other, the price level will be twice as high (the value of money will be half as high) in the economy with the larger money stock. Prices adjust to the stock of money. Now consider two economies that are identical except that the gold stock in one is twice as large as in the other. If gold is never consumed but serves solely as a commodity money, prices will simply be twice as high in the economy with the larger stock of gold, just as it was in the case of fiat money. But if gold is consumed at the margin in both countries, with a trading value just equal to its intrinsic value, then the economy with a larger gold stock will consume gold until gold's trading value equals gold's intrinsic value. After the consumption of gold, the amount of gold used as money will be the same in the two economies. The intrinsic value of gold sets a minimum value for the trading value of gold, preventing higher nominal prices. If we consider the *initial* stock of gold in the two economies, the quantity theory does not hold because the price level in the economy with the larger initial stock of gold is not twice as high as the economy with the smaller gold stock. However, if we consider the stocks of gold actually *used as money* in the two economies, then the quantity theory does hold. In this case, the quantity theory holds because the stock of gold used as money adjusts to the price level and not because the price level adjusts to the stock of gold.

We see then that the price of gold will equal or exceed its intrinsic value if it is used as a medium of exchange – in other words, as a commodity money. This is a general feature of monetary systems, including commodity money systems; the trading value of a money may exceed its intrinsic value. This is not puzzling in light of the conclusions of Chapter 1. In the monetary equilibrium of that chapter, we saw that fiat money is valued even though it has an intrinsic value of zero. Like gold, fiat money, when used as a medium of exchange, may also have a price in excess of its intrinsic value. Money – whether it be fiat money or commodity money – may have value in excess of its intrinsic value because it provides a means of trading for goods desired (c_2) but otherwise unattainable.

Because the use of a commodity as money may raise its value, what serves as a medium of exchange in an economy has implications for the distribution of wealth. For example, if a commodity money system with $v_t^g > \tilde{v}$ were replaced with a fiat money system, the price of gold would fall to its intrinsic value of $\tilde{v}$. For this reason, owners of gold or other possible commodity monies are very interested in the medium of exchange used in their economy.

The Inefficiency of Commodity Money

Economists have often stated that commodity monies are inefficient.[9] What is meant by this statement? From the development of this chapter, we can gain useful insights into this claim.

[9] See, for example, Friedman (1960).

It is useful to compare the economy developed in this chapter with the fiat money economy of Chapter 1. In that chapter, we considered a monetary equilibrium where there was a constant population and a constant money supply. Hence, that environment was similar to the environment of the commodity money economy of this chapter.

Recall the combined budget constraint governing individual choices in our commodity money economy (Equation 2.3, with stationarity imposed):

$$c_1 + \left[\frac{v_t^g}{v_{t+1}^g}\right] c_2 \leq y. \tag{2.10}$$

We found that, in this economy, the price of gold is constant over time, which implies a rate of return on gold of 1 ($v_{t+1}^g / v_t^g = 1$). Substituting this result, we find

$$c_1 + c_2 \leq y. \tag{2.11}$$

This represents the budget set available to future generations. Reference to Figure 1.8 shows that the budget set in the commodity money economy is identical to that in the comparable fiat money economy. The choices open to individuals of future generations are the same. Given identical preferences between the two economies, we expect individuals to choose the same (c_1^*, c_2^*) combination. With regard to future generations, the commodity money regime provides no advantages (or disadvantages) relative to the fiat money regime. All consumption possibilities that are attainable in the commodity money economy are also available in the fiat money economy. From the viewpoint of future generations, the inefficiency of commodity money systems is not apparent.

It is the initial old who are better off if our commodity money economy switches to the use of fiat money as a medium of exchange. The initial old could use their holdings of fiat money to purchase some of the consumption good. The amount of the consumption good they could purchase with fiat money would be identical to the amount that could be purchased in the commodity money regime. In addition, they could consume all their holdings of gold, which gives them even more utility. Clearly, then, the consumption and utility of the initial old are higher in the fiat money regime than in the commodity money regime. It is important to keep in mind that this is accomplished without diminishing the welfare of future generations.

The intuition is that with a commodity money system, resources that have intrinsic value are tied up in order to provide a medium of exchange. The fiat money system utilizes intrinsically worthless resources to provide the same services. In the case of a gold standard, precious metal that could be used to make jewelry or aeronautical equipment is used as money and is unavailable for these purposes. In this way, commodity money systems are inefficient. A fiat money system allows the same trading patterns while freeing up a commodity that is useful for nonmonetary purposes.

Summary

The major goal of this chapter was to compare the efficiency of trade using barter or commodity money with that of trade using fiat money. This analysis is interesting because of the historical importance of barter and commodity money.

We found that search costs of barter exceed those of money when many types of goods are present in the economy. Intuitively, money facilitates trade by solving the double coincidence of wants problem that is inherent in barter. The search cost advantage of using money expands as the number of types of goods becomes larger.

It is important to remember that the search cost advantage of money over barter holds whether the money we are considering is fiat money or commodity money. However, search costs are only part of the story. The use of money (trading goods for money and money for goods) requires twice as many exchanges as barter (trading goods for goods). Therefore, the exchange costs associated with using money may be higher than those associated with barter, partially offsetting money's lower search costs. To minimize exchange costs, a medium of exchange should be easily recognized and measured.

In the last part of this chapter, we compared welfare under two different monetary standards – a commodity money system and a fiat money system – assuming identical costs of search and exchange. We found that a commodity money system needlessly reserves as a medium of exchange goods that would give people utility if consumed. The switch to a fiat money system improves welfare by freeing those goods for individual consumption.

Exercises

2.1 Consider a fiat money/barter system like that portrayed in this chapter. Suppose that the number of goods J is 100. Each search for a trading partner costs an individual 2 units of utility.

- **a.** What is the probability that a given random encounter between individuals of separate islands will result in a successful barter?
- **b.** What are the average lifetime search costs for an individual who relies strictly on barter?
- **c.** What are the average lifetime search costs for an individual who uses fiat money to make exchanges?

Now let us consider exchange costs. Suppose that it costs 4 units of utility to verify the quality of goods accepted in exchange and 1 unit of utility to verify that money accepted in exchange is not counterfeit.

d. What are the total exchange costs of someone utilizing barter?

e. What are the total exchange costs of someone utilizing money?

2.2 Consider a commodity money model economy like the one described in this chapter but with the following features. There are 100 identical people in every generation. Each individual is endowed with 10 units of the consumption good when young and nothing when old. To keep things simple, let us assume that each young person wishes to acquire money balances worth half of his endowment, regardless of the rate of return. The initial old own a total of 100 units of gold. Assume that individuals are indifferent between consuming 1 unit of gold and consuming 2 units of the consumption good.

a. Suppose the initial old choose to sell their gold for consumption goods rather than consume the gold. Write an equation that represents the equality of supply and demand for gold. Use it to find the number of units of gold purchased by each individual, m_t^g, and the price of gold, v_t^g.

b. At this price of gold, will the initial old actually choose to consume any of their gold?

c. Would the initial old choose to consume any of their gold if the total initial stock of gold were 800? In this case, what would be the price of gold and the stock of gold after the initial old consume some of their gold? Compare your answer in this part with your answer in part a. Does the quantity theory of money hold?

d. Suppose it is learned that a gold discovery will increase the stock of gold from 100 to 200 units in period t^*. Assume the government uses the newly discovered gold to buy bread that will not be given back to its citizens. Find the price of gold at $t^* - 1$ and at t^*. Find also the rate of return of gold acquired at $t^* - 1$.

2.3 **(advanced)** Suppose the consumption of gold offers people a marginal utility that diminishes as that person consumes more gold. Assume also that gold can be mined in unlimited amounts at the constant marginal cost, χ, units of the nongold consumption good.

a. Can the trading value of gold exceed χ in equilibrium? Explain. What is the effect on gold consumption and mining of an increased use of gold as money?

b. Suppose instead that the marginal mining cost increases with the amount mined. What is now the effect on gold consumption and mining of an increased use of gold as money?

2.4 **(advanced)** Consider again the model economy described in Exercise 2.2, but suppose there is a second storable good, silver. Silver is as easy to exchange and store as gold. The initial old own a total of 50 units of silver. There can be no additions to the stock of silver. Individuals are indifferent between consuming 1 unit of silver and 1 unit of the consumption good. Let v_t^s denote the trading value of a unit of silver.

a. Find the market clearing condition if both silver and gold are used as money. Can there be an equilibrium in which both silver and gold are used only as money (are not consumed) and $v_t^s = 1.5$? ... $v_t^s = 2$? In each case use the market-clearing condition to find the corresponding equilibrium trading value of gold. For what range of values of v_t^s is there an equilibrium in which both silver and gold are used only as money (are not consumed)?

b. What would happen to the value of silver if the government passed a law banning the use of gold as money?

c. If one member of the initial old owned the entire stock of silver, would that person prefer that gold alone, silver alone, or both gold and silver be used as money? Explain.

d. If each member of the initial old owned $1/2$ unit of silver and 2 units of gold, would the initial old prefer that gold alone, silver alone, or both gold and silver be used as money? Explain.

Chapter 3

Inflation

IN CHAPTER 1, where the basic overlapping generations model was presented, we concentrated on factors that affected the demand for fiat money. For example, we considered a case in which the population was growing at a constant rate and analyzed the effects of such a situation. In this chapter, we focus on the supply of fiat money.

What are the consequences of an increasing stock of fiat money? What effect does such a policy have on the welfare of individuals in the economy? Can a government raise revenue merely by printing money at a faster rate? These are some of the questions we address in this chapter.

We have seen that we can find a role for money with either the simple, single-good model of Chapter 1 or the more complex multiple-good model of Chapter 2. It can be verified that both models have essentially the same implications for the subject of this chapter, inflation, and for the subjects of later chapters.[1] If two models have the same implications for a topic of interest, then it is generally preferable to work with the simpler model. For this reason, we use the single-good model of Chapter 1 as the framework for this and following chapters.

A Growing Supply of Fiat Money

Let us now study the effects of an expansion of the supply of fiat money. First we consider money supply expansion in the simplest overlapping generations model with a constant population and a nonstorable consumption good. Contrary to Chapter 2, no commodity money is present in the economy.

Let the money supply growth be such that

$$M_t = zM_{t-1} \tag{3.1}$$

[1] See Freeman (1989).

for each period t where z, the gross rate of money supply expansion, is greater than 1. This implies that

$$M_t - M_{t-1} = M_t - \frac{M_t}{z} = \left(1 - \frac{1}{z}\right) M_t \tag{3.2}$$

units of new fiat money are printed each period. This new money is introduced into the economy by means of lump-sum subsidies (transfers) to each old person in every period t worth a_t units of the consumption good; that is,

$$N_{t-1} a_t = \left(1 - \frac{1}{z}\right) v_t M_t$$

or

$$a_t = \frac{\left(1 - \frac{1}{z}\right) v_t M_t}{N_{t-1}}. \tag{3.3}$$

(To find a_t we multiplied the newly created money by the value of money to find its real value and then divided it by the number of old people among whom it will be distributed to find its value per old person.)

Equation 3.3 is our first example of the **government budget constraint**, an equilibrium condition that will prove essential in the analysis of government policy. The government budget constraint simply says that the government (like an individual) cannot spend more than it takes in. In this case, the expenses of government are its gifts to old people and its revenue is the new fiat money it has printed.

It is important that these subsidies be made in a lump-sum fashion so that we can study the effect of money supply expansion in isolation. A subsidy (or tax) is **lump sum** if the amount given to (or taken from) any individual does not depend on any decision made by that particular individual. The subsidy returns the new money to the public. In this way, we ensure that the expansion of the money stock does not represent a transfer of resources from the public to the government, a case we will consider later in this chapter.

The budget constraints of the individual are now

$$c_{1,t} + v_t m_t \leq y, \tag{3.4}$$

and

$$c_{2,t+1} \leq v_{t+1} m_t + a_{t+1}. \tag{3.5}$$

The resulting budget line is now

$$c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} \leq y + \left[\frac{v_t}{v_{t+1}}\right] a_{t+1}. \tag{3.6}$$

The equality of supply and demand in the market for money is

$$v_t M_t = N_t(y - c_{1,t}). \tag{3.7}$$

Using stationarity,[2] we can solve this for v_t to get

$$v_t = \frac{N_t(y - c_1)}{M_t}. \tag{3.8}$$

Then the rate of return of fiat money is given by

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{M_t}{zM_t} = \frac{1}{z}. \tag{3.9}$$

Because the population is constant, the N terms in Equation 3.9 cancel out.

Equation 3.9 tells us that when $z > 1$ the value of money declines over time. Furthermore, the larger the value of z, the lower the rate of return on money. In other words, expansion of the money supply creates inflation as more dollars (for example) bid for the same number of goods. The resulting inflation is easily seen by recalling that $p_t = 1/v_t$ and analyzing how the price level evolves over time. This is done by looking at the ratio of next period's price level to this period's price level (this ratio is the **gross inflation rate**) and using the results of Equation 3.9:

$$\frac{p_{t+1}}{p_t} = \frac{\frac{1}{v_{t+1}}}{\frac{1}{v_t}} = \frac{v_t}{v_{t+1}} = z, \tag{3.10}$$

$$\Rightarrow p_{t+1} = zp_t. \tag{3.11}$$

When $z > 1$, Equation 3.11 predicts that the price level increases over time at the same rate as the fiat money stock. For example, if $z = 1.05$, the price level grows at the same 5 percent net rate at which the fiat money stock is growing. In this way, the price level remains proportional to the size of the fiat money stock, as predicted by the quantity theory of money.

The Budget Set with Monetary Growth

We found in Equation 3.9 that the rate of return of fiat money (v_{t+1}/v_t) in a stationary equilibrium is $1/z$. Substituting this into the lifetime budget set (Equation 3.6), we find

$$c_1 + zc_2 \leq y + za. \tag{3.12}$$

[2] In Equation 3.33 of the appendix, it is verified that a stationary equilibrium is consistent with a constant subsidy, a.

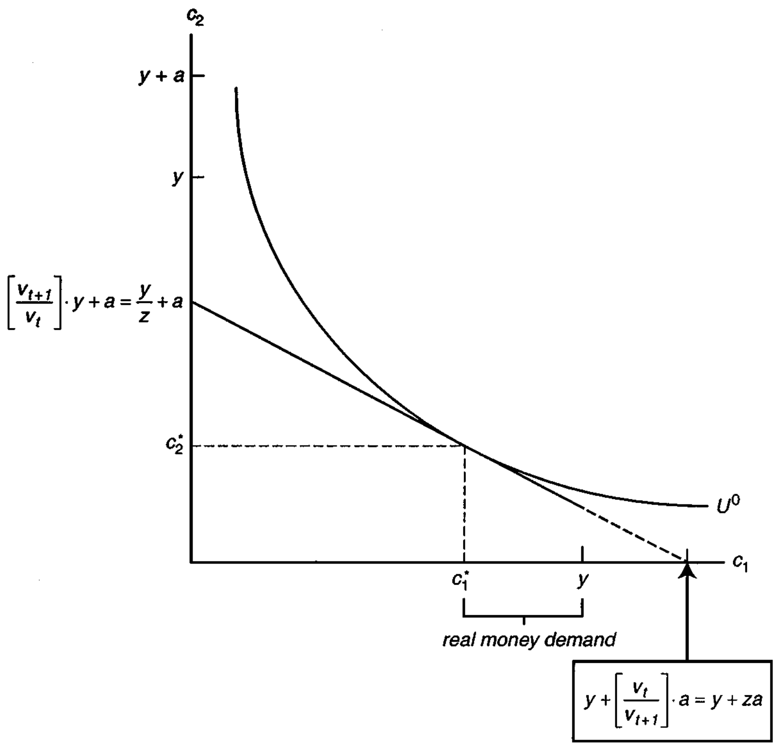


Figure 3.1. Equilibrium with growth of the money supply. The lifetime budget line is drawn for the case in which the fiat money stock is growing at the rate z and the newly printed money is introduced in the form of a lump-sum transfer to the old. Individuals will choose the consumption bundle where the budget line is tangent to the indifference curve labeled U^0. The individual's real money demand is marked in the diagram.

In Figure 3.1, the budget set with inflation is graphed with a typical indifference curve that indicates the monetary equilibrium (c_1^*, c_2^*). Note that inflation $(z > 1)$ has altered our graph of the budget set in two ways. First, the budget line is flatter. This means that to get a unit of goods when old, an individual must give up more units when young than when there was no inflation. This reflects the lower rate of return offered by money when new money is being created. Second, the budget set intercepts the horizontal axis at $y + za$ instead of y, because an individual's income now includes both the endowment and the subsidy.[3]

Common sense tells us that in order to make gifts (subsidies) to individuals, a government that owns no goods can raise revenue for the gifts only by taking goods from private citizens (i.e., through taxation). Money creation may seem to be a way

[3] Note that the budget line from $c_1 = y$ to $c_1 = y + za$ (the intercept) is dashed. For people to consume more than y when young $(c_1 > y)$, they must hold no money balances and also borrow from others, promising to repay the loan from the subsidy they will receive when old. Although any single person has this choice, no one is willing to lend when everyone is alike, so this option is never actually used. Therefore, although we mention this option here for completeness, we hereafter ignore it when presenting the budget equations and lines.

to raise revenue without taxation. Is this really so? The government can create fiat money out of thin air (or cheap paper), but the real value of the government subsidy must come from somewhere. The feasible set is not magically expanded when the government decides to print additional intrinsically useless pieces of paper. Because the total number of goods in the economy is fixed at the total endowment ($N_t y$), the gifts to old people can come only from losses sustained by them or by others.

Who loses goods when the government expands the fiat money stock? When the government expands the stock of fiat money, the stock of money currently held by private citizens falls in value. The new money competes with the old money to purchase the goods of the young and drives down the value of all money. The loss sustained by the owners of the old money works as a tax on their money holdings.

Note that the value lost to the "tax" effected by the expansion of the money stock is proportional to the amount of money held (the more money held, the more one loses through inflation). In other words, the expansion of the money stock lowers the rate of return on fiat money. To reduce one's exposure to this tax on money balances, one can reduce one's use of money. In this way, inflation induces people to conserve on their use of money; the incentive for holding money is reduced.

The Inefficiency of Inflation

Let us return to the question of the optimality of expanding the money stock. To judge whether the equilibrium with inflation is optimal, we must compare it with the other possible alternatives. As in Chapter 1, this translates into comparing the budget set, which shows the options available to individuals in a monetary equilibrium, with the feasible set, which details the consumption allocations that are feasible for the economy. If the budget set coincides with the feasible set, as it did in Chapter 1, then the golden rule allocation is attainable under the monetary equilibrium.

