Exercises 5^{th} Week

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Exercise 1

Political Science, Inc. (PSI) specializes in voter polls and surveys designed to keep political office seekers informed of their position in a race. Using telephone surveys, interviewers ask registered voters who they would vote for if the election were held that day. In a recent election campaign, PSI found that 220 registered voters, out of 500 contacted, favored a particular candidate. PSI wants to develop a 95% confidence interval estimate for the proportion of the population of registered voters that favors the candidate.

Exercise 2

Let X be a random variable with p.d.f.:

$$f(x|\theta) = \frac{exp(-(x-\theta))}{(1+exp(-(x-\theta)))^2} \qquad -\infty < x < \infty \quad -\infty < \theta < \infty$$

Use the pivotal method to verify that if $0 < \alpha_1 < 0.5$ and $0 < \alpha_2 < 0.5$, then

$$\left(X - log\left(\frac{1-\alpha_2}{\alpha_2}\right), X - log\left(\frac{\alpha_1}{1-\alpha_1}\right)\right)$$

is a confidence interval for θ with coverage probability $1-(\alpha_1+\alpha_2)$.

Exercise 3

Compare the asymptotic efficiency of method of moments estimator of parameter α of the Pareto distribution with maximum likelihood estimator (assume x_m known). Remind that a r.v. X with a Pareto distribution has the following density:

$$f(x;\alpha,x_\mathrm{m}) = \alpha\, x_\mathrm{m}^\alpha x^{-(\alpha+1)} \quad \text{for } x \geq x_m$$

Exercise 4

The life times of the neon lamps in the Uni Mail rooms can be modelled by an exponential distribution, i.e. we suppose that $(X_1, \ldots, X_n)$ are from the exponential distribution $Exp(\lambda)$ with probability distribution function:

$$f(x|\lambda) = \lambda exp(-\lambda x) I_{(0,\infty)}(x)$$

We would like to construct point and interval estimates of the median survival time:

1. Compute the median $Me(\lambda)$ of the model,
2. Find $\hat{Me}(\lambda)_{MLE}$: the maximum likelihood estimator for $Me(\lambda)$, the median of this distribution,
3. Give an approximate (asymptotic) confidence interval for $Me(\lambda)$ based on $\hat{Me}(\lambda)_{MLE}$,
4. Show that $2\lambda \sum_i X_i$ is a pivotal statistic and give the exact confidence interval for $Me(\lambda)$.
5. Suppose that for a random sample of $n = 100$ neon lamps you observe $\bar{x} = 5$, compare the exact and approximate confidence interval and discuss results found.

Exercise 5

To study the mean of a population variable X, $\mu = E(X)$, a simple random sample of size n is considered. Imagine that we do not trust the first and the last data, so we consider the following statistics:

$$\tilde{X} = \frac{1}{n-2} \sum_{j=2}^{n-1} X_j = \frac{X_2 + \ldots + X_{n-1}}{n-2}$$

Calculate the expectation and the variance of this statistic. Calculate the mean square error (MSE) and its limit when n tends to infinite. Study the consistency. Compare the previous error with that of the ordinary sample mean.