$p(\theta)$, it is enough to offer the Agent a menu of contracts. If the Agent announces that his type is θ, he will receive the allocation $q(\theta)$ and will pay the transfer $p(\theta)$.

Direct truthful mechanisms are very simple but rely on messages that are not explicit. In the example of the wine seller, one can hardly expect the buyer to come into the shop and declare "I am sophisticated" or "I am frugal." A second result sometimes called the *taxation principle* comes to our aid in showing that these mechanisms are equivalent to a nonlinear tariff $\tau(.)$ that lets the Agent choose an allocation q and pay a corresponding transfer $p = \tau(q)$. The proof of this principle again is simple. Let there be two types θ and θ' such that $q(\theta) = q(\theta')$; if $p(\theta)$ is larger than $p(\theta')$, then the Agent of type θ can pretend to be of type θ', and the mechanism will not be truthful. Therefore we must have $p(\theta) = p(\theta')$, and the function $\tau(.)$ is defined unambiguously by

$$\text{if} \quad q = q(\theta), \quad \text{then} \quad \tau(q) = p(\theta)$$

In our earlier example the wine seller only needs to offer the buyer two wine bottles that are differentiated by their quality and price. This is, of course, more realistic; although most retailers do not post a nonlinear tariff on their doors, they often use a system of rebates that approximates a nonlinear tariff.

2.2 A Discrete Model of Price Discrimination

In section 2.3, we will obtain the general solution for the standard adverse selection model with a continuous set of types. Here we learn first to derive the optimum in a simple two-type model by way of heavily graphical techniques and very simple arguments.

To simplify things, we will reuse the example of a wine seller who offers wines of different qualities (and at different prices) in order to segment a market in which consumers' tastes differ. This is therefore a model that exhibits both vertical differentiation and second-degree price discrimination.[8]

2.2.1 The Consumer

Let the Agent be a moderate drinker who plans to buy at most one bottle of wine within the period we study. His utility is $U = \theta q - t$, where q is the quality he buys and θ is a positive parameter that indexes his taste for quality. If he decides not to buy any wine, his utility is just 0.

Note that with this specification,

$$\forall \theta' > \theta, \quad u(q, \theta') - u(q, \theta) \quad \text{increases in } q$$

This is the discrete form of what I call the Spence-Mirrlees condition in section 2.3. For now, just note its economic significance: At any given quality level, the more sophisticated consumers are willing to pay more than the frugal consumers for the same increase in quality. This is what gives us the hope that we will be able to segment the market on quality.

There are two possible values for θ: $\theta_1 < \theta_2$; the prior probability that the Agent is of type 1 (or the proportion of types 1 in the population) is π. In the following, I will call "sophisticated" the consumers of type 2 and "frugal" the consumers of type 1.

2.2.2 The Seller

The Principal is a local monopolist in the wine market. He can produce wine of any quality $q \in (0, \infty)$; the production of a bottle of good quality q costs him $C(q)$. I will assume that C is twice differentiable and strictly convex, that $C'(0) = 0$ and $C'(\infty) = \infty$.

8. The classic reference for this model is Mussa-Rosen (1978), who use a continuous set of types.

The utility of the Principal is just the difference between his receipts and his costs, or $t - C(q)$.

2.2.3 The First-Best: Perfect Discrimination

If the producer can observe the type θ_i of the consumer, he will solve the following program:

$$\max_{q_i, t_i} (t_i - C(q_i))$$

$$\theta_i q_i - t_i \geq 0$$

The producer will therefore offer $q_i = q_i^*$ such that $C'(q_i^*) = \theta_i$ and $t_i^* = \theta_i q_i^*$ to the consumer of type θ_i, thus extracting all his surplus; the consumer will be left with zero utility.

Figure 2.1 represents the two first-best contracts in the plane (q, t). The two lines shown are the indifference lines corresponding to zero utility for the two types of Agent. The curves tangent to them are isoprofit curves, with equation $t = C(q) + K$. Their convexity is a consequence of our assumptions on the function C. Note that the utility of the Agent increases when going southeast, while the profit of the Principal increases when going northwest.

Both q_1^* and q_2^* are the "efficient qualities." Since $\theta_1 < \theta_2$ and C' is increasing, we get $q_2^* > q_1^*$, and the sophisticated consumer buys a higher quality wine than the frugal consumer. This type of discrimination, called first-degree price discrimination, is generally forbidden by the law, according to which the sale should be anonymous: You cannot refuse a consumer the same deal you prepared for another consumer.[9] However, we are interested in the case where the seller cannot observe directly the consumer's type. In this case perfect discrimination is infeasible no matter what is its legal status.