The government's expansion of the fiat money stock should have no effect on what is feasible in this economy. Merely printing more pieces of paper does not alter the stock of goods available for distribution between the consumption of the young and old. The feasible set is therefore exactly the one we found in Equations 1.4 to 1.6 of Chapter 1:

$$N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y,$$

which, for a constant population and a stationary allocation, simplifies to

$$Nc_1 + Nc_2 \leq Ny,$$

or

$$c_1 + c_2 \leq y.$$

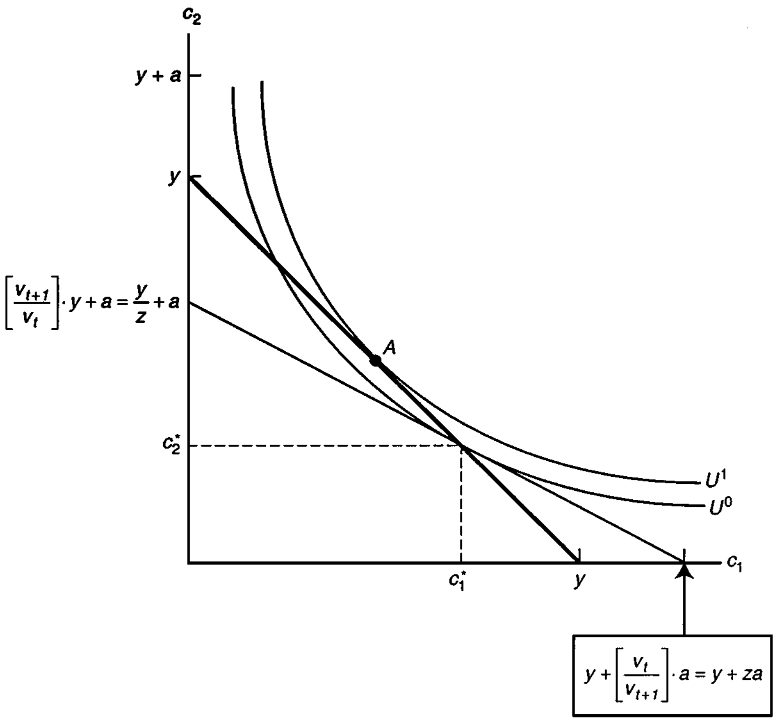


Figure 3.2. The inefficiency of inflation. By comparing the budget line (thin line) and the feasible set line (thick line), we discover that the monetary equilibrium is inefficient when there is inflation. Point *A* yields a higher level of utility for both future generations and the initial old than the monetary equilibrium (c_1^*, c_2^*). Point *A* is feasible but unattainable in the inflationary equilibrium; it lies outside the budget set. Point *A* could be attained in a monetary equilibrium by keeping the fiat money stock constant.

To compare the monetary equilibrium with its feasible alternative allocations, in Figure 3.2 we superimpose the feasible set line on the monetary equilibrium graphed in Figure 3.1.[4] In this diagram, the feasible set line is represented by the thick line and the budget line is represented by the thin line. The feasible set line starts at y on the vertical axis and intersects the budget line at (c_1^*, c_2^*) as shown in Figure 3.2. If (c_1^*, c_2^*) lay in the interior of the feasible set, it would imply that someone was throwing goods away, an action not consistent with utility maximization. If it lay outside the feasible set, people would be consuming more goods than exist, which is impossible. Therefore, the equilibrium consumption bundle (c_1^*, c_2^*) must lie on the edge of the feasible set; that is, the feasible set line passes through (c_1^*, c_2^*).[5]

In examining Figure 3.2, recall that, because (c_1^*, c_2^*) represents the maximum utility possible in the budget set, the consumption bundle (c_1^*, c_2^*) is located where some indifference curve (U^0 in Figure 3.2) is tangent to the budget line at (c_1^*, c_2^*).

[4] This graph and its proof of the inefficiency of inflation are taken from Wallace (1980).
[5] See the appendix of this chapter for a formal proof of this statement.

Note also that the absolute value of the slope of the budget line is $1/z$ and that the absolute value of the slope of the feasible set is 1. Given that $(1/z) < 1$, the budget line is flatter than the feasible set. Because the feasible set line goes through (c_1^*, c_2^*) but at a different slope, it cannot also be tangent to the indifference curve U^0 but must intersect it. This tells us that in the feasible set there are points of higher utility for the future generations than the monetary equilibrium (c_1^*, c_2^*). One such point is A on indifference curve U^1.

Point A is preferred by the future generations over (c_1^*, c_2^*) because it lies on a higher indifference curve. Furthermore, because second-period consumption is higher at point A than at (c_1^*, c_2^*), the initial old also prefer point A over (c_1^*, c_2^*).

Because point A is preferred by future generations over (c_1^*, c_2^*), why did the future generations not choose it? The answer is that point A is not in their budget set. The rate of return on fiat money is too low for the future generations to be able to consume at point A. If individuals were to consume the amount of first-period consumption associated with point A, their money holdings would be too small to afford the level of second-period consumption associated with that point. This is due to the low rate of return on fiat money. We know that the best the future generations can do, given this policy of monetary expansion, is to choose (c_1^*, c_2^*) where their budget line is tangent to an indifference curve.

Recall from Chapter 1 (Figure 1.8) that, in the absence of money creation, the budget set was identical to the feasible set. To see this, realize that if there is no expansion of the money stock, $z = 1$ and $a = 0$. For these values of z and a the budget set is identical to the feasible set, as drawn in Figure 3.2. Therefore, when the fiat money stock is fixed, individuals are free to choose A, the best feasible point for future generations. It follows that future generations prefer the monetary equilibrium without an expanding money supply.

Figure 3.2 can help us uncover the welfare cost of expanding the money stock. The inflation caused by money creation does not destroy any goods; individuals still consume at the boundary of the feasible set. However, they consume a different combination of c_1 and c_2 with inflation than they would consume without it. They choose to consume less of the good c_2, whose purchase requires the use of fiat money (and more of the other good, c_1) because of the lower rate of return on fiat money. In other words, the tax on money balances induces future generations to reduce their demand for money $(y - c_1)$ to a level below the optimum. Moreover, the drop in the demand for fiat money reduces the value of the initial money balances owned by the initial old, thus also reducing their utility.

We should be careful about interpreting this model. A literal interpretation may lead us to conclude that the cost of inflation is that people are induced to consume too much when young and not enough when old.

What, then, is the cost of inflation? People are induced by fiat money's low rate of return to consume needlessly less of goods that require the use of money. In our model, c_2 represents a market good, whose acquisition requires the use of fiat money and c_1 is a nonmarket good that can be acquired without the use of money (leisure is a good real-world example).

To make this clearer, one can interpret the model as follows. Let the endowment in the first period of life be an endowment of time, which can be spent in any combination of leisure or labor. Leisure is that part of the time endowment consumed immediately, c_1 in the notation we have been using. Each unit of labor produces 1 unit of goods, which can be sold to the old for fiat money. The worker then spends the money in the second period of life. In this interpretation, the key economic decision of the model is not one of an individual saving for retirement but one of an individual who works during the week to acquire money to spend on the weekend.

What is the cost of inflation under this interpretation of the model? Inflation discourages the consumption of the market good c_2 in favor of the consumption of leisure c_1, which an individual can acquire without the use of money. By discouraging the use of money, inflation also discourages the supply of labor to be exchanged for money. In this way inflation may affect aggregate output in addition to the timing of consumption.

More generally, we might say that inflation causes people to economize needlessly on the benefits offered by the use of money to conduct transactions. Therefore, inflation will reduce welfare in any model or real economy where money offers benefits of any sort to those who use it and people face a nontrivial choice of how much money to hold.[6]

The Golden Rule Monetary Policy in a Growing Economy

Up to this point in the chapter, we have held the population constant. We would like to see how the results of this chapter change if we allow for a growing population. With such a modification of the environment, we can then analyze an economy where fiat money supply and demand both change over time.

Consider our basic overlapping generations model when the consumption good cannot be stored and the economy is growing so that $N_t = nN_{t-1}$ for every period t, where n is a constant greater than 1. Let $M_t = zM_{t-1}$. Any increases in the fiat money stock will finance a lump-sum gift of a_{t+1} goods to each old person in period $t + 1$. Hence, this setup will be identical to the one just covered, except that we now allow for a growing population. What will be the rate of return on fiat money in this economy?

[6] The literature's first formal discussion of the welfare cost of inflation was by Bailey (1956). For a more modern survey, see Abel (1987).

If we set the supply of money equal to its demand in periods t and $t+1$, we find the expression for the real rate of return of money like those we found previously in Equations 1.24 and 3.9:

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{\frac{N_{t+1}}{M_{t+1}}}{\frac{N_t}{M_t}} = \frac{N_{t+1}}{N_t}\frac{M_t}{M_{t+1}} = \frac{nN_t}{N_t}\frac{M_t}{zM_t} = \frac{n}{z}. \tag{3.13}$$

As before, we are making use of the fact that in a stationary equilibrium $a_{t+1} = a$ and $c_{1,t} = c_1$ for all t. The other cancelations occur because of assumptions about how the fiat money stock and the population change over time.

Because we restrict ourselves to stationary equilibria, in which money demand per person is the same in every period, the only source of change in total money demand in our model is the growth in population.

The budget line in this economy is the same one we found in Equation 3.6:

$$c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right]c_{2,t+1} \leq y + \left[\frac{v_t}{v_{t+1}}\right]a_{t+1}, \tag{3.14}$$

but with $v_t/v_{t+1} = z/n$ and stationarity:

$$c_1 + \left[\frac{z}{n}\right]c_2 \leq y + \left[\frac{z}{n}\right]a. \tag{3.15}$$

Again, we must compare the budget set with the feasible set. The printing of money does not alter what is feasible, so the feasible set remains

$$N_t c_{1,t} + N_{t-1}c_{2,t} \leq N_t y, \tag{3.16}$$

which, in a stationary allocation with a growing population, simplifies to

$$c_1 + \left[\frac{1}{n}\right]c_2 \leq y. \tag{3.17}$$

Again, note that the expansion of the money stock does nothing to alter what is feasible (neither z nor a appears in Equation 3.17).

To compare the monetary equilibrium with the feasible set, we graph the two together (the feasible set line is the thick line). As before, we take advantage of our knowledge that the point of maximum utility in the budget set (point B in Figure 3.3) must lie on the edge of the feasible set.

In Figure 3.3 we see that there are many feasible points (such as A) that offer greater utility to both the future generations and the initial old than does the monetary equilibrium (point B). Point A lies on a higher indifference curve, indicating that the future generations prefer it, and it offers more c_2, indicating that the initial old prefer it.

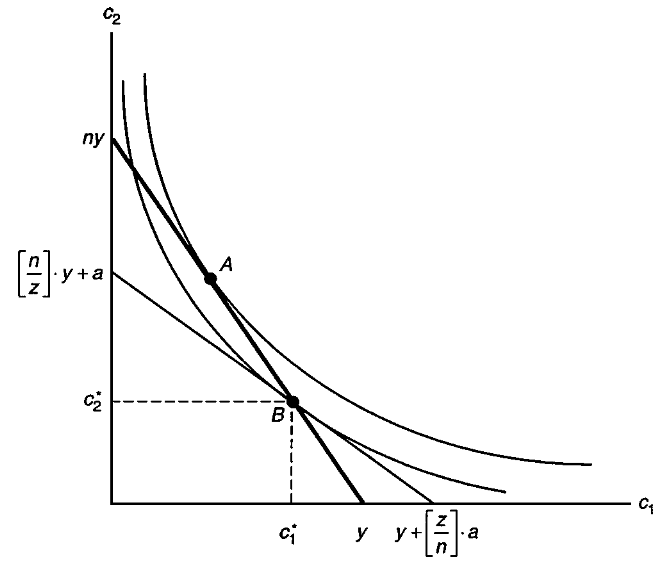


Figure 3.3. An economy with a growing population and monetary expansion. The monetary equilibrium when the fiat money stock grows at the rate of z is represented by point B in the diagram. The monetary equilibrium is inefficient because an allocation like that represented by point A is attainable and is preferred by all to the monetary equilibrium.

The expansion of the money stock ($z > 1$) distorts the budget set by changing its slope from n to n/z. In this way the budget set is no longer the same as the feasible set. This means that the budget set no longer offers an individual a choice of all feasible allocations. In Figure 3.3, for example, we can see that, although allocation A is feasible and is preferred to allocation B, it is not available within the individual's budget set. People cannot choose allocation A because the expansion of the money stock lowers the rate of return of fiat money below n, taxing people's money balances. A person who holds more money in an effort to get to allocation A would find himself not at A but at a point below A on the budget line. He does not get to A because the more money he holds, the more value he loses to the newly printed money.

A Government Policy to Fix the Price Level

In the case just analyzed, the population grew at the rate n, implying that the total endowment of the economy also grew at this rate. We saw in Chapter 1 that the value of a unit of money rises with time when the economy is growing but the money stock is fixed. Many economists[7] have suggested that if the economy is growing, the money supply should grow at the same rate in order to keep the value

[7] Notably, Friedman (1960). Friedman (1969) no longer supported this view in "The Optimum Quantity of Money."

of money constant. Let us examine this policy suggestion in two steps. First, let us ask what rate of fiat money creation will maintain constant prices. Second, let us ask whether such a policy will make individuals better off.

From Equation 3.13 we see that, to keep the value of money (and thus the price level) constant, the rate of expansion of the fiat money stock z must be set to equal the rate of growth of money demand, which is the rate of growth of the population n. Speaking more generally, we will maintain a constant value of money when the stock of fiat money expands at the same rate as the demand for fiat money.

The question that remains is whether it is desirable to increase the money stock at the same rate at which money demand is growing. To answer this question, we must compare the monetary equilibrium with $z = n$ to the feasible set when $n > 1$. When z is equal to n, the lifetime budget set in a stationary monetary equilibrium (Equation 3.15) becomes

$$c_1 + c_2 \leq y + a. \tag{3.18}$$

This budget set, along with the feasible set (which is still given by Equation 3.17), is displayed in Figure 3.4. As we can see from the diagram, there are many points, like point A, that everyone prefers to the monetary equilibrium (represented by point B). Point A is attainable and is preferred to point B by both the future

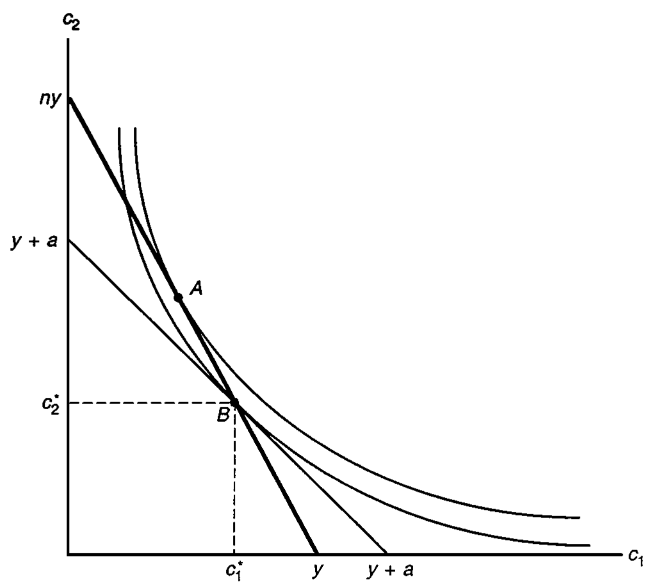


Figure 3.4. A monetary equilibrium when the government fixes the price level. When the government sets the growth rate of the money supply equal to the growth rate of the economy, the monetary equilibrium is at point B. The monetary equilibrium is inefficient because an allocation like point A is feasible and preferred by all.

generations and the initial old. Future generations prefer point A because it lies on a higher indifference curve. The initial old prefer point A because it represents more second-period consumption.

What is wrong with this policy of setting z equal to n? When the price level is fixed, an individual's budget set has a slope equal to -1. This tells the individual that, by consuming one less good today, he will receive one more good in the next period. In other words, the budget set tells the individual that goods are equally available in every period. However, this is not the true state of the economy. The economy is growing. Therefore, if in each generation young people consume one less good when young, there will be n extra goods available for old people in each generation. In other words, the economy can provide n goods for old people for each single good not consumed by young people. For this reason, the feasible set has the slope $-n$.

The message that the economy can offer n goods to the old for each good not consumed by the young is not conveyed through the budget set if prices are constant over time. Because the rate of return on money is 1, people see instead that giving up one good when young will get them only one good when old. As a result, at the monetary equilibrium B, people consume more when young and less when old than at the best feasible allocation A.

How, then, can we convey to individuals the message of the extra availability of goods for the old? The budget set faced by individuals must be identical to the feasible set. We saw in Chapter 1 that the budget and feasible sets (Equations 1.26 and 1.21, respectively) are identical when there is no expansion of the fiat money stock. When there is no change in the fiat money stock, fiat money offers the rate of return n, which signals to people the true state of the growing economy; for each good not consumed by a young person, an old person can consume n goods. In this case, the budget set is identical to the feasible set, so that people who choose the highest level of utility afforded by their budget are selecting the point with the highest feasible utility. For this reason, the golden rule monetary policy is to maintain a fixed stock of fiat money, whatever the growth rate of the economy. Although the policy prescription that the growth rate of the money supply should be set equal to the growth rate of the economy (here, $z = n$) keeps the price level constant, this policy does not maximize the utility of future generations.

Financing Government Purchases

In the preceding section, we found that the government was able to print costlessly new units of fiat money that were valued by the public. It follows that a government that needs to raise revenue for government purchases of goods may do so by printing new units of fiat money. The use of money creation as a revenue device is called **seigniorage**. Let us examine the welfare effects of such a policy.

Again, let $M_t = zM_{t-1}$ for every period t, where z is a constant greater than 1. This implies that

$$M_t - M_{t-1} = (z-1)M_{t-1} = \left(1 - \frac{1}{z}\right)M_t \tag{3.19}$$

units of new fiat money are created each period. This rate of money creation can finance the government's acquisition of

$$G_t = \left[1 - \frac{1}{z}\right]v_t M_t \tag{3.20}$$

goods per period. Denote (constant) government purchases per old person as $g = G_t/N_{t-1}$. Equation 3.20 is the government's budget constraint when the revenue from printing money is used to finance government purchases of goods (in contrast to the government subsidies already studied).

We assume that the goods the government acquires from its seigniorage revenue are used in such a way as not to affect an individual's consumption bundle choice. We might think of such an expenditure as foreign aid or defense expenditures, which may be necessary or desirable but have no direct effect on the relative desirability of c_1 and c_2. For simplicity, we could even think of the government as merely dumping the acquired goods into the ocean. We make this assumption so that we may study the effects of acquiring revenue for the government in isolation from the benefits of the government purchases.

The problem of the individual is the same as it was in the case with no subsidy in that the budget line is still $c_{1,t} + (v_t/v_{t+1})c_{2,t+1} = y$ as in Equation 1.10.

We can again use the equality of supply and demand in the money market $v_t M_t = N_t[y - c_{1,t}]$ (Equation 1.11) and stationarity to get an equation for v_t,

$$v_t = \frac{N_t(y - c_1)}{M_t} \tag{3.21}$$

Assume, for now, that the population is constant ($N_t = N$ for every period t). Then

$$\frac{v_{t+1}}{v_t} = \frac{\frac{N_{t+1}(y-c_1)}{M_{t+1}}}{\frac{N_t(y-c_1)}{M_t}} = \frac{M_t}{M_{t+1}} = \frac{1}{z}. \tag{3.22}$$

Note that because the money supply increases at the same rate in each period, we again looked at the stationary solution ($c_{1,t} = c_1$ for all t). Through cancelation of terms, we learned that the value of money declines when money is created in a

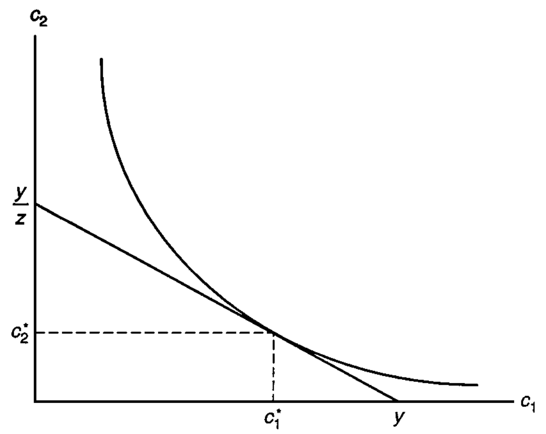


Figure 3.5. A monetary equilibrium with seigniorage revenue. The monetary equilibrium when a growing fiat money stock is used to finance government expenditures is represented by (c_1^*, c_2^*). The rate of fiat money creation z determines the slope of the budget line.

nongrowing economy. In other words, money creation causes inflation because an increasing number of dollars bid for the same number of goods.

Given that the rate of return on fiat money is $1/z$, the individual's lifetime budget constraint becomes

$$c_1 + zc_2 \leq y. \tag{3.23}$$

In Figure 3.5 the resulting budget set is graphed with an arbitrarily drawn indifference curve indicating the monetary equilibrium (c_1^*, c_2^*). Note two effects of an increase in z. As before, the slope of the budget line has been made flatter, which implies that an individual must give up more of c_1 to get a unit of c_2 in the presence of inflation because money has a lower rate of return. In addition, we now find that the budget set has shrunk; it lies inside the budget set without inflation. This occurs because the goods acquired by the expansion of the money stock are now being used up by the government instead of being returned to individuals as a subsidy.

Is Inflation an Efficient Tax?

As before, to discuss the optimality of this monetary equilibrium, we need to find the feasible set to see if any feasible allocations are preferred to the monetary equilibrium (c_1^*, c_2^*). To find the feasible set, we look at the total resources available and require that they not be exceeded by the goods used up. However, now we must be sure to include the goods used up by the government so that we compare the utility of individuals given the same level of government purchases G_t. Therefore, the feasible set for stationary allocations is now given by

$$N_t c_1 + N_{t-1} c_2 + G_t \leq N_t y. \tag{3.24}$$

To get the per capita form, divide through by N_t:

$$c_1 + \left[\frac{N_{t-1}}{N_t}\right] c_2 + \frac{G_t}{N_t} \leq y$$
$$\Rightarrow c_1 + \frac{c_2}{n} + g \leq y. \tag{3.25}$$

For $N_t = N$ (constant population so that $n = 1$),

$$c_1 + c_2 + g \leq y. \tag{3.26}$$

From Equation 3.26 we see that the new feasible set touches the horizontal axis at $c_1 = y - g$. We also know that the monetary equilibrium (c_1^*, c_2^*) lies on the line defining the feasible set because, after the government has taken its share, no consumer will elect to throw goods away.

We can use this information to add the feasible set to Figure 3.5, as we do in Figure 3.6. Note that, because the indifference curve is tangent to the budget line with a slope of $-1/z$, the feasible set line going through (c_1^*, c_2^*) with a slope of -1 must intersect the indifference curve if $z \neq 1$. This implies that the feasible set can reach a higher indifference than can the budget set. Therefore,

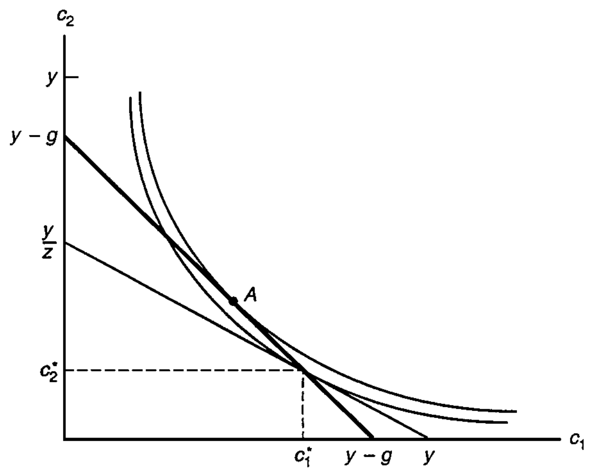


Figure 3.6. The inefficiency of an inflation tax. When a government raises seigniorage revenue to finance government purchases, the monetary equilibrium is (c_1^*, c_2^*). As we have seen before, this equilibrium is inefficient because there exist many points, such as point A, which are feasible, provide the same level of government revenue, and are preferred to (c_1^*, c_2^*).

a move from (c_1^*, c_2^*) to A, as shown in Figure 3.6, benefits the current young and future generations. Also, because this move increases second-period consumption c_2 it also benefits the initial old. Therefore, the monetary equilibrium in this case is not optimal because point A, among other allocations, will make everyone better off.

A Nondistorting Tax

Can we get a budget set of a monetary equilibrium to reflect the feasible set and so make point A attainable? Yes. Consider a fixed tax of τ goods collected from each old person. We refer to such a tax as a *lump-sum tax* because the amount paid to the government is not affected by any actions the individual may undertake. The equations defining this budget when young and old become

$$c_{1,t} + v_t m_t = y \quad \text{and} \quad c_{2,t+1} = v_{t+1} m_t - \tau, \tag{3.27}$$

or, combined,

$$c_{1,t} + \left[\frac{v_t}{v_{t+1}}\right] c_{2,t+1} = y - \left[\frac{v_t}{v_{t+1}}\right] \tau. \tag{3.28}$$

If the entire amount of government purchases is raised through lump-sum taxation ($\tau = g$), the money supply can be held constant. As we found before, the rate of return on money (v_{t+1}/v_t) in a stationary equilibrium will equal 1 when both population (i.e., money demand) and the stock of money are fixed over time. (You will be asked to study the case of a growing population in Example 3.1.) The budget set for $\tau = g$ and $z = 1$,

$$c_1 + c_2 = y - g, \tag{3.29}$$

is identical to the per capita feasible set. Therefore, the point of the maximum feasible utility for the future generations (point A) also lies within the budget set and is thus attainable by individuals. By using lump-sum taxes, the government raised the desired revenue with no distortion of the budget set – that is, without inducing people to reduce their money balances in an effort to avoid inflation's implicit tax on those money balances. Moreover, with lump-sum taxation, the demand for fiat money is greater than when revenue is raised through inflation, implying a greater real value of the money balances owned by the initial old. This, in turn, implies an improvement in the welfare of the initial old.

We see from the previous work that money creation is inferior to lump-sum taxation as a revenue device. Indeed, any tax on an economic activity (unless the activity is socially undesirable) is inferior to a lump-sum tax because it reduces the incentive to undertake that activity. Given that we do not see lump-sum taxes in the real world (perhaps because societies want the rich to pay more than the poor), seigniorage may just be one of many imperfect taxes in an imperfect world.

An obvious advantage of printing money to raise revenue is the ease with which it may be done. It requires no army of accountants or police; the only administrative costs are the costs of printing the notes. It costs pennies to produce a $1000 bill (or a $1 bill). This may explain the heavy use of money creation in poorer nations that may be lacking the extensive informational infrastructure required to enforce income taxes.

The burden of seigniorage falls on those who hold currency. Although everyone uses currency to make purchases, most U.S. currency is held by nonresidents or by people engaged in illegal activities, who do not want their transactions observed.[8] Seigniorage may then be desirable as a way to tax these groups.[9]

The use of seigniorage as a source of government revenue varies from country to country.[10] For most developed countries during normal times, seigniorage contributes very little to government revenue. In the United States, during the period 1948–89, on average seigniorage accounted for less than 2 percent of total federal government revenues and for around 0.3 percent of gross national product (GNP). On the other hand, Fischer (1982) found significant reliance on seigniorage in high-inflation countries like Argentina, Uruguay, Chile, and Brazil. As an example, seigniorage accounted for around 46 percent of Argentinian government revenues (6.2 percent of GNP) for the period 1960–75. Figure 3.7 presents data on seigniorage revenue as a percentage of total government revenue for several countries.

An extreme case in point is provided by Germany during its hyperinflation of the early 1920s. To help finance subsidies to workers in the French-occupied Ruhr and other government expenditures after World War I, Germany turned to the printing press. As a result, seigniorage revenue was eventually 10 to 15 percent of GNP.[11]

Example 3.1 Let $N_t = nN_{t-1}$ and $M_t = zM_{t-1}$ for every period t, where z and n are both greater than 1. The money created in each period is used to finance government purchases of g goods per old person. Prove that the monetary equilibrium does not maximize the utility of future generations. *Hint:* Follow the steps of the example just completed. Explain but do not formally prove why the feasible set line goes through the monetary equilibrium (c_1^*, c_2^*).

[8] See Avery et al. (1987).
[9] The case for seigniorage is made by Aiyagari (1990).
[10] For an excellent cross-country accounting of revenue from seigniorage, see Fischer (1982). See Barro (1982) for seigniorage estimates for the United States.
[11] Barro (1982).

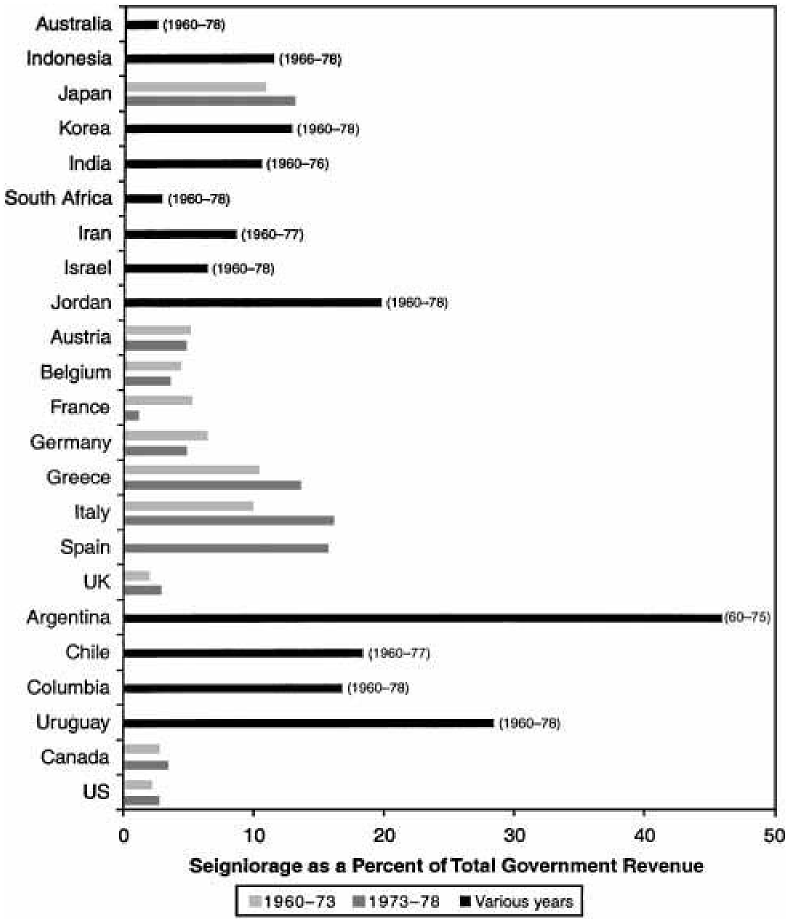


Figure 3.7. Seigniorage revenue as a percentage of government expenditures. Reliance on seigniorage as a source of government revenue varies dramatically across countries. Although the use of seigniorage varies most significantly across regions of the world, substantial variation exists within regions. For example, within Europe during the period 1973–78, seigniorage as a percentage of government revenue ranged from to 1 (France) to 16 (Italy) percent. *Source:* Fischer (1982, Tables A1 and A2, pp. 308–12).

The Limits to Seigniorage

Does seigniorage represent an unlimited source of government revenue? Can the government simply print enough money to pay all its bills without the bother of direct taxation? Although the government is able to print any number of dollars, the value of those dollars shrinks as the government prints more fiat money. Therefore, government revenue in terms of real goods is limited by the real value of the fiat money stock.

To see this, recall that real government revenue from seigniorage at t can be written as

$$(M_t - M_{t-1})v_t = \left[1 - \frac{1}{z}\right] v_t M_t. \tag{3.30}$$

The term $v_t M_t$ in Equation 3.30 represents the real value of the fiat money stock. Because this is the object being taxed, we may consider this the seigniorage **tax base**. The term $1 - (1/z)$ represents the fraction of the value of the real fiat money stock that winds up as government revenue; therefore, it may be considered the seigniorage **tax rate**.

Assume for a moment that the real value of the fiat money stock $v_t M_t$ remains constant as the rate of money creation z is increased. This assumes that people desire the same level of real balances of fiat money whatever the rate of inflation. If this is the case, real seigniorage revenue is always increasing in z. It is nevertheless bounded. As z is driven to infinity, the seigniorage tax rate goes to $1 - (1/\infty) = 1$ and the entire real value of money balances $v_t M_t$ is acquired by the government. But this quantity is finite, limited to the real value of desired money balances by the equality of supply and demand for money (Equation 1.11):

$$v_t M_t = N_t[y - c_{1,t}]. \tag{3.31}$$

There is in fact a more severe limit on the real value of seigniorage revenue. Suppose that a fixed amount of government expenditure is raised through some combination of lump-sum taxes and seigniorage. As the rate of inflation increases, each individual will choose to reduce the real balances of money held $(y - c_{1,t})$ in an attempt to reduce the amount of goods lost to the government through inflation.

To see this reduction in the demand for fiat money, let us examine the budget set when a fixed amount of government purchases is raised through some combination of lump-sum taxes and seigniorage. (You are asked to find this budget set in Exercise 3.6.) Figure 3.8 graphs the budget set and the monetary equilibrium for two alternative policies raising the same government revenue: policy A, in which all revenue is raised through lump-sum taxes ($\tau = g; z = 1$) and policy B, in which some revenue is raised through an expansion of the fiat money supply ($z > 1$). It illustrates how the seigniorage tax base $N_t(y - c_1^*)$ falls as the rate of money creation z increases. The reduction of the demand for fiat money reduces the real value of fiat money balances and thus the real value of the fiat money the government is printing.

The effect of the rate of fiat money creation on the real demand for fiat money can be seen by looking at data from the hyperinflationary episodes after World War

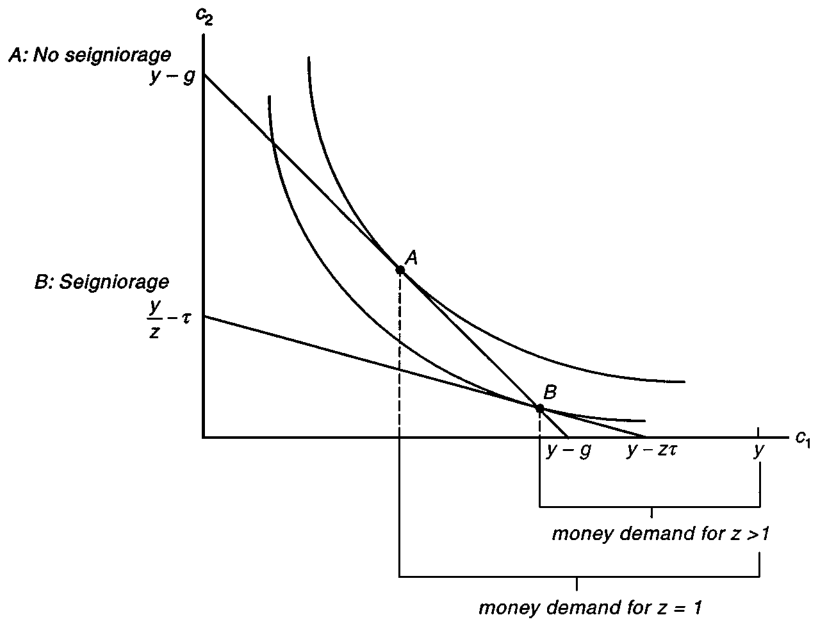


Figure 3.8. The decline in real money balances resulting from an increase in the rate of monetary expansion. Policy *B*, where the government provides for some of its purchases by printing fiat money, results in a smaller real demand for fiat money than policy *A*, where the government provides for its expenditures through a lump-sum tax. This illustrates the reduction in the seigniorage tax base from an increase in the rate of monetary growth.

I studied by Sargent (1986a). Austria, to illustrate such a case, printed fiat money at extremely high rates during the early 1920s in order to finance government deficits. For example, Austrian notes in circulation increased by over 70 percent from July to August 1922. This rapid increase in fiat money creation led to annual inflation rates that approached 10,000 percent per year. As shown in Figure 3.9, data from this episode demonstrate the tendency for real money balances to fall as the inflation rate increases.

We see from Figure 3.8 that, for a given level of government purchases, there is a more severe limit on the real value of seigniorage revenue. An increase in the rate of fiat money expansion discourages people from using money, which reduces the demand for fiat money $(y - c_1)$. In this way, an increase in the rate of fiat money expansion reduces the seigniorage tax base as it increases the seigniorage tax rate. It follows that, if the government inflates the stock of fiat money too rapidly, it may raise less revenue in real terms than it could raise with a lower rate of money creation. Although the exact shape of the revenue function depends on

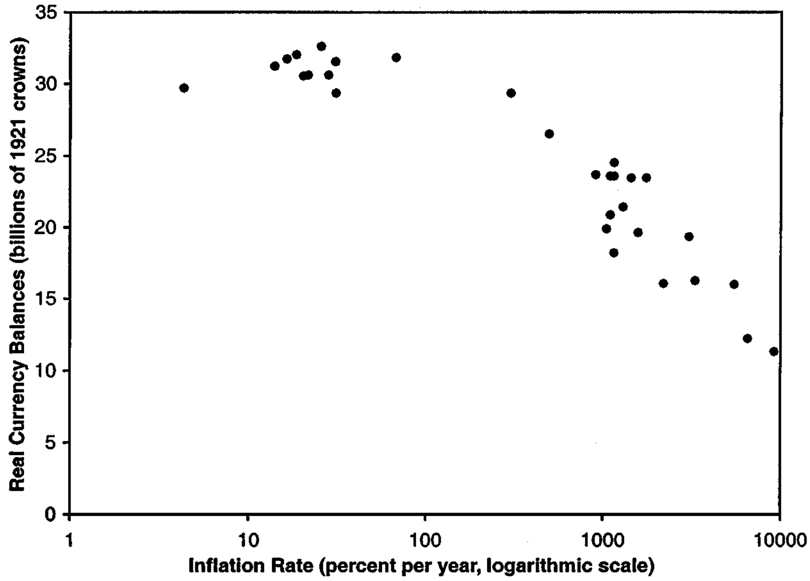


Figure 3.9. Real money balances during the Austrian hyperinflation. During this hyperinflationary episode of the 1920s, real currency balances tended to fall as the rate of inflation increased. *Source:* Authors' calculations using data from Young (1925) as published by Sargent (1986a, Tables 3.2 and 3.3, pp. 49 and 51).

the utility function of individuals and anything else that affects the demand for fiat money, the general shape of the revenue function may be something like that in Figure 3.10.[12]

Such a relationship between tax rates and tax revenues may sound familiar. The Laffer curve hypothesizes a similar relationship between income tax rates and income tax revenue.[13] The notion that a government might increase tax revenues by cutting income tax rates is analogous to the possibility that the government might increase seigniorage revenue by decreasing the rate of money creation.

In Chapter 6, we will see that the introduction of alternative forms of saving will place additional limitations on the amount of seigniorage revenue that can be generated by the government.

As shown in Figure 3.11, data from the Austrian hyperinflation show that after a certain point real seigniorage revenue declines at higher rates of fiat money creation.

[12] See Bailey (1956).
[13] Economists discussed this relationship between tax rates and tax revenue long before its popularization by Arthur Laffer during the promotion of *supply-side economics* by the Reagan administration.

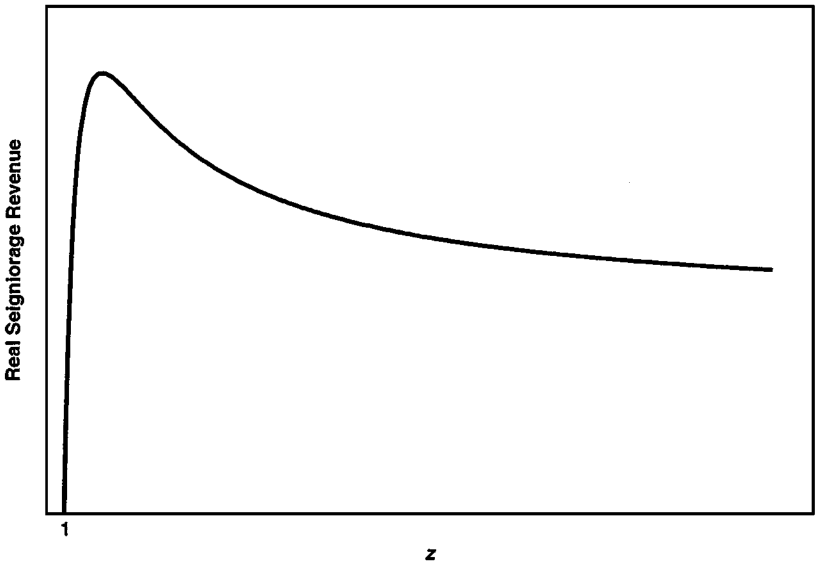


Figure 3.10. Seigniorage revenue and the growth rate of the money supply. As the government increases the rate of monetary expansion above 1, seigniorage revenue increases as the seigniorage tax rate increases. However, as shown in Figure 3.8, the seigniorage tax base falls as z increases. Eventually, this effect may dominate so that seigniorage revenue actually falls as z continues to increase.

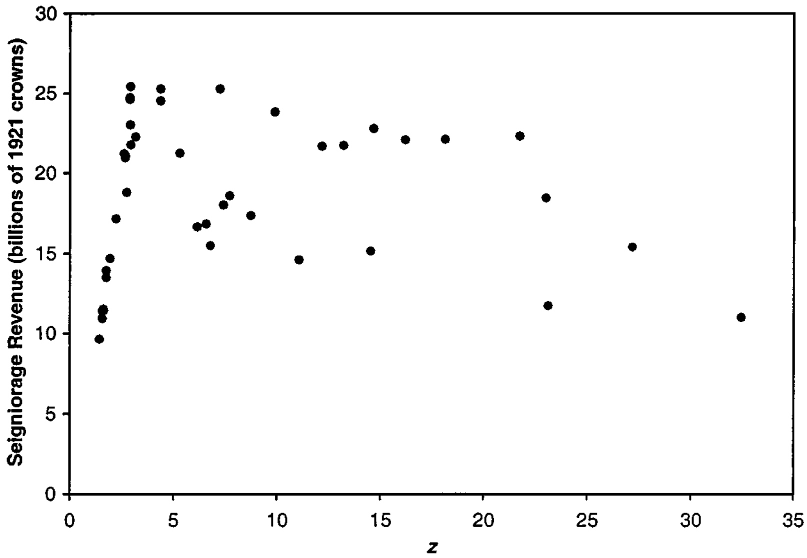


Figure 3.11. Seigniorage revenue during the Austrian hyperinflation. Continual increases in the rate of fiat money creation eventually correspond to lower levels of real seigniorage revenue. *Source:* Authors' calculations using data from Young (1925) as published by Sargent (1986a, Tables 3.2 and 3.3, pp. 49 and 51).

Summary

Whereas Chapter 1 concentrated on the demand for fiat money, this chapter analyzed the effects of a changing supply of fiat money. We concentrated on increases in the fiat money stock that were used to finance government policies such as lump-sum subsidies and government purchases of goods.

The models of this chapter had one overriding theme. In each of the cases considered, the monetary equilibria with an increasing fiat money stock did not attain the golden rule. An increasing fiat money stock acts as an implicit tax on money holdings, causing individuals to economize on their holdings of fiat money. By economizing on their money holdings, individuals do not fully take advantage of the benefits that fiat money provides. Real money holdings fall below the optimal level. The consumption pattern of individuals is altered, tilting it away from the good (c_2) that requires fiat money for its acquisition and toward the good (c_1) that does not. This results in a lower level of utility than could be attained without monetary expansion.

Exercises

3.1 Let $N_t = nN_{t-1}$ and $M_t = zM_{t-1}$ for every period t, where z and n are both greater than 1. The money created each period is used to finance a lump-sum subsidy of a_t^* goods to each *young* person.

a. Find the equation for the budget set of an individual in the monetary equilibrium. Graph it. Show an arbitrary indifference curve tangent to the budget set and indicate the levels of c_1 and c_2 that would be chosen by an individual in this equilibrium.

b. On the graph you drew in part a, draw the feasible set. Take advantage of the fact that the feasible set line goes through the monetary equilibrium (c_1^*, c_2^*). Label your graph carefully, distinguishing between the budget and feasible sets.

c. Prove that the monetary equilibrium does not maximize the utility of the future generations. Support your assertion with references to the graph you drew of the budget and feasible sets.

3.2 Consider an economy with a shrinking stock of fiat money. Let $N_t = N$, a constant, and let $M_t = zM_{t-1}$ for every period t, where z is positive but less than 1. The government taxes each old person τ goods in each period, payable in fiat money. It destroys the money it collects.

a. Find and explain the rate of return in a monetary equilibrium.

b. Prove that the monetary equilibrium does not maximize the utility of the future generations. *Hint:* Follow the steps of the equilibrium with a subsidy, noting that a tax is like a negative subsidy.

c. Do the initial old prefer this policy to the policy that maintains a constant stock of fiat money? Explain.

3.3 Consider an overlapping generations model with the following characteristics. Each generation is composed of 1,000 individuals. The fiat money supply changes according to $M_t = 2M_{t-1}$. The initial old own a total of 10,000 units of fiat money ($M_0 = \$10,000$). Each period, the newly printed money is given to the old of that period as a lump-sum transfer (subsidy). Each person is endowed with 20 units of the consumption good when born and nothing when old. Preferences are such that individuals wish to save 10 units when young at the equilibrium rate of return on fiat money.

a. What is the gross real rate of return on fiat money in this economy?

b. How many goods does an individual receive as a subsidy?

c. What is the price of the consumption good in period 1, p_1, in dollars?

3.4 Consider the following economy. Individuals are endowed with y units of the consumption good when young and nothing when old. The fiat money stock is constant. The population grows at rate n. In each period, the government taxes each young person τ goods. The total proceeds of the tax are then distributed equally among the old who are alive in that period. (The tax is less than the real balances people would choose to hold in the absence of the tax.)

a. Write down the first- and second-period budget constraints facing a typical individual at time t. (*Hint:* Be careful; remember that more young people than old people are alive at time t.) Combine the constraints into a lifetime budget constraint.

b. Find the rate of return on fiat money in a stationary monetary equilibrium.

c. Does the monetary equilibrium maximize the utility of future generations?

d. Does this government policy have any effect on an individual's welfare?

e. Does your answer to part d change if the tax is larger than the real balances people would choose to hold in the absence of the tax?

f. Suppose that tax collection and redistribution are (very) costly, so that, for every unit of tax collected from the young, only 0.5 unit is available to distribute to the old. How does your answer to part d change?

3.5 Describe the essential features of a model economy of rational people for which each of the following statements is true. These features might include the pattern of population growth, monetary growth, endowments, and government policies. Note that there may be more than one model that yields the given results.

a. The gross rate of return on fiat money is 1. The monetary equilibrium also maximizes the utility of the future generations.

b. The price level doubles from period to period. The monetary equilibrium also maximizes the utility of the future generations.

c. The gross rate of return on fiat money is 1. The monetary equilibrium does not maximize the utility of future generations.

3.6 Assume that people face a lump-sum tax of τ goods when old and a rate of expansion of the fiat money supply of $z > 1$. The tax and the expansion of the fiat money stock are used to finance government purchases of g goods per young person in every period. There are N people in every generation.

a. Find the individual's budget constraints when young and when old. Combine them to form the individual's lifetime budget constraint and graph this constraint.

b. Find the government's budget constraint.

c. Graph together the feasible set and the stationary monetary equilibrium.

d. Find the stationary monetary equilibrium when $z = 1$ and add it to the graph in part c.

e. Use a ruler on your graph to compare the real balances of fiat money when $z > 1$ to the values when $z = 1$.

3.7 **(advanced, requires calculus)** Assume that the utility function of people in the economy described in Exercise 3.6 is $\log(c_{1,t}) + \log(c_{2,t+1})$.

a. Find the real demand for money ($q = v_t m_t$) in a stationary equilibrium as a function of z and τ. *Hint:* See the appendix to Chapter 1 for a discussion of solution techniques.

b. Find the government budget constraint in a stationary equilibrium. Solve it for τ as a function of z. (The expression will also involve y and g.)

c. Substitute your expression for τ from the government budget constraint (part b) into the demand for money (part a). Use this to find seigniorage as a function of z alone. Graph seigniorage as a function of z. For the graph, use the following parameter values: $N = 1{,}000$, $y = 100$, and $g = 10$.

3.8 Consider an economy with a constant population of $N = 1{,}000$. Individuals are endowed with $y = 20$ units of the consumption good when young and nothing when old. All seigniorage revenue is used to finance government expenditures. There are no subsidies and no taxes other than seigniorage. Suppose that preferences are such that each individual wishes to hold real balances of fiat money worth

$$\frac{y}{1 + \frac{v_t}{v_{t+1}}} \text{ goods.}$$

a. Use the equality of supply and demand in the money market to find the total real balances of fiat money in a stationary equilibrium as a function of the rate of fiat money creation z.

b. Use your answer in part a to find total seigniorage revenue as a function of z. Graph this function and explain its shape.

3.9 **(advanced)** Suppose the monetary authority prints fiat money at the rate z but now does not distribute the newly printed money as a lump-sum subsidy. Instead, the government distributes the newly printed money by giving each old person α new dollars for each dollar acquired when young. Assume that there is a constant population of people endowed only when young.

a. Use the government budget constraint to find α as a function of z.

b. Find the individual's budget constraints when young and old. Combine them to form the individual's lifetime budget constraint.

c. What is the rate of inflation p_{t+1}/p_t? What is the real rate of return on fiat money? *Hint:* The real rate of return on a unit of fiat money is not simply v_{t+1}/v_t in this case.

d. Compare the individual's lifetime budget constraint with the feasible set. Demonstrate that the monetary equilibrium satisfies the golden rule regardless of the rate of inflation. Explain why inflation does not induce people to reduce their real balances of fiat money in this case.

Appendix: Equilibrium Consumption Is at the Edge of the Feasible Set

We wish to prove algebraically that all goods are consumed in equilibrium – i.e., that the monetary equilibrium consumption bundle (c_1^*, c_2^*) is on the line defining the feasible set. From the work done previously, we know that the following equations – the lifetime budget constraint, the definition of the subsidy a, and the market clearing condition – describe the stationary monetary equilibrium:

$$c_1^* + \left[\frac{z}{n}\right]c_2^* = y + \left[\frac{z}{n}\right]a, \tag{3.32}$$

$$a = \frac{\left[1 - \frac{1}{z}\right] v_t M_t}{N_{t-1}}, \tag{3.33}$$

$$v_t M_t = N_t(y - c_1^*). \tag{3.34}$$

From Equations 3.33 and 3.34 we have that

$$a = \frac{\left[1 - \frac{1}{z}\right] v_t M_t}{N_{t-1}} = \frac{\left[1 - \frac{1}{z}\right] v_t M_t n}{N_t} = \left(1 - \frac{1}{z}\right) n[y - c_1^*]. \tag{3.35}$$

Substituting Equation 3.35 into the lifetime budget constraint, Equation 3.32, we find

$$c_1^* + \left[\frac{z}{n}\right]c_2^* = y + \left[\frac{z}{n}\right]\left(1 - \frac{1}{z}\right) n[y - c_1^*]. \tag{3.36}$$

Collecting and canceling terms, we find

$$zc_1^* + \left[\frac{z}{n}\right]c_2^* = zy. \tag{3.37}$$

Dividing through by z, we find that

$$c_1^* + \left[\frac{1}{n}\right]c_2^* = y, \tag{3.38}$$

proving that (c_1^*, c_2^*) is on the line defining the feasible set.

Chapter 4

International Monetary Systems

UP TO THIS point, we have examined only closed monetary economies – economies that operate entirely in isolation with a single fiat money. Trade and financial links between countries are increasingly important in the modern world, raising the importance of monetary links. Therefore, in this chapter we examine the role of money in economies that encompass more than one country and currency. We examine how exchange rates are determined and seek to explain observed exchange rate changes, especially the dramatic fluctuations of recent decades. We then go on to ask what kind of international monetary system should be in place. In particular we ask the question now facing the European Community: Should trading partners agree to fix their exchange rates or, going even further, adopt a single currency?

A Model of International Exchange

To address these international issues we assume that there exist two countries, a and b, each with its own fiat money. As in Chapter 3, people live two-period lives in overlapping generations. They are endowed with goods when young but not when old, yet they want to consume in both periods of life. The endowments in each country consist of the same goods (a good in country a is indistinguishable from a good in country b). People are indifferent to the origin of the goods they purchase. We use superscripts a and b to identify the parameters and variables of each country; for example, countries a and b have population growth rates n^a and n^b and money growth rates z^a and z^b, respectively. Assume that all changes in the fiat money stock are used to purchase goods for the government. We assume there is free international trade in goods.

The monies of the two countries can be traded at the **exchange rate** e_t, which is defined to be the units of country b money that can be purchased with one unit of

Table 4.1. *Options available to an owner of 1 unit of country* a *money*

Option A	Option B
Keep the country a money Buy v_t^a goods	Trade for e_t units of country b money Buy $e_t v_t^b$ goods

Options available to an owner of 1 unit of country b *money*

Option A	Option B
Trade for $1/e_t$ units of country a money Buy v_t^a/e_t goods	Keep the country b money Buy v_t^b goods

country a money. For example, suppose country a is the United States and country b is Japan. Then the exchange rate is

$$e_t = \frac{\text{Japanese yen}}{\text{U.S. dollar}},$$

the number of Japanese yen per U.S. dollar or, alternatively, the number of yen that can be bought with a dollar. (There is, of course, a second exchange rate, the number of U.S. dollars that can be bought with a Japanese yen, which is simply the inverse of the first exchange rate. It does not matter which one we study.)

As in our single country model, old people seek to trade their fiat money for the goods owned by young people. Naturally, the old people wish to purchase the most goods possible with the money they have. By definition, the owner of a unit of country a money at time t can buy v_t^a goods and the owner of a unit of country b money at time t can buy v_t^b goods. If people are free to trade monies at the exchange rate e_t, then the owner of a unit of country a money has the option of purchasing v_t^a goods with country a money or trading a unit of country a money for e_t units of country b money, which will buy $e_t v_t^b$ goods. Similarly, an owner of a unit of country b money has the option of purchasing v_t^b goods with country b money or trading a unit of country b money for $1/e_t$ units of country a money, which will buy v_t^a/e_t goods. These options are depicted in Table 4.1.

If $v_t^a > e_t v_t^b$, everyone prefers country a money (option A). Owners of country b money will want to trade for country a money to make their purchases, but owners of country a money will not want to trade their money for country b money. Because owners of country b money are not content with the form of their money balances, this cannot be an equilibrium in which both fiat monies are valued. The exchange rate e_t must be higher or v_t^a/v_t^b must be lower. Similarly, if $v_t^a < e_t v_t^b$, everyone prefers country b money (option B). Owners of country a money will want to trade for country b money to make their purchases, but owners of country b money will not want to trade their money for country a money. This also is not an equilibrium in which both fiat monies are valued because the owners of country a money are not content with the form of their money balances.

Only if $v_t^a = e_t v_t^b$ will owners of both countries' monies be indifferent between their two options and thus satisfied with the form of their money balances. Therefore, if both fiat monies are valued, in equilibrium it must be that

$$v_t^a = e_t v_t^b \quad \text{or} \quad e_t = \frac{v_t^a}{v_t^b}. \tag{4.1}$$

We wish to determine the behavior of this exchange rate under alternative international monetary arrangements.

Foreign Currency Controls

The first international monetary system we will study is one that completely separates the monetary sectors of the two countries through a policy of **foreign currency controls** and flexible exchange rates. By foreign currency controls, we mean that the citizens of each country are permitted to hold over time only the fiat money of their own country. Foreign currency controls do not rule out the possibility of trade between the two countries. An old citizen who wishes to buy goods from another country may exchange his money for the foreign currency and then make the purchase. However, the young of each country can hold only their country's money from one period to the next.

The imposition of foreign currency controls implies that each country has its own money supply and demand that independently determine the value of its fiat money:

$$v_t^a M_t^a = N_t^a \left(y^a - c_{1,t}^a\right), \tag{4.2}$$

$$v_t^b M_t^b = N_t^b \left(y^b - c_{1,t}^b\right), \tag{4.3}$$

The exchange rate $e_t = v_t^a / v_t^b$ is therefore

$$e_t = \frac{v_t^a}{v_t^b} = \frac{\frac{N_t^a\left(y^a - c_{1,t}^a\right)}{M_t^a}}{\frac{N_t^b\left(y^b - c_{1,t}^b\right)}{M_t^b}} = \frac{N_t^a\left(y^a - c_{1,t}^a\right)}{N_t^b\left(y^b - c_{1,t}^b\right)} \frac{M_t^b}{M_t^a}. \tag{4.4}$$

Note that the exchange rate, the value of country a money in terms of country b money, depends simply on the relative values of the demand for money and the supply of money in the two countries. The greater the demand for country a money relative to the demand for country *b* money, the higher the value of country *a* money (the exchange rate). The greater the supply of country *a* money relative to the supply of country *b* money, the lower the value of country *a* money.

Following the steps described in Equation 3.13, we can use Equations 4.2 and 4.3 to find the rates of return of the two monies to be

$$\frac{v_{t+1}^a}{v_t^a} = \frac{n^a}{z^a} \quad \text{and} \quad \frac{v_{t+1}^b}{v_t^b} = \frac{n^b}{z^b}. \tag{4.5}$$

Essentially, everything here is just what we found in the one-country case of Chapter 3 (but with superscripts now attached for each country).

Let us now determine the path of the exchange rate over time. The rate of change of the exchange rate is e_{t+1}/e_t. Using the definition of the exchange rate (Equation 4.1), we can express this in terms of the values of the two countries' monies at t and $t+1$,

$$\frac{e_{t+1}}{e_t} = \frac{\frac{v_{t+1}^a}{v_{t+1}^b}}{\frac{v_t^a}{v_t^b}}, \tag{4.6}$$

at which point we can make use of the expressions for the rates of return of the two monies (Equation 4.5) to find

$$\frac{e_{t+1}}{e_t} = \frac{\frac{v_{t+1}^a}{v_{t+1}^b}}{\frac{v_t^a}{v_t^b}} = \frac{v_{t+1}^a}{v_t^a}\frac{v_t^b}{v_{t+1}^b} = \frac{n^a}{z^a}\frac{z^b}{n^b} = \frac{n^a}{n^b}\frac{z^b}{z^a}. \tag{4.7}$$

From Equation 4.7 we can determine how the exchange rate will change over time: the greater the growth rate of country *a*'s population relative to country *b*'s, the greater the rate of growth of the exchange rate, the relative value of country *a* money. This happens because the growth of a country's population causes an increase in its demand for fiat money. Indeed, any increase in the demand for money in a country will drive up its relative value. An increase in a country's endowments (in y, the output of young people), for example, would have the same effect. If both countries expand the money stock at the same rate ($z^a = z^b$) but country *a* grows faster (in output or population), the relative value of country *a*'s money will increase over time; country *a* will experience an **appreciation** of its exchange rate.

We can also see from Equation 4.7 that the greater the growth rate of country *a*'s money supply relative to country *b*'s, the lower the rate of growth of the exchange rate, the relative value of country *a* money. Suppose, for example, that the two countries have equal rates of growth in the demand for money ($n^a = n^b$); then if country *a* expands its money at a faster rate than does country *b*, the value of country a's money will fall relative to country b's money; country a will experience a **depreciation** of its exchange rate.

Fixed Exchange Rates

We see from Equation 4.7 that the exchange rate will not change over time ($e_{t+1} = e_t$) if

$$z^a = \frac{n^a}{n^b} z^b. \tag{4.8}$$

A commitment to fix the exchange rate therefore requires that one or both of the countries choose rates of fiat money creation that satisfy Equation 4.8. Of course, a monetary authority committed to a fixed exchange rate can no longer freely set the rate of money creation in order to raise a chosen level of seigniorage revenue. A country can choose the rate of money creation to fix the exchange rate or to acquire its preferred level of seigniorage revenue, but it cannot meet both objectives.

Suppose, for example, that country a desires to keep a fixed exchange rate with country b. It will then set its growth rate of fiat money creation according to Equation 4.8. If country b now increases its fiat money creation growth rate, country a will be forced to follow suit and increase z^a if it wants to keep the exchange rate fixed.

Note also that Equation 4.8 implies that the fiat monies of both countries will have the same rate of return ($n^a/z^a = n^b/z^b$) under fixed exchange rates. Alternatively stated, they will have the same inflation rates. If country a wishes to maintain a fixed exchange rate and the monetary authority of country b inflates, country a's monetary authority will be forced to inflate too. Country a loses its independence in monetary policy by following its fixed exchange rate policy.[1]

Example 4.1 Suppose that the United States (country a) and Great Britain (country b) have foreign currency controls in effect. The demand for money is growing at 10.25 percent in the United States and at 2 percent in Great Britain (net rates) each period. The fiat money supplies in the United States and Britain are growing at 5 percent and at 6.25 percent net rates in each period, respectively.

a. Defining the exchange rate (e_t) as in the text, what are the units in which the exchange rate is measured, U.S. dollars per British pound or British pounds per U.S. dollar?
b. What is the rate of return on fiat money in the United States? In Great Britain?
c. In a system of flexible exchange rates, what is the time path of the exchange rate between the United States and Great Britain (e_{t+1}/e_t)?

[1] Countries with a history of overusing seigniorage may actually choose to fix their exchange rate with respect to a country that is not likely to inflate. Chapter 16 examines why countries may need to make commitments that limit their ability to print money at will.

d. Suppose the United States desires to fix the exchange rate. How can the United States government set its gross rate of fiat money creation z^a to accomplish this goal?

Example 4.2 Suppose that the (gross) rate of return on fiat money in the United States (country *a*) is 2.0 and that of Canada (country *b*) is 1.0. The (gross) growth rate of the Canadian population (n^b) is 1.2. Foreign exchange controls are in effect.

a. What is the time path of the exchange rate (e_{t+1}/e_t)?
b. Suppose that Canada wishes to maintain a fixed exchange rate with the United States. To accomplish this goal Canada must set its gross rate of fiat money creation (z^b) to what value?

The Costs of Foreign Currency Controls

We have assumed that people don't care where goods come from. Suppose instead that people want to consume at least some goods from another country. Foreign currency controls require that when an old person of country *a* buys a good from a young person of country *b*, the young person of *b* cannot simply keep the country *a* money and use it to make a purchase in old age. Because he is allowed to hold only his own country's money, he must either require that the country *a* person exchange his country *a* money and pay in country *b* money, or accept the country *a* money and immediately exchange it himself. In either case, an exchange of monies occurs that would not be necessary in the absence of foreign currency controls.

In the model of an international economy just described, there seems to be little cost to the money changing that results from the imposition of foreign currency controls. It was assumed that people could exchange one money costlessly for another in order to purchase goods from another country. Anyone who has traveled abroad, however, knows that the exchange of one money for another is not costless. Money changers incur expenses in providing the offices and labor required to conduct the exchanges and charge for this service.[2]

The Indeterminacy of the Exchange Rate[3]

Because foreign currency controls force people to exchange money in order to buy the goods of another country, they impose extra costs on international trade in a

[2] There is a second reason for the inefficiency of foreign currency controls. If the monies of the two nations have different rates of return, their citizens differ in their willingness to trade c_1 for c_2 (have different marginal rates of substitution). This is inefficient because the separation of the two economies prevents citizens from making mutually beneficial trades. See Kareken and Wallace (1977).

[3] The ideas expressed in this section are drawn from the work of Kareken and Wallace (1981). The exposition owes much to Wallace's (1979) article "Why Markets in Foreign Exchange Are Different from Other Markets."

world of costly money exchange. Therefore, let us consider our two-country model economy when people are free to hold and use the money of any country.

To find the exchange rate in such a world, we will turn, as before, to the equality of money supply and demand. Because people are now allowed to hold the money of either country, we can no longer determine the money supply and demand of each country separately but must examine the world's supply of and demand for money. The world supply of fiat money, measured in goods, is $v_t^a M_t^a + v_t^b M_t^b$ and the world demand for fiat money is $N_t^a(y^a - c_{1,t}^a) + N_t^b(y^b - c_{1,t}^b)$. Setting supply equal to demand, we have that

$$v_t^a M_t^a + v_t^b M_t^b = N_t^a\big(y^a - c_{1,t}^a\big) + N_t^b\big(y^b - c_{1,t}^b\big). \tag{4.9}$$

A serious problem now appears in our effort to find the exchange rate. We have the single Equation 4.9 with which to determine two variables, v_t^a and v_t^b. Such an equation has an infinite number of solutions. Because $e_t = v_t^a/v_t^b$, this means that we can find an equilibrium in which world money supply equals world money demand for any positive exchange rate e_t.

This indeterminacy of the exchange rate did not appear when foreign currency controls limited citizens to their own country's money. In that case, the equality of money supply and money demand determined the value of fiat money in each country; the two market-clearing equations, Equations 4.2 and 4.3, determined the two variables v_t^a and v_t^b, which, in turn, determined the exchange rate.

Now, however, we have only a single market-clearing condition with which to try to determine the value of two monies. The right-hand side of Equation 4.9 tells us the total world demand for money, but it cannot tell us whether the dollars of country *a* are worth more or less than the yen of country *b*.

Substitute $e_t v_t^b$ for v_t^a in Equation 4.9. We find

$$e_t v_t^b M_t^a + v_t^b M_t^b = N_t^a\big(y^a - c_{1,t}^a\big) + N_t^b\big(y^b - c_{1,t}^b\big)$$

or

$$v_t^b\big[e_t M_t^a + M_t^b\big] = N_t^a\big(y^a - c_{1,t}^a\big) + N_t^b\big(y^b - c_{1,t}^b\big). \tag{4.10}$$

The term $[e_t M_t^a + M_t^b]$ in Equation 4.10 is the world money supply (measured in units of country *b* money), and $v_t^b[e_t M_t^a + M_t^b]$ is therefore the real value of the world money supply.

Note that, because people are free to hold either country's money, the size of one nation's money demand affects the real value of the world money supply. However, it no longer determines the rate of exchange because a nation is no longer restricted to using only its own money. Similarly, the supply of money printed by any one country does not determine the exchange rate because this money can be used in any country.

To understand this indeterminacy better, suppose that a single government issued two types of currency (say, green and blue) in a single, unified economy but neglected to put any numbers on the bills, choosing instead to let the free market determine the rate of exchange between the two. What would be the exchange rate? Would people value the green bills more or less than the blue? It is impossible to say. Either bill could be worth more than the other. There is nothing to pin down the rate at which people will exchange two intrinsically useless fiat currencies.

Now suppose that the green bills are printed in New York and the blue bills are printed in Des Moines, Iowa. Does this change our answers? No. If the two bills can be traded freely in all parts of the country, their rate of exchange is still undetermined. Printing the bills in two different locations does not end the indeterminacy as long as they are acceptable in trade everywhere. Note that neither the size of the city nor the number of bills printed in the city matters to the exchange rate.

Finally, suppose the blue bills are printed in Toronto, but the United States and Canada allow the holding and use of both colors of money. The political border should not make any difference to our answer. If the two colors of bills are perfect substitutes for each other within North America, nothing pins down their rate of exchange.[4]

Exchange Rate Fluctuations

In the absence of the government determination of the exchange rate, the exchange rate in a unified world economy can be whatever people believe it to be. It follows that, if these beliefs fluctuate, the exchange rate will also fluctuate because there is nothing to pin it down. These fluctuations need not be tied to changes in real economic conditions. Therefore the dollar may fall against another currency simply because everyone believes it will fall, regardless of whether U.S. output or some other real factor has changed.

Since 1971, when President Richard Nixon announced the abandonment of all U.S. efforts to control exchange rates, the world has seen tremendous volatility in exchange rates. It has become common for a currency to gain or lose 20 percent or more of its value in a matter of months. This volatility is clearly shown in Figure 4.1, which displays the U.S. exchange rate against six major currencies for the past four decades.

[4] On both sides of the U.S.–Canadian border, the currencies of both countries do circulate, but there remain some exchange controls that make the currencies less than perfect substitutes – for example, the restriction that only U.S. dollars can be used as reserves for U.S. bank deposits. (Reserve requirements are studied in Chapter 7.)

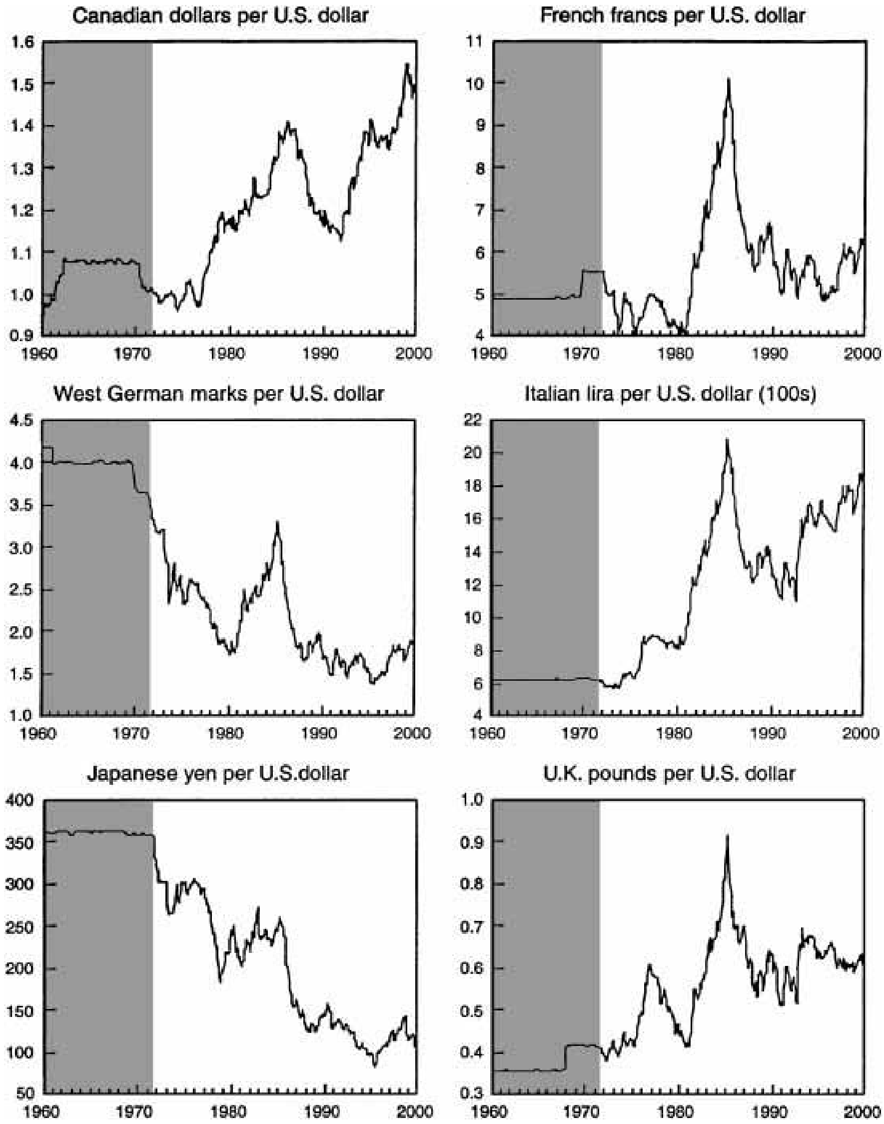


Figure 4.1. The U.S. exchange rate against six major currencies. Since the United States abandoned efforts to stabilize exchange rates in 1971, there has been marked volatility in the exchange rates between major currencies. This is seen in the U.S. exchange rates with Canada, France, former West Germany, Italy, Japan, and the United Kingdom. Shaded regions portray the period when the United States attempted to stabilize exchange rates. *Source:* Exchange rate data are from the Federal Reserve Bank of St. Louis FRED database (http://www.stls.frb.org/fred/index.html).

Table 4.2. *Exchange rate fluctuations*

Country	Months	Exchange Rate Movement
France	Dec 1973–Jan 1974	9.4% depreciation of the franc
Germany	Jun 1973–Jul 1973	9.4% appreciation of the mark
Italy	Sep 1992–Oct 1992	11.3% depreciation of the lira
Japan	Sep 1998–Oct 1998	10.0% appreciation of the yen
United Kingdom	Sep 1992–Oct 1992	11.7% depreciation of the pound

For a sense of the volatility of the data presented in Figure 4.1, we could calculate month to month changes in the exchange rates. Table 4.2 presents the extreme values for these calculations and the month in which they occurred.

The fluctuations in exchange rates cannot be readily traced to changes of similar magnitude in a country's money supply or its demand for money. None of the countries in Table 4.2 printed or destroyed over 9 percent of its money stock in a single month, nor did it have a one-month change in real economic activity of that magnitude, nor did the combination of one-month changes in money supply and demand across the two countries reach the magnitude of these changes in the exchange rates. A possible explanation for this exchange rate volatility may be the existence of sufficiently large sectors of the world economy that are free to hold multiple currencies. Although you or I may not be part of this group, there exist multinational institutions that certainly are.

International Currency Traders

Is there a cost to large random fluctuations in exchange rates? An individual can hedge against the fluctuations if he were able to costlessly hold a perfectly balanced portfolio of different currencies. In real life this option, although open to multinational institutions, does not seem to be costlessly open to ordinary people with small money balances (at least, we do not observe the holding of balanced money portfolios). The nuisance and costs of determining and acquiring a balanced portfolio of monies may be the reason or it may be that people are subject to government regulations that force them to use the local currency. As a result, fluctuations in the exchange rate put the value of people's money balances at risk.

To make this point more precisely, consider a model economy suggested by King, Wallace, and Weber (1992), in which there are three types of people:

1. citizens of country a, forced by law to hold only country a's money;
2. citizens of country b, forced by law to hold only country b's money;
3. multinational people, free to hold either currency.

Let N_t^a, N_t^b, and N_t^c, respectively, represent the number of people of each type in a generation born in period t. (We use superscripts to indicate a person's type for all variables.)

As always, the value of each country's currency (and thus the exchange rate) is affected by the demand for it. Each country's money is held by its own citizens and perhaps by multinational people as well. Let's let λ_t represent the fraction of the multinational people's money balances that is held in the form of country a's money. We can now write the two equations that represent the markets for the currencies of countries a and b, respectively,

$$v_t^a M_t^a = N_t^a\big(y^a - c_{1,t}^a\big) + \lambda_t N_t^c\big(y^c - c_{1,t}^c\big), \tag{4.11}$$

$$v_t^b M_t^b = N_t^b\big(y^b - c_{1,t}^b\big) + (1-\lambda_t) N_t^c\big(y^c - c_{1,t}^c\big). \tag{4.12}$$

It is obvious from Equations 4.11 and 4.12 that, the more the multinational people (type c) want to hold country a's money (i.e., the greater the value of λ_t), the greater will be the value of country a's money and the lower will be the value of country b's money. This, in turn, implies that the greater the value of λ_t, the greater will be the exchange rate e_t. However, because the multinational people are free to hold any fraction of their money balances in each country's money, there are many possible equilibrium exchange rates. To see this point, note that from Equations 4.11 and 4.12, the exchange rate in this world economy is

$$e_t = \frac{v_t^a}{v_t^b} = \frac{\frac{N_t^a\left(y^a - c_{1,t}^a\right) + \lambda_t N_t^c\left(y^c - c_{1,t}^c\right)}{M_t^a}}{\frac{N_t^b\left(y^b - c_{1,t}^b\right) + (1-\lambda_t) N_t^c\left(y^c - c_{1,t}^c\right)}{M_t^b}}. \tag{4.13}$$

As an illustration, consider a simple case in which the total real demand for currency is identical across the different types of people. In other words, suppose that $N_t^a(y^a - c_{1,t}^a) = N_t^b(y^b - c_{1,t}^b) = N_t^c(y^c - c_{1,t}^c)$. We can then factor those terms out of Equation 4.13. We find that the exchange rate is

$$e_t = \frac{v_t^a}{v_t^b} = \frac{\frac{1+\lambda_t}{M_t^a}}{\frac{1+(1-\lambda_t)}{M_t^b}} = \frac{\frac{1+\lambda_t}{M_t^a}}{\frac{2-\lambda_t}{M_t^b}}. \tag{4.14}$$

Equation 4.14 illustrates that, for given stocks of fiat money in countries a and b, changes in λ_t will cause fluctuations in the exchange rate. An increase in λ_t will cause the exchange rate to rise and a decrease in λ_t will cause the exchange rate to fall. As an example, verify to yourself that, if the two countries issue the same nominal number of notes (i.e., $M_t^a = M_t^b$), the exchange rate can take on any value between 1/2 and 2. (Hint: What is the range of values for λ_t?)

Example 4.3 Suppose there are three types of people in our model of two countries and two currencies. Type a people can hold only the money of country a, type b can hold only the money of country b, and type c can hold the money of either country. Every person wants to hold 10 goods worth of money. There are 300 type a people, 200 type b people, and 100 type c people. There are 100 units of country a money and 200 units of country b money.

a. Find the range of stationary equilibrium values for v^a, v^b, and e.
b. Now suppose that 100 type a people and 100 type b people become type c people (able to hold the money of either country). Now find the range of stationary equilibrium values for v^a, v^b, and e. Has the range of equilibrium exchange rates expanded or contracted? Explain this change.

As we saw earlier, the multiplicity of exchange rates that satisfy the conditions for a stationary equilibrium suggests that exchange rates may fluctuate dramatically as multinationals change the composition of their money balances. These fluctuations make each currency a risky asset.[5] Those who have access to only a single currency, however, will see the real value of their money balances, and thus their consumption, rise or fall with the exchange rate. Multinationals can free themselves from this risk if they hold a balanced portfolio of both monies so that if the exchange rate changes, the decreased value of one currency is offset by the increased value of the other. Although this balancing of currency balances may free multinationals from risk, it may be bothersome or otherwise costly to hold perfectly balanced stocks of both countries' currency.

Monetary authorities may therefore wish to stabilize the exchange rate to free their citizens from the risk of a decline in the value of their money balances or from the bother of perfectly balancing their money balances.

Fixing the Exchange Rate

Cooperative Stabilization

How can we organize the world to provide a stable exchange rate? For a solution to the indeterminacy of the exchange rate in the absence of foreign currency controls, let us take a cue from the monetary organization of national economies. What determines the exchange rate between two different bills in a single national economy? Quite simply, the government tells us the rate of exchange by printing the denomination on each bill and standing ready to exchange the bills at that rate. In the United States a bill with a picture of Alexander Hamilton trades for 10 bills with pictures of Washington because the monetary authority of the United States, the

[5] Fluctuating exchange rates also make risky the real value of any contract denominated in a single country's currency.

Federal Reserve, will exchange the bills at a rate of ten to one. This exchange rate does not depend on how many pictures of Washington have already been printed.

The exchange rate in a national economy also fails to depend on where the bills are printed. Each piece of U.S. currency carries the name of one of the 12 Federal Reserve banks, but the bills always trade one for one. No merchant in California sells goods for a higher dollar price if the dollars happen to be marked with the name of the Boston Federal Reserve Bank. No bank trades two pictures of Washington marked "Federal Reserve Bank of New York" for one picture of Washington marked "Chicago." When the Texas economy is in a slump, the value of bills marked "Dallas" does not fall.[6]

What is true for a single national economy is also true for a world economy unified in its use of currencies. If the two governments stand ready to exchange currencies at some given rate, they may determine the exchange rate. If the central banks of all countries stood ready to give $2 whenever presented with a British pound, people would be indifferent between £1 and $2. In this way the exchange rate would become determined.[7]

The exchange rate would also be fixed over time. In the absence of foreign currency controls, fiat currencies are held voluntarily. However, no currency will be held voluntarily if its value will fall over time relative to the value of other currencies. Such a currency offers a lower rate of return than the others, inducing everyone to switch to other currencies.

This solution seems so easy that one wonders why we rarely see fixed exchange rates. The European Economic Community (EEC), for example, although an advanced and integrated international economy, had tremendous difficulties in maintaining fixed exchange rates despite the pledges of the European governments.[8] During 1992, several countries in the EEC encountered difficulties maintaining fixed exchange rates with one another. As one example, after attempts to fix the value of the British pound relative to the German mark, Britain abandoned such measures in September 1992, allowing the pound to fall more than 10 percent in value relative to the mark. We will now examine two major impediments to the stabilization of exchange rates – speculative attacks on currencies and the strong incentive to inflate when exchange rates are fixed.

A key part of fixing the exchange rates among different forms of a national money is the willingness of the monetary authority to accept any amount of one

[6] Rolnick and Weber (1989) discuss the notion that Federal Reserve notes are distinct currencies trading at fixed exchange rates. See their paper for an excellent comparison of fixed and floating exchange rates.

[7] A recent example of fixing the exchange rate in this way came during the reunification of Germany, when the German central bank announced that it would accept East German marks at a one-for-one rate of exchange with West German marks, despite the fact that they were trading well below that rate of exchange before the announcement.

[8] Actually, the members of the European Monetary System agreed to keep exchange rates between pairs of member countries within narrow bands (±2.25 percent) of a fixed exchange rate. These bands were increased to ±15 percent after several countries abandoned attempts to maintain the narrow bands during September 1992.

form of money in exchange for money at a different form at the fixed rate. No matter how many Hamiltons you wish to trade for Washingtons, the Federal Reserve will exchange them at the rate of 10 Washingtons per Hamilton. And no matter how many bills with the stamp of Dallas you wish to trade for bills with the stamp of Boston, they can be had at the rate of one for one. How can the monetary authority make such an unbounded promise? What if they run out of Washingtons or bills with the stamp of Boston?

No one worries about a scarcity of Washingtons or bills with the stamp of Boston. If for any reason people want more of any type of bill, the Federal Reserve can simply have more printed. There is no limit to the exchanges the Federal Reserve can make; and if they burn the bills turned in, there is no inflationary consequence. Because people know this, no one ever worries about a shortage of any particular bill or believes that a bill stamped with one city's name will sell at a premium relative to that with another city's name. As a result there is never any reason to avoid any type of bill. Indeed most people never even look at the stamp indicating a city's name.

So why might there be any problem with fixing the exchange rates of any two fiat monies, such as those of two different countries? They are just bills with the names of countries instead of cities. If there is a monetary authority that can print any amount of one nation's currency for that of another, there is indeed no problem in maintaining a fixed exchange rate between the currencies of the two countries.

Unilateral Defense of the Exchange Rate

But does such an unlimited commitment exist between sovereign nations? Suppose that every holder of the British pound decided to turn in his pounds for German marks. Will the central bank of Germany (the Bundesbank) actually print all the marks necessary? Might they not be afraid that the United Kingdom would later decide to reimpose foreign currency controls that would send all those marks back to Germany in an inflationary tidal wave?[9]

How can the fixed exchange rate be supported without the full cooperation of foreign central banks? Is there another manner in which a government can keep its promise to exchange foreign currency for the domestic currency at a fixed exchange rate? One option is a government commitment to tax its citizens to acquire goods that may be sold in order to purchase the foreign currency demanded.[10]

[9] See Exercise 4.2.

[10] Another option is to dedicate a stockpile of storable goods like gold as reserves for the defense of the currency. Government stockpiles, of course, do not materialize out of thin air; they come from an earlier taxation of the people or an earlier decision not to distribute the stocks among the people. Interest-bearing assets may also function as reserves, as we will see in Chapter 10.

If such a commitment is believed and no foreign currency controls are imposed, there will be little incentive for anyone to turn in one form of money for the other. Both currencies can be used and held in either country (because of the absence of foreign currency controls) and neither loses value relative to the other (because of the fixed exchange rate). The two "national" currencies function essentially as two denominations of a single internationally accepted currency. People will be indifferent between the two types of currencies. Thus, if the commitment is believed, the government may never be obliged to actually tax its citizens or spend its stockpile.

To be believable in all circumstances, the government commitment to tax must be large enough to acquire enough goods to redeem all its money that might be turned in to it – all of that nation's money in the hands of those who are free to exchange one currency for another. This quantity could be quite large.[11] One must ask if it is believable that the government would actually tax its citizens to defend a fixed exchange rate in the circumstance in which a large number of people are trying to exchange the domestic currency for another.

Consider our two-country model economy with no foreign currency controls and no cooperation between central banks. The government of country *a* pledges to tax the old in order to defend a fixed exchange rate. (The tax is levied on the old because they are the citizens who will lose if the nation's money loses value.) Because of the absence of foreign currency controls we will assume that some of each country's currency is held by the old of each country. Recall that the world market for currency is given by

$$v_t^a M_t^a + v_t^b M_t^b = N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right),$$

or

$$\bar{e} v_t^b M_t^a + v_t^b M_t^b = N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right). \tag{4.15}$$

where $\bar{e}$ denotes the fixed exchange rate.

Now suppose that the entire world arbitrarily decides to exchange a large part of its holdings of country *a* money for country *b* money. If the government honors these requests, M_t^a falls, increasing the value of all currency by reducing the world's nominal stock of currency. (Note that M_t^b does not rise as a consequence of this action because no additional country *b* money is printed. The government of country *a* purchases the country *b* money from people currently holding it.) This increases the wealth of all holders of this money, whatever their citizenship.

[11] Restricting our attention to currency, we find an extreme example in the United States, whose currency is used worldwide in official and unofficial transactions. Porter and Judson (1996) estimate that two-thirds of the stock of U.S. currency is held abroad.

Where does this wealth come from? The government of country a is obliged under its pledge to tax its old citizens in order to acquire the foreign currency demanded by those turning in country a money. Thus the reduction of country a money comes from the taxation of the old citizens of country a. In this way the taxpayers of a alone pay for an increase in the value of money that benefits moneyholders in all countries. The net effect of the policy is therefore to transfer wealth from citizens of a to citizens of b. Although the people of a may want a fixed exchange rate, they will be made worse off if the government must actually tax them to defend the currency.

To better understand the differences between cooperative stabilization versus unilateral defense of the exchange rate, let us consider a specific example. Suppose countries a and b are identical. In each country, the population of every generation is 100 ($N_t^a = N_t^b = 100$), and each young person wants real money balances worth 10 goods. This implies that aggregate real money balances in each country are

$$N_t^a\left(y^a - c_{1,t}^a\right) = N_t^b\left(y^b - c_{1,t}^b\right) = (100)(10) = 1{,}000.$$

Also assume that the total fiat money stock of country a is \$800 and that of country b is £600. We assume that there are no foreign currency controls in effect and that each money is held in both countries. In particular we assume that the fiat money stocks are equally dispersed among the initial old of both countries. Because there are 100 individuals born in each generation, there are 200 initial old people across the two countries. This implies that each member of the initial old holds \$4 (=\$800/200) and £3 (=£600/200), regardless of citizenship. Finally, assume that the exchange rate is fixed at $\bar{e} = 1/2$; \$1 trades for £0.5.

From the world money market-clearing condition (Equation 4.13), we can find the value of each country's fiat money in a stationary equilibrium.

$$\begin{gathered}
\bar{e}v_t^b M_t^a + v_t^b M_t^b = N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right) \\
\left(\frac{1}{2}\right)v_t^b(800) + v_t^b(600) = 1{,}000 + 1{,}000 \\
1{,}000v_t^b = 2{,}000 \\
v_t^b = 2.
\end{gathered}$$

Because the exchange rate is fixed at $1/2$, we can derive the value of country a money:

$$v_t^a = \bar{e}v_t^b = \frac{1}{2}(2) = 1.$$

The consumption by each old person in both countries is equal to the real value of that person's total money holdings. In other words,

$$c_2^a = c_2^b = v_t^a(4) + v_t^b(3) = (1)(4) + (2)(3) = 10 \text{ goods.}$$

Now suppose that every member of the initial old of both countries decides to cut their real balances of country a money in half. Each member of the initial old therefore turns in \$2 to the monetary authority of country a in order to acquire country b money. Assume that the monetary authority of country b has agreed to cooperate by printing as much of its currency as demanded. This is an example of cooperative stabilization. Because the exchange rate is fixed at $1/2$, this means that country b must print £0.5 for every dollar turned in by the old, or £1 per old person. At the end of the currency exchange, the stock of dollars has shrunk by \$400 and the stock of pounds has grown by £200. In this situation, the total fiat money stocks of each country have become \$400 and £800.

As we did earlier, when we solve the world money market-clearing condition for the value of country b money, we find that its value is unchanged:

$$\begin{gathered}
\bar{e}v_t^b M_t^a + v_t^b M_t^b = N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right) \\
\left(\frac{1}{2}\right)v_t^b(400) + v_t^b(800) = 1{,}000 + 1{,}000 \\
1{,}000v_t^b = 2{,}000 \\
v_t^b = 2.
\end{gathered}$$

With the exchange rate fixed at $1/2$, we see that v_t^a is still equal to 1. The consumption of each old person is equal to

$$c_2^a = c_2^b = v_t^a(2) + v_t^b(4) = (1)(2) + (2)(4) = 10 \text{ goods.}$$

We see that the consumption by each old person is unchanged under a policy of cooperative stabilization. Each is unaffected in real terms by holding fewer dollars and more pounds.

Now let us see how the results differ when cooperative stabilization is absent and country a attempts a unilateral defense of the exchange rate. Suppose country b refuses to print fiat money to accommodate the desires of the old to trade in their dollars for pounds. Assume that the government of a decides to honor its pledge to exchange currency through an equal tax on every old citizen of its country. To do this, the government of country a must raise tax revenue sufficient to honor its pledge to provide all the country b money demanded. With 200 individuals across the two countries exchanging \$2 for pounds at the exchange rate of $1/2$, the total number of pounds that must be acquired by country a is equal to (200)\$2$\bar{e}$ = (200)£1 = £200.

The real value of the tax on the old is v_t^b(£200) goods. Because country a can tax only its own citizens, each old person of country a will be required to pay a tax of

$$\frac{200v_t^b}{100} = 2v_t^b.$$

To completely see the impact of this tax, we need to determine the new value of country b money. Since each of 200 old people has turned in \$2 of his initial holding of \$4, the total fiat money stock of country a has fallen to

$$M_t^a = \left(N_t^a + N_t^b\right)\$2 = (100 + 100)\$2 = \$400,$$

which is half its previous level. However, unlike the case of cooperative stabilization, the fiat money stock of country b is unchanged, because country b refuses to print additional money. Country b's total fiat money stock remains at £600. Using the world money market-clearing condition, we find that the value of country b money is

$$\begin{aligned}
\bar{e}v_t^b M_t^a + v_t^b M_t^b &= N_t^a\left(y^a - c_{1,t}^a\right) + N_t^b\left(y^b - c_{1,t}^b\right)\\
\left(\frac{1}{2}\right)v_t^b(400) + v_t^b(600) &= 1{,}000 + 1{,}000\\
800v_t^b &= 2{,}000\\
v_t^b &= 2.5.
\end{aligned}$$

Given the fixed exchange rate of 1/2, the value of country a money rises to 1.25. This verifies our earlier statement that the value of all currency will increase under a unilateral defense of the exchange rate. This stands in marked contrast to the cooperative stabilization solution, where we found that the value of each currency remained unchanged.

Now that we have found the value of country b money, we can see that to pay for the defense of its currency each old person of country a must be taxed

$$2v_t^b = 2(2.5) = 5 \text{ goods.}$$

Now let us see the effect of this policy on the consumption by each old person the two countries. After each person has traded \$2 to get £1, each person owns \$2 and £4 before taxes. The old of country b have no taxes to pay, permitting them to consume

$$c_2^b = v_t^a(2) + v_t^b(4) = (1.25)(2) + (2.5)(4) = 12.5 \text{ goods.}$$

The old of country b benefit from the unilateral defense policy because the real value of their currency holdings increases and they are not subject to a tax.

Because the old of country a must pay a tax to defend their currency, their consumption is equal to the real value of their money holdings less the tax:

$$c_2^a = v_t^a(2) + v_t^b(4) - (\text{tax}) = (1.25)(2) + (2.5)(4) - 5 = 7.5 \text{ goods.}$$

Because of the tax, the old of country a are made worse off by this policy of unilateral defense than they were under the cooperative stabilization policy, where their consumption was 10.

In effect, the unilateral defense policy has resulted in a transfer of 2.5 goods from each old person of country a to each old person of country b. Only the citizens of country a pay the tax that increases the value of all moneyholders, transferring wealth from the taxpayers of the country defending the exchange rate to the moneyholders of the other country.

Example 4.4 Consider two identical countries in our standard overlapping generations model. In each country the population of every generation is 100 and each young person wants money balances worth 18 goods. Each member of the initial old starts with \$3 of country a money and £3 of country b money regardless of citizenship. The exchange rate is fixed at 2: \$1 is worth £2. There are no foreign currency controls.

a. Find the value (measured in goods) of a unit of each country's money in a stationary equilibrium with unchanging money stocks. [Use the world money market-clearing condition (Equation 4.13).] What is the consumption of each old person? (Remember that each old person owns currency from both countries.)
b. Suppose each member of the initial old of both countries decides to cut his real balances of country a money by one-third (to 8 goods). He turns in \$1 to the monetary authority of country a in order to acquire more country b money. Assume that the monetary authority of country b has agreed to cooperate by printing as much of its currency as is demanded. What will be the total nominal stock of the each country's money? What will be the value of a unit of each country's money?
c. Suppose each member of the initial old turns in \$1 to the monetary authority of country a in order to acquire more country b money at the fixed exchange rate, but the monetary authority of country b refuses to cooperate. Assume that the government of country a decides to honor its pledge through an equal tax on every old citizen. What is the value of a unit of each country's money? How many goods must each old citizen of a be taxed? What is the consumption of the old in each country? (Remember to include the tax.) Who prefers this policy to the policy in part b? Who does not?
d. Suppose each member of the initial old decides to cut his real balances of country a money by one-third (to 8 goods), and the government decides not to intervene to fix the exchange rate. What is the new exchange rate? What is the consumption of each old person? Why doesn't the exchange rate change hurt anyone? Who prefers this policy to the policy in part c? Who does not?

Speculative Attacks on Currencies

A unilateral policy of fixing the exchange rate relies on the government's willingness to take actions (taxation) that make its citizens worse off. People may quite rationally question the government's commitment to follow through with a policy that hurts its own citizens. If the government lacks the will to take any of the actions it promises, people will rationally anticipate the promise of a fixed exchange rate as meaningless, returning the economy to an equilibrium of undetermined exchange rates.

It may be, however, that the government is prepared to take limited action to defend the exchange rate. Suppose, for example, that the government is willing to tax its citizens a limited amount – say F goods, where F is less than the total value of the country's stock of currency. The government is committed to exchange foreign for domestic currency until the tax bill of this policy has reached F goods, at which point it will abandon its efforts and let the exchange rate fluctuate. If fewer than F goods worth of domestic currency are turned in for exchange, the fixed exchange rate is maintained.

As pointed out by Salant and Henderson (1978) and Krugman (1979),[12] a limited government commitment may encourage speculative attacks in foreign currency markets in a way that does not occur when the government commitment is total. European countries (e.g., Britain and Sweden) in 1992–93 and East Asian countries (e.g., South Korea and Indonesia) in 1997 experienced recent waves of such speculative attacks.

Suppose you are holding some currency balances of a country with a limited commitment to defend its exchange rate. You decide to exchange that currency for the money of another country. If the commitment of that country is sufficient to meet the entire demand for foreign exchange, the exchange rate does not change and you are no worse off than before. If that country's commitment is too small to meet the entire demand for foreign exchange, its currency will fall in value, and the foreign currency will gain in value. If you are one of the lucky ones who arrive at the foreign exchange window before the government's limit is reached, you will profit by acquiring the currency that is about to gain in value. This is a can't-lose proposition for speculators: they either win or are not hurt.[13] Faced with these possible outcomes, every holder of that country's currency will want to rush to the foreign exchange window.[14]

This is also a can't-win policy for taxpayers. If a speculative attack occurs and the commitment proves sufficient, taxpayers have still been taxed to meet the attack.

[12] See also Krugman and Rotemberg (1991).
[13] Of course there is a chance of a loss if the currency they purchase is also subject to a speculative attack.
[14] Your only cost is the cost of making the transaction, which may be small for large traders of foreign currency.

If the commitment proves insufficient, the taxpayers are taxed and the currency depreciates nevertheless.

Inflationary Incentives

In the absence of foreign currency controls, the exchange rate is independent of national money stocks. Look again at the world money market-clearing condition (Equation 4.13). The value of a unit of money is determined by the total world money supply and not the money supply of the issuing nation. Therefore, an increase in the stock of one money reduces the value of all money and not just the money whose supply is expanded.

Let us examine this implication of the absence of foreign currency controls in the context of a national economy. If the monetary authority prints and distributes a large number of new \$1 bills, the real (goods) value of the \$1 bills will fall, but the real (goods) value of \$10 bills will also fall. Why? The two are perfect substitutes for each other and have a fixed rate of exchange. Therefore, if inflation reduces the real value of \$1 bills, it also reduces the real value of \$10 bills. Similarly, an increase in the number of Federal Reserve notes marked Boston will reduce the value of all Federal Reserve notes in every part of the United States.

For the same reasons, in an international economy of perfectly substitutable currencies trading at a fixed exchange rate, an increase in the stock of one country's money reduces the real value of all monies. This can occur because people, indifferent between currencies in the absence of foreign currency controls, treat the different currencies as simply different denominations of a world money free to circulate in all nations. Therefore, it does not matter which denomination (which nation's money) is increased during an expansion of the world stock of money; all currencies will fall in real value.

The expansion of one nation's money stock does not affect the real value of other currencies when foreign currency controls are in effect because the currencies are not perfect substitutes and do not trade at a fixed exchange rate. Citizens hold only their own country's money and thus are not affected by the inflation of some other country.

The transmission of inflation across countries in the absence of foreign currency controls raises an important political problem. We learned in the previous chapter that a nation that expands its money stock acquires revenue by lowering the value of the outstanding money stock, in effect by taxing moneyholders. In the presence of foreign currency controls, a nation willing to see the value of its money fall by half can raise seigniorage equal to half the value of the nation's money balances. In the absence of foreign currency controls, however, a nation willing to see the value of its money fall by half can raise seigniorage equal to half the value of the world's money balances; the seigniorage tax base is greatly expanded, and with it, seigniorage revenue. In this way, seigniorage can be collected from the citizens of other countries.

The political incentives created by a single world demand for currency in the absence of foreign currency controls are obvious. Imagine the inflation that would result if local governments were free to issue nationally accepted money. If any tax is favored by politicians, it's a tax collected in large part from people unable to vote against them in the next election. The same logic applies to the international case. Because every national government will wish to inflate to collect seigniorage from the citizens of other countries, a large inflation of the world's money stock will result.

This inflation can be prevented if governments are willing to agree to limit the rate at which each is allowed to expand its fiat money stock. Such coordination may work if each government wishes to rely on seigniorage to roughly the same degree. If, however, some countries want to rely on seigniorage far more than others, it may be difficult to reach an agreement.

If it is not possible to coordinate monetary policies, a nation can avoid the politically induced inflation only by separating the demand for its currency from that of the others – that is, by imposing foreign currency controls that prevent the currency of other countries from substituting for their own currency. Of course, under foreign currency controls, the citizens incur the costs of exchanging money whenever they trade with the people of another nation.

The Optimal International Monetary System

If political coordination were not a problem, what sort of international monetary system would we want? Let us answer this by first asking what monetary system we would want within a nation (a politically coordinated entity). Would we want each city and town to have its own money? If they did, imagine the costs of learning the current exchange rate and changing money as one makes purchases in different towns. The obvious way to eliminate these transaction costs and facilitate trade is to have only a single money for the entire nation. This is the monetary system selected by every nation.

How do these nations prevent their cities from issuing money to tax each other through seigniorage? They simply authorize a single national authority as the only issuer of fiat money. This means that the cities within any nation are not free to pursue distinct seigniorage policies. Nevertheless, cities seem willing to yield their sovereignty over monetary policy in order to reduce the costs of trade among themselves.

The same solution suggests itself to the world economy. The costs of conducting trade between nations would be minimized if a single money were used worldwide. People would not have to exchange their money to make purchases from other countries, nor would they have to fear that their money would suddenly lose its value because of an exchange rate change. A single world money would require that nations surrender their sovereignty over monetary policy to some trusted nation[15] or international institution, preferably with strict instructions about the rate of money expansion and the disposition of the revenue from seigniorage. This solution, in the form of a single European currency with a single European monetary authority, is currently being implemented by the European Economic Community (EEC). Adoption of the U.S. dollar, long established in Panama, has been considered in Argentina and is commonly discussed in other countries in the Americas.

If a world money is too much to ask, most of the benefits of a world money can be acquired if there are multiple currencies trading at fixed exchange rates with no currency controls. In this case, the different currencies function as different denominations of the world money supply, freely traded everywhere. This requires that monetary policies be coordinated to prevent speculative attacks and also to prevent the temptation for each national government to tax the entire world through inflation.

In actuality, political coordination may not be a trivial prerequisite. If countries considering a monetary union differ greatly over whether seigniorage is an important source of government revenue or over some other aspect of monetary policy, the gains to reducing the costs of international trade may not be worth foregoing an independent monetary policy. It follows that monetary union is more likely among countries with similar economies, like the countries of the EEC. Even these, however, differ significantly in their reliance on seigniorage. Seigniorage as a percentage of tax revenue ranged from 1 to 16 percent during the period 1973–1978.[16] Figure 3.7 of Chapter 3 presents data on seigniorage revenue for the countries of the EEC.

Summary

The goal of this chapter has been to understand the implications of different international monetary systems. This study is important in today's world, where countries

[15] At the close of World War II, the Western nations and others pledged at Bretton Woods, New Hampshire, to conduct their monetary policies in a way that maintained a fixed rate of exchange with the U.S. dollar, which pledged to redeem dollars in gold. Although this era is not strictly an example of a world money, its political implications are similar because the fixed exchange rates required that nations maintain rates of money creation compatible with that of the United States. The agreement broke down in the Vietnam War era when the United States effectively printed dollars to help finance the war. In 1971, President Nixon announced that the United States would no longer maintain a fixed exchange rate or its commitment to redeem dollars for gold.

[16] See Fischer (1982). See Canzoneri and Rogers (1990) for a discussion of the trade-off faced by the EEC.

are considering adopting widespread reforms of the systems under which they operate.

We first looked at a system in which currency controls are in effect. We found that the exchange rate between two countries' currencies is determined by the factors affecting the relative supply and demand of those currencies. With floating exchange rates and currency controls, the value of each country's money is unaffected by the other country's money supply or demand.

Currency controls require a potentially costly exchange of money in order to make a purchase in another country. These costs of the exchange of currencies can be avoided if people are free to hold and use any country's money. In this case, however, the exchange rate becomes indeterminate. This indeterminacy may give rise to erratic fluctuations in exchange rates, fluctuations that expose moneyholders to the risk of a sudden drop in the value of the money they hold.

The indeterminacy problem can be solved if countries agree to fix the exchange rate. When all monies are perfect substitutes, however, there exists the temptation to tax the citizens of other countries through seigniorage. This implies that countries fixing their exchange rate must also coordinate their monetary policies.

Exercises

1. Suppose that Germany (country a) and France (country b) do not have foreign currency controls in effect. The total demand for money is always 2,000 goods in Germany and 1,000 goods in France. The fiat money supplies are 100 marks in Germany and 300 francs in France.

- **a.** Find the value of each country's money if the exchange rate e_t (as defined in the text) is 3. Do the same if $e_t = 1$. Is one exchange rate more likely than the other? Explain.
- **b.** Suppose the exchange rate is 3 and that France triples its fiat money stock, whereas Germany prints no new money. How many goods will France gain in seigniorage? What fraction of this seigniorage comes from German citizens?

4.2 Consider two identical countries in our standard overlapping generations model. In each country the population of every generation is 100 and each young person wants money balances worth 10 goods. There are \$400 of country a money and £100 of country b money. The exchange rate is fixed at 1. There are no foreign currency controls and the monetary authorities do not cooperate. Each country is willing to raise up to 500 goods in taxes on their old citizens in order to defend the exchange rate.

- **a.** What is the value in goods of a dollar? Of a pound?
- **b.** Find the value of a dollar if people abandon use of the pound and the value of a pound if people abandon use of a dollar.
- **c.** To be free from a speculative attack, a country's commitment to defend the exchange rate must be sufficient to purchase all its currency if it is offered for foreign exchange.

Which of these two countries is subject to a speculative attack? (*Hint:* In answering you will need to use your answers to part b, not to part a.)

4.3 Consider two identical countries, a and b, in our standard overlapping generations model. In each country the population of every generation is 200 and each young person wants money balances worth 50 goods. Assume that the money of country a is the only currency that currently circulates in the two countries. There is \$800 of country a money split equally among the initial old of both countries.

a. Find the value of a country a dollar and the consumption of the initial old.

b. Suppose country b issues its own money, giving £10 to each of the initial old of country b. To ensure a demand for this currency, country b imposes foreign exchange controls. Find the value of a pound and the value of a dollar. Find the consumption of the initial old in country a and in country b. Who has been made better off by this policy switch?

Chapter 5

Price Surprises

TO THIS POINT we have examined only anticipated increases in the fiat money stock. In this chapter we examine the effects of monetary surprises – unanticipated fluctuations in the fiat money stock – on output, in particular. As we do so, we also study the more general question of how data correlations resulting from policy surprises may mislead naive policy makers about the effects of the sustained implementation of their policies.

The Data

The Phillips Curve

In 1958, A.W. Phillips discovered a significant statistical link between inflation and unemployment for the United Kingdom over a century.[1] Subsequent work uncovered the same correlation for many other economies. Although it was not understood why such a correlation existed, this discovery excited many in the economics profession by suggesting that there may be an exploitable trade-off between inflation and unemployment – that by increasing inflation, the government might achieve lower unemployment and greater output. The apparent inverse relationship between inflation and unemployment rates that existed in the United States data between 1948 and 1969 is illustrated in Figure 5.1.

In the next decades, many governments tried to use monetary policy to stimulate the economy. Suddenly, the Phillips curve, a stable relation over a century, disappeared. Inflation occurred with no gains in output or employment. The disappearance of the stable relationship between the inflation rate and the unemployment

[1] Actually, Phillips investigated a relationship between *wages* and the unemployment rate. Although the statistical correlation between the inflation rate and the unemployment rate bears Phillips' name, Fisher (1926) originally pointed out such a relationship.

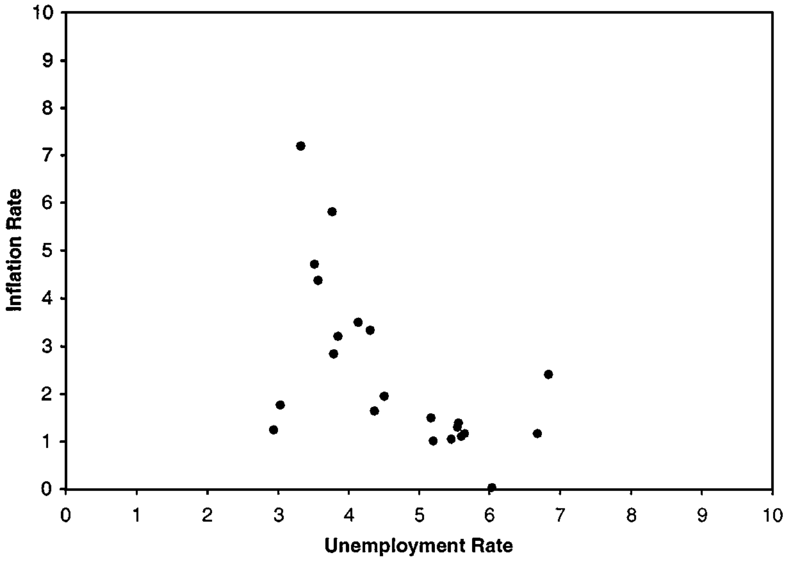


Figure 5.1. The Phillips curve (1948–1969). Before the 1970s, there appeared to be a stable inverse relationship between the inflation rate and the unemployment rate, often referred to as the Phillips curve. *Source:* The Federal Reserve Bank of St. Louis FRED database (http://www.stls.frb.org/fred/index.html).

rate becomes obvious when we look at U.S. data on these variables for the period from 1970 to the present, as in Figure 5.2.

What happened? Did some malevolent god, in order to frustrate the progress of humanity, suddenly alter the "laws" of economics at the very moment we discovered the way to end recessions?

Cross-Country Comparisons

Comparisons across countries add to the puzzle. Lucas (1973), for example, found that, if anything, inflation rates are on average higher in countries with lower average real growth rates as shown in Figure 5.3. How can these seemingly contradictory correlations come from a single world?

Expectations and the Neutrality of Money

In "Expectations and the Neutrality of Money," Lucas (1972) addressed this puzzle, proposing a model economy consistent with

- a positive short-run correlation between inflation and output,
- the disappearance of that correlation when policy makers attempt to exploit it, and
- a negative correlation between long-run inflation and output across countries.

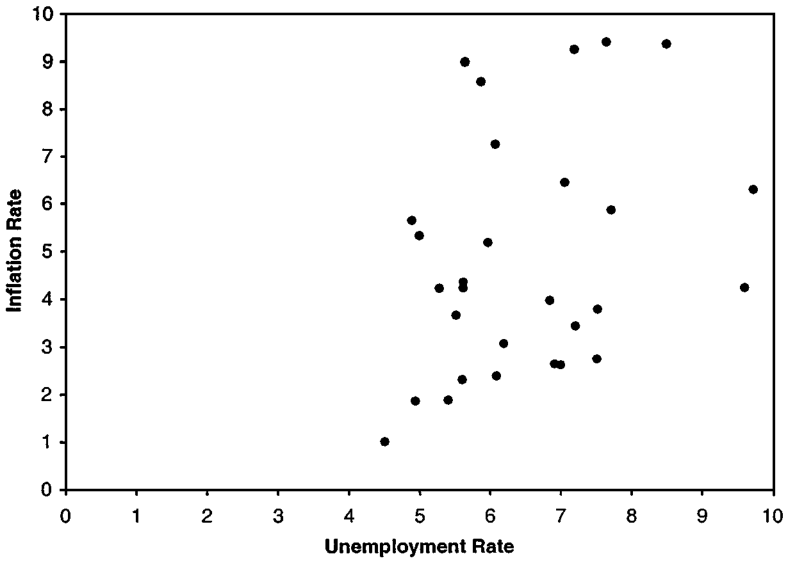


Figure 5.2. The Phillips curve (1970–present). Data on the unemployment rate and the inflation rate from the period after the 1960s display no apparent relationship between these two variables. *Source:* The Federal Reserve Bank of St. Louis FRED database (http://www.stls.frb.org/fred/index.html).

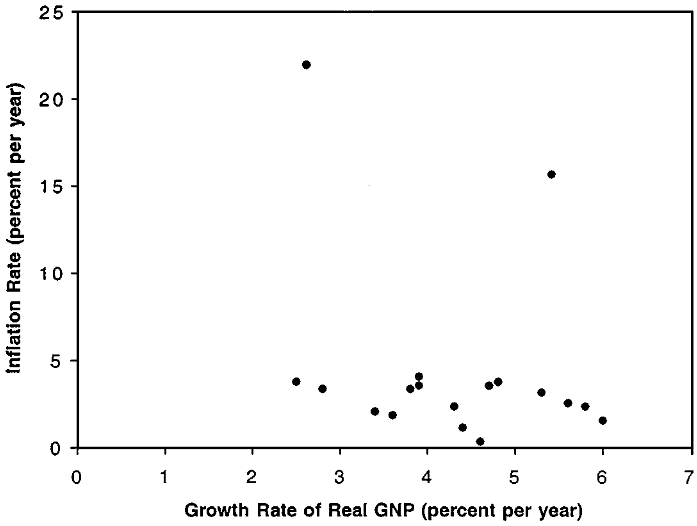


Figure 5.3. Inflation rate versus real output across countries. Data on inflation rates and real output demonstrate the weak tendency for average inflation to be high in countries with low average growth rates of real output. *Source:* Lucas (1973).

With this model as an illustration, Lucas revolutionized the methods of modern macroeconomic theory and practice.

The Lucas Model

For his model, Lucas adopted the standard overlapping generations model of money, adding the assumption that individuals live on two spatially separated islands. The total population across the two islands is constant over time. Half of the old individuals in any period live on each of the islands. The old are randomly distributed across the two islands, independently of where they lived when young. The young, however, are distributed unequally across the islands, with two-thirds of the young living on one island (and one-third on the other), in our simplified version of the original model.[2] In any single period, each island has an equal chance of having the large population of young. The outcome of this random assignment of population in any period has no effect on the outcome in any other period.

The stock of fiat money grows according to the rule $M_t = z_t M_{t-1}$. As in Chapter 3, increases in the fiat money stock are effected through lump-sum subsidies to each old person in every period t worth $a_t = [1 - (1/z_t)](v_t M_t/N)$ units of the consumption good.[3]

Informational assumptions are critical to individual behavior in this model. In any period, the young can directly observe neither the number of young people on their island nor the size of the subsidies to the old. The nominal stock of fiat money balances is known with a delay of one period. The price of goods on an island is observed but only by the people on that island. No communication between islands is possible within a period.

Although individuals are assumed to be unable to observe directly the realization of a variable of importance, the population of young people on their island, we do not assume that these people are stupid or irrational. They are assumed to know the possible outcomes they face and the probability of each outcome. They are free to infer whatever they can from the price they observe. We assume they make the most correct inference possible given the explicitly specified limits on what they can observe. The assumption that people understand the probabilities of outcomes important to their welfare was introduced by Muth (1961) as **rational expectations.**

While working with the overlapping generations model in this chapter, we will reinterpret an individual's problem to better reflect the difference between market and nonmarket goods. People are endowed when young with y units of time, which can be used in leisure, c_1, or as labor. The young work (give up leisure)

[2] We draw some of our exposition from a similarly simplified version of the Lucas model presented by Wallace (1980).

[3] Lucas (1972) assumes subsidies proportional to an individual's balances of fiat money.

to produce goods to sell to the old. We will let $l_t^i = l(p_t^i)$ represent the choice of labor by an individual born in period t for a given price of goods p_t^i on island i. Each unit of labor produces one unit of goods, implying that $l(p_t^i)$ also represents the individual's production of goods. Note that the amount of labor supplied by an individual depends on the price the individual receives on the goods produced. The individual's budget constraint when young in period t on island i can now be written as

$$c_{1,t}^i + l_t^i = c_{1,t}^i + v_t^i m_t^i = y. \tag{5.1}$$

The notation is that used in previous chapters, but with i superscripts to denote the island on which the individual was born. A young individual's holdings of fiat money (in units of the consumption good $v_t^i m_t^i$) is equal to the amount of goods that individual produces and sells on the market l_t^i. This represents the individual's real demand for fiat money. These holdings of fiat money, along with the lump-sum government transfer, will serve to finance consumption when old. In terms of notation, the budget constraint of an old person in period $t+1$ may be represented by

$$c_{2,t+1}^{i,j} = v_{t+1}^j m_t^i + a_{t+1} = \left[\frac{v_{t+1}^j}{v_t^i}\right] l_t^i + a_{t+1} = \left[\frac{p_t^i}{p_{t+1}^j}\right] l_t^i + a_{t+1}. \tag{5.2}$$

Note second-period consumption depends on the island i where the individual is born and on the island j where the individual is randomly assigned when old. People choose their work effort l_t^i to maximize their expected utility for a given local price, p_t^i. Preferences are restricted to the case in which the young will choose to work more the greater the rate of return to their work. For a given future price of goods, the greater the current price of goods, the greater the rate of return to labor p_t^i / p_{t+1}^j. Therefore, we are assuming that an increase in the current price of goods, *other things equal*, will induce the young to work more.[4] In the words of standard microeconomic theory, the *substitution effect* of an increase in price (a high relative price of goods encourages output) is assumed to dominate the *wealth*, or *income*, *effect* (a high relative price of goods makes people wealthier and thus more desirous of reducing work in order to consume more leisure).

Nonrandom Inflation

Let us start by examining the behavior of individuals when the money stock grows at a fixed rate $z_t = z$ in all periods. In this case rational individuals can easily determine the current money stock by multiplying last period's money stock, which they are assumed to know, by z.

[4] See Lucas (1972) for the exact restrictions assumed on preferences.

Now we examine the market-clearing condition on an island with N^i young people. In period t, each young person's demand for fiat money is $l_t^i = l(p_t^i) = v_t^i m_t^i$ goods. Because there are N^i young people on island i, the total demand for fiat money is $N^i l(p_t^i)$. Because the old people are equally distributed across islands regardless of their island of birth, half the fiat money stock is brought to each island. Equating the total real supply of fiat money in period t, $v_t^i(M_t/2)$, we obtain the following condition clearing the market of fiat money for goods

$$N^i l\big(p_t^i\big) = v_t^i \frac{M_t}{2}. \tag{5.3}$$

Because the value of fiat money v_t^i is equal to the inverse of the price level p_t^i, we can rewrite Equation 5.3 as

$$N^i l\big(p_t^i\big) = \frac{M_t/2}{p_t^i}. \tag{5.4}$$

N^i is either $(1/3)N$ or $(2/3)N$, depending on whether island i has a small or large number of young people, respectively. Rearranging Equation 5.4, we find that

$$p_t^i = \frac{M_t/2}{N^i l\big(p_t^i\big)}. \tag{5.5}$$

Because the population of the young on each island is the only random variable, the market-clearing condition implicitly expresses the price level as a function of the population of the young (N^i). Therefore, observing the price of goods p_t^i allows all of the young to infer the number of the young on their island. Letting p_t^A and p_t^B denote the price of goods when the population is small $[N^A = (1/3)N]$ and large $[N^B = (2/3)N]$, respectively, we find from Equation 5.5 that on island A

$$p_t^A = \frac{M_t/2}{N^A l\big(p_t^A\big)} = \frac{M_t/2}{\frac{1}{3}N l\big(p_t^A\big)}, \tag{5.6}$$

and on island B

$$p_t^B = \frac{M_t/2}{N^B l\big(p_t^B\big)} = \frac{M_t/2}{\frac{2}{3}N l\big(p_t^B\big)}. \tag{5.7}$$

We can see that $p_t^A > p_t^B$, revealing that the price of goods is high when the population is low. The price of goods is driven up by the scarcity of young people producing goods. (We present a proof that $p_t^A > p_t^B$ in the appendix of this chapter.) Because the price of goods in the next period is independent of the price of goods this period, the greater the price this period, the greater the rate of return to producing goods, p_t^i/p_{t+1}^j. In sum, when the population on an island is low, people want to work more because the price of their goods and thus the rate of return on their labor is greater.

Put another way, those young people on an island with plenty of young people face a relatively low demand for their product; there are many young people available to produce for the old. A low price of goods results. Those young people on an island with few young people face a relatively high demand for their product; there are few young people available to produce for the old. A high price of goods results.

Our assumption that the substitution effect dominates the wealth effect ensures that the young respond to favorable rates of return by working more. This means that when there are few young people to produce for the old, each young person produces more; where there are many young people, each produces less. Of course, because there is always one island with $(2/3)N$ people and another with $(1/3)N$ young people, aggregate output does not depend on which of the islands has the larger number of young.

Prices here do the job we expect of them in market economies. They signal the true state of the world so that people can choose the quantity of their output that maximizes their well-being, given their true situation.

Will the young react to high prices in the same way if they know the high prices are caused by a once-and-for-all higher level of the fiat money stock? No. Look at the rate of return to work when the money stock is higher in both this period and the next.

$$\frac{v_{t+1}^j}{v_t^i} = \frac{p_t^i}{p_{t+1}^j} = \frac{\frac{M_t/2}{N^i l(p_t^i)}}{\frac{M_{t+1}/2}{N^j l\left(p_{t+1}^j\right)}} = \frac{N^j l\left(p_{t+1}^j\right)}{N^i l\left(p_t^i\right)} \frac{M_t}{M_{t+1}}. \tag{5.8}$$

A permanent increase in the money stock raises both M_t and M_{t+1} by the same portion and so fails to affect the relative price of goods in this period and the next. Therefore, a high current price caused by a permanent increase in the money stock does not at all affect the rate of return to labor and thus the desire to work. As we saw in Chapters 1 and 3, money is neutral in this economy.

What is the effect of anticipated inflation on work? Is money superneutral? No. Look again at Equation 5.8, this time with $M_{t+1} = zM_t$. As z increases, $M_t/M_{t+1} = M_t/zM_t = 1/z$ decreases, and the rate of return to work falls, discouraging work because the money balances earned from labor are taxed by the expansion of the money stock.[5] The decline in work effort as z increases translates into lower output.

[5] Lucas (1972) assumed subsidies proportional to an individual's money balances. In this case, output is unaffected by rate of expansion z of the fiat money stock.

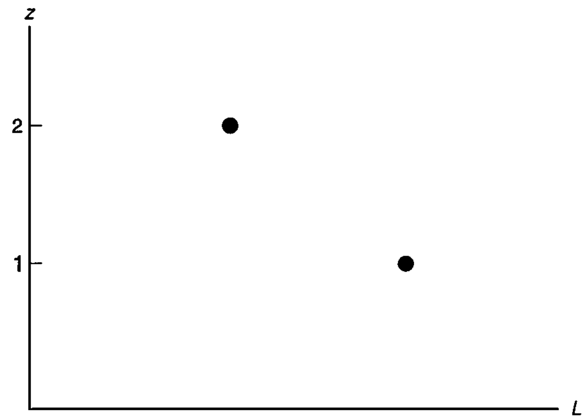


Figure 5.4. Inflation and output across economies in the Lucas model. This figure illustrates the output predicted by the Lucas model for two economies, one with a high rate of expansion of the fiat money stock and one with a low rate.

Let us now construct a graph comparing output as a function of the (steady) rate of expansion z of the fiat money stock.

Figure 5.4 shows a negative correlation between inflation and output, the opposite of the Phillips curve correlation. [Not exactly having unemployment rates in this model, we use total labor supplied (denoted by L in the diagram), or equivalently aggregate output, which we expect to be negatively correlated with the unemployment rate.] It is important to keep in mind that Figure 5.4 represents a *cross section*, a comparison of two distinct economies, each with a different fixed inflation rate. In this way Figure 5.4 is better compared with Lucas's (1973) study of the correlation of average inflation and output across countries, which, like Figure 5.4, shows a negative correlation between inflation and output.

In contrast, the Phillips curve was a *time-series* comparison of inflation and unemployment in different periods of the same economy. Therefore, to judge whether our model is also consistent with the Phillips curve, we must introduce variations in the inflation rate over time.

Random Monetary Policy

Now let us consider a single two-island economy with the following random monetary policy:

$$M_t = \begin{cases} M_{t-1} & \text{with probability } \theta \quad\quad (z_t = 1) \\ 2M_{t-1} & \text{with probability } 1-\theta \; (z_t = 2) \end{cases}. \tag{5.9}$$

The realization of monetary policy (the realized value of z_t) is kept secret from the young until all purchases have occurred – i.e., individuals do not learn M_t until period t is over.

As before, in order to determine their preferred work effort, the young wish to know whether they live with many or few other young people. Prices are the only thing directly observable by the young. Can they still deduce the population of young on the island by observing prices as they were able to do in the case in which z was nonrandom? Look again at the market clearing condition (Equation 5.3):

$$N^i l(p_t^i) = v_t^i (M_t/2), \tag{5.10}$$

or

$$p_t^i = \frac{M_t/2}{N^i l(p_t^i)} = \frac{z_t(M_{t-1}/2)}{N^i l(p_t^i)}. \tag{5.11}$$

Because both the island population N^i and the money stock are unknown to individuals, it is no longer always possible to infer the number of young just by looking at the price of goods. A high price, for example, may result from either a low population of young workers or a high fiat money stock. The distinction is important to the young. If the high price comes from a small number of young people, all of the young will want to work hard because they anticipate a good average return to their labor. On the other hand, if the high price comes from an increase in the fiat money stock, there is no reason to work especially hard. A high current money stock does not affect the anticipated rates of return to money and labor because it does not affect expectations of the future rate of money printing M_{t+1}/M_t; the monetary shocks are independent over time ("serially uncorrelated").

Is there anything about N^i that the young can learn from the price of goods? In our simplified version of the model with two possible population sizes and two possible rates of money printing, there are four possible states of the world represented by the various combinations of young people on the island and the realized value of z. Making use of Equation 5.11, let us look at what happens to the price level in each of those four cases.

Note from Table 5.1 that (for a given l) $p_t^a < p_t^b = p_t^c < p_t^d$. Therefore two of the four possible prices are unique: each can have occurred in only one particular combination of events. The price p_t^a can occur only when the money stock is small and the population is large, and the price p_t^d can occur only when the money stock is large and the population is small.

Therefore, if the young observe the price p_t^d, they can infer that the population on their island must be small. This implies that on average they can expect a good return to work, which encourages them to work hard, supplying l_t^d units of labor. Note that the price p_t^d is observed only when the fiat money stock is large ($z_t = 2$).

Table 5.1. *The four possible prices when the money stock is random*

	Number of Young People	
Growth Rate of Fiat Money Stock	$\frac{2}{3}N$	$\frac{1}{3}N$
$z_t = 1$	$p_t^a = \dfrac{M_{t-1}/2}{\frac{2}{3}Nl\left(p_t^a\right)}$	$p_t^b = \dfrac{M_{t-1}/2}{\frac{1}{3}Nl\left(p_t^b\right)}$
$z_t = 2$	$p_t^c = \dfrac{2(M_{t-1}/2)}{\frac{2}{3}Nl\left(p_t^c\right)}$	$p_t^d = \dfrac{2(M_{t-1}/2)}{\frac{1}{3}Nl\left(p_t^d\right)}$

Note: With a random money stock and population, there are four possible values for the price of output, only two of which are unique. The low price p_t^a can occur only when the growth rate of money is low and the population is large. The high price p_t^d can occur only when the growth rate of money is large and the population is small. However, when the intermediate price $p_t^b = p_t^c$ is observed, individuals cannot infer the particular values of the population and the growth rate of the money.

Similarly, if the young observe the price p_t^a, they can infer that the population on their island must be large. This implies that on average they can expect a poor return to work, which encourages them to work little, supplying l_t^a units of labor. Note that the price p_t^a is observed only when the fiat money stock is small ($z_t = 1$).

What happens in cases b and c? In these two cases the young are unable to infer the number of young on their island. They cannot tell if they are on an island with a small number of young people and a small money stock (case b) or on an island with a large number of young people but also a large money stock (case c). Unable to infer anything about the number of young on their island, each young worker in this situation will produce l^*, less than he would if he knew the population to be small and more than if he knew the population to be large. This will result in an intermediate price level, p^*, which is higher than p^a and lower than p^d.

Note that this randomized monetary policy does not always increase output. Although in case c people produce more than they would have if they knew their actual situation, in case b they produce less, imagining that the price they see may signal an increase in the money stock instead of an increase in the demand for their product. This output behavior is summarized in Figure 5.5.

In an economy, there is always one island with a large population of young and another with a small population of young. Therefore, in periods when the money stock is large ($z_t = 2$), one island will be in case c and another will be in case d, and total output will be a weighted average of l^c and l^d. Similarly, in periods when the money stock is small ($z_t = 1$), one island will be in case a and another will be in case b, and total output will be a weighted average of l^a and l^b. A graph of total output L will look something like Figure 5.6. This results in a relationship similar

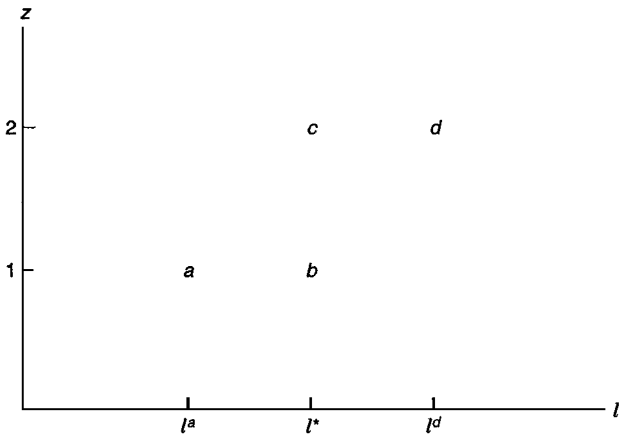


Figure 5.5. Inflation and output across islands. This figure illustrates the output predicted by the Lucas model for islands in a single economy with randomly high and low rates of expansion of the fiat money stock.

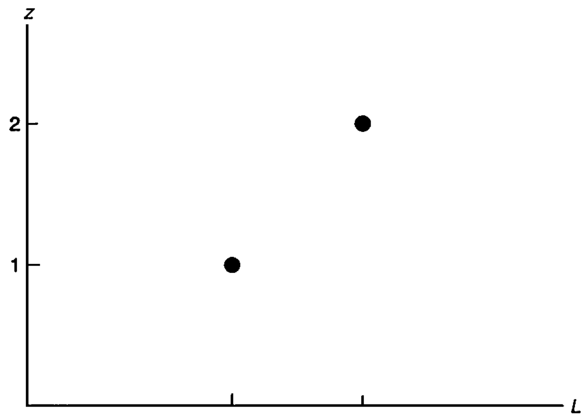


Figure 5.6. Inflation and aggregate output. This figure illustrates the total output predicted by the Lucas model in a single economy with randomly high and low rates of expansion of the fiat money stock.

to the Phillips curve. Output is high (unemployment is low), when the inflation rate is high (a high value of z).

The Lucas Critique of Econometric Policy Evaluation

Suppose that economists look at the time series plotted in Figure 5.6, the economy's experience over, say, 100 years, with no understanding of the model economy that generated it. The historical record clearly demonstrates that output is higher in the periods in which the fiat money stock is expanded. What might the economists be tempted to infer? . . . that money printing causes increased output?

The government controls the fiat money stock. Does this historical correlation suggest that the government can control aggregate output through its control of the money stock? If economists believe their government always to be more concerned with achieving high output than low rates of inflation, what policy might they then be tempted to propose? Print money to stimulate output in every period?

Will this policy work? What happens to output in this economy if the money stock is expanded in every period? We have already worked out the answer in Figure 5.4. Output is reduced, not increased. When the government inflates the fiat money stock in every period, people will no longer be confused about the state of the world. They know that cases a and b will no longer occur. Therefore, if they see the price p^c they will know there is a large number of people on their island, leading them to work less and create less output. Because they observe that the government always inflates, no longer will they imagine that they might be in case b, with a small number of young people and a small money stock. Inflation's boost to output under a random monetary policy no longer works because people are no longer fooled about the state of the world.

[Inflating almost always will not do the trick either. Suppose the government inflates in 99 of 100 periods. Although it is possible that people may find themselves in case a and b under this policy, anyone observing an intermediate price, p^*, knows that there is a 99 percent chance that this is caused by a high money stock and only a 1 percent chance that it is caused by a low number of young people. Although they may shade their labor decision a tiny bit to reflect the 1 percent chance of being in case b, young people observing p^* will base their labor reaction to the far likelier possibility that the price is almost surely the result of case c, a large population and a large stock of money.]

Our atheoretical economists have egg on their faces. They went to the monetary authorities with a well-intentioned policy designed to permanently stimulate output and wound up reducing it instead. A correlation stable for 100 years changed the very moment the government tried to exploit it. What went wrong? Why did the inflation/output curve change the sign of its slope?

The correlation of money and output, or indeed any set of variables, results from the reaction of decision makers to the environment they face. An important feature of this environment is government policies. In particular, the relation between money and output depends on the monetary policy being followed. When in this economy the policy changed from one of random inflation to one of steady inflation, the reactions of producers also changed.

A correlation between variables that is the result of the equilibrium interactions of an economy can be called a reduced-form correlation. In our example this would be represented by the slope of a line connecting the points in Figure 5.6. The "Lucas critique" points out that these reduced-form correlations are subject to change when the government changes its policies and thus the rules under which decision makers operate. The example we have studied is particularly startling in that when the government changes from a policy of random inflation to a policy of steady inflation, the correlation (slope) not only changes, it also changes sign – from positive to negative.

How then can we evaluate policies? We need some understanding about how people will react to the new policy: we need a theory. If we understand people's motives (preferences) and constraints (physical limitations, informational restrictions, government policies), we can predict how people will react to changes in a policy. Lucas's point is not that econometric policy evaluation is impossible, but that it cannot be done without a theory, an understanding of how the economy works. It is not sufficient just to look at the data. The correlations found in the data are subject to change when government policy changes.

Optimal Policy

What is the best policy? Should the government play dice with the economy? Let us look at the welfare consequences of a randomized monetary policy.

By randomizing the rate of expansion of the fiat money stock, the government creates confusion about the meaning of prices. In essence, isn't the government withholding information about the true state of the world? People are not always sure whether a price increase signals an increase in the demand for their product, in which case they benefit by producing more, or an increase in the money stock, in which case they will not make themselves better off by increasing production. The more often the government expands the money stock, the more people believe that any observed price increase is just the government playing with the money stock. It follows that a major cost of randomizing the money stock is that people fail to take advantage of actual increases in the demand for their output.

Even if the government could fool people, should it? Why should the government want to fool people into producing more than they would choose to produce if they knew their actual situation? A baseball pitcher or a soccer player will randomize the location of the ball to fool the batter or goalie, but these players are on opposing teams. Isn't the government on the same team as the public it represents? Is the proper goal of a government manipulation of output or the welfare of its citizens?

Summary

We began this chapter by noting the observed relationship between the unemployment rate and the inflation rate and the subsequent breakdown of that relationship. This chapter presented a model consistent with these observations – a simplified version of Lucas's 1972 model.

In this model, young people cannot directly observe a real variable important to their output decision, the number of other young people producing on the island on which they were born. We first considered a case of nonrandom inflation, where the monetary authority adheres to a fixed growth rate of the fiat money stock. In this case, agents could infer the number of young people producing on their island by observing the price of goods. An increase in inflation, which in this case is always known, lowers the rates of return on labor, discouraging work effort and lowering output. This is consistent with Lucas's observation that average inflation rates and output are negatively correlated across countries.

When we examined random inflation in this model, the relation of inflation to output dramatically changed, generating a Phillips curve. Random inflation complicated an individual's work effort decision because agents could no longer always infer the number of young people on their island by observing the price of the good. If, for example, a high price was caused by a small number of young people on the island, the young would want to work hard because they expect a high average return to their labor. On the other hand, if a high price is caused by an increase in the fiat money stock, there is no incentive to work hard. Because of the randomness of the money stock, prices are less informative about the true state of the world. This inability of individuals to determine the true state of the world causes them, at times, to work harder and produce more output than they would choose to do if they were able to determine their true situation. At other times, they mistakenly work less than they would choose to do if they were able to determine their true situation. When we observe this economy over time, we find that high inflation rates are associated with high levels of output (low unemployment rates) – the Phillips curve.

This relation between output and inflation depends crucially on the assumption of random inflation. A government attempting to exploit this relation by inflating in any systematic way will find that the positive correlation between inflation and output disappears.

The importance of the Lucas model lies not primarily in its explanation of the money/output correlation, as interesting as it may be. There are certainly other explanations, some of which we will study in later chapters, that may or may not do a better job of explaining that correlation.

Lucas's paper changed macroeconomics by demonstrating that the correlations among macroeconomic aggregates are subject to change when economic policy changes. This showed macroeconomists the pitfalls in evaluating policy by looking simply at correlations in the data, without a working theory of how people may react to policy changes. Those macroeconomists who open their eyes to Lucas's critique are thereafter compelled to fully specify the environment in which the economic agents studied make their decisions. It is with the Lucas critique in mind that this book endeavors to present only explicit models, specifying all our assumptions about people's preferences and constraints.

Exercises

5.1 Consider the following version of the model of this chapter. The number of young individuals born on island i in period t, N_t^i, is random according to the following specification:

$$N_t^i = \begin{cases} \frac{4}{5}N & \text{with probability } 0.5 \\ \frac{1}{5}N & \text{with probability } 0.5 \end{cases}.$$

Assume that the fiat money stock grows at the fixed rate $z_t = z$ in all periods.

a. Set up the budget constraints of the individuals when young and when old in terms of l_t^i. Also set up the goverment budget constraint and money market-clearing condition. Find the lifetime budget constraint (combine the budget constraints of the young and old by substituting for l_t^i).

b. On which island would you prefer to be born? Explain with reference to the rate of return to labor.

c. Show how the rate of return to labor and the individual's labor supply depend on the value of z.

For the following parts, assume that the growth rate of the fiat money stock z_t is random according to

$$z_t = \begin{cases} 1 & \text{with probability } \theta \\ 4 & \text{with probability } 1 - \theta \end{cases}.$$

The realization of z_t is kept secret from the young until all purchases of goods have occurred (individuals do not learn M_t until period t is over). Given these changes in assumptions, answer the following questions.

d. How many states of the world would the agents be able to observe if information about every variable were perfectly available? Describe those possible states.

e. How many states of the world are the agents able to distinguish when there is limited information (they do not know the value of z_t)?

f. Draw a graph of labor supply and the growth rate of the fiat money stock in each possible state of the world when there is limited information. What is the correlation observed between money creation and output?