9. As we will see shortly, the sophisticated consumer envies the frugal consumer's deal.

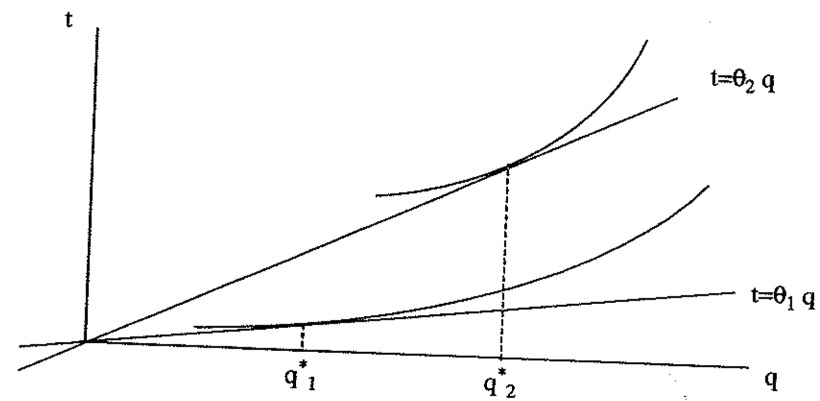

Figure 2.1
The first-best contracts

2.2.4 Imperfect Information

Now in the second-best situation information is asymmetric. The producer now only knows that the proportion of frugal consumers is π. If he proposes the first-best contracts (q_1^*, t_1^*), (q_2^*, t_2^*), the sophisticated consumers will not choose (q_2^*, t_2^*) but (q_1^*, t_1^*), since

$$\theta_2 q_1^* - t_1^* = (\theta_2 - \theta_1) q_1^* > 0 = \theta_2 q_2^* - t_2^*$$

The two types cannot be treated separately any more. Both will choose the low quality deal (q_1^*, t_1^*).

Of course, the producer can get higher profits by proposing (q_1^*, t_1^*) the point designated A in figure 2.2, since A will be chosen only by the sophisticates and only by them. Note that A is located on a higher isoprofit curve than (q_1^*, t_1^*), and therefore it gives a higher profit to the seller.

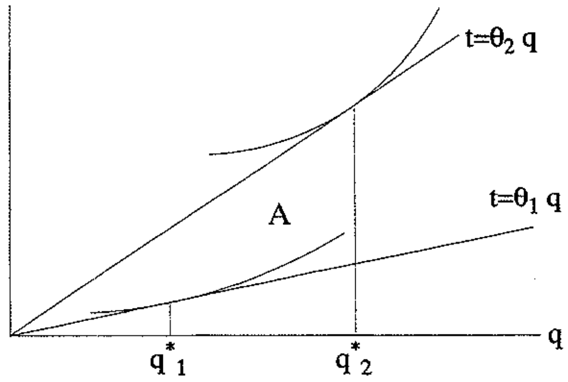

Figure 2.2
A potentially improving contract

A number of other contracts are better than A. Our interest is in the best pair of contracts (the second-best optimum). This is obtained by solving the following program:

$$\max_{t_1, q_1, t_2, q_2} \{\pi[t_1 - C(q_1)] + (1 - \pi)[t_2 - C(q_2)]\}$$

subject to

$$\begin{cases} \theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2 & (IC_1) \\ \theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 & (IC_2) \\ \theta_1 q_1 - t_1 \geq 0 & (IR_1) \\ \theta_2 q_2 - t_2 \geq 0 & (IR_2) \end{cases}$$

The constraints in this program are identified as follows:

- The two (IC) constraints are the *incentive compatibility* constraints; they state that each consumer prefers the contract that was designed for him.
- The two (IR) constraints are the *individual rationality*, or *participation* constraints; they guarantee that each type of consumer accepts his designated contract.

We will prove that at the optimum:

1. (IR_1) is active, so $t_1 = \theta_1 q_1$.
2. (IC_2) is active, whence

$t_2 - t_1 = \theta_2(q_2 - q_1)$.

3. $q_2 \geq q_1$.
4. (IC_1) and (IR_2) can be neglected.
5. Sophisticated consumers buy the efficient quality

$q_2 = q_2^*$.

Proofs We use (IC_2) to prove property 1:

$$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 \geq \theta_1 q_1 - t_1$$

since $q_1 \geq 0$ and $\theta_2 > \theta_1$. If (IR_1) was inactive, so would be (IR_2), and we could increase t_1 and t_2 by the same amount. This would increase the Principal's profit without any effect on incentive compatibility.

Property 2 is proved by assuming that (IC_2) is inactive. Then

$$\theta_2 q_2 - t_2 > \theta_2 q_1 - t_1 \geq \theta_1 q_1 - t_1 = 0$$

We can therefore augment t_2 without breaking incentive compatibility or the individual rationality constraint (IR_2). This obviously increases the Principal's profit, and therefore the original mechanism cannot be optimal.

To prove property 3, let us add (IC_1) and (IC_2). The transfers t_i cancel out, and we get

$$\theta_2(q_2 - q_1) \geq \theta_1(q_2 - q_1)$$

and

$$q_2 - q_1 \geq 0$$

since $\theta_2 > \theta_1$.

By property 4, the (IC_1) can be neglected, since (IC_2) is active. By property 3,

$$t_2 - t_1 = \theta_2(q_2 - q_1) \geq \theta_1(q_2 - q_1)$$

The proof of assertion 1 shows that (IR_2) can be neglected.

Finally, by property 5, we can prove that $C'(q_2) = \theta_2$. If $C'(q_2) < \theta_2$, for instance, let ε be a small positive number, and consider the new mechanism (q_1, t_1), $(q_2' = q_2 + \varepsilon, t_2' = t_2 + \varepsilon\theta_2)$. It is easily seen that

$$\theta_2 q_2' - t_2' = \theta_2 q_2 - t_2 \text{ and } \theta_1 q_2' - t_2' = \theta_1 q_2 - t_2 - \varepsilon(\theta_2 - \theta_1)$$

so the new mechanism satisfies all four constraints. Moreover

$$t_2' - C(q_2') \simeq t_2 - C(q_2) + \varepsilon(\theta_2 - C'(q_2))$$

This tells us that the new mechanism yields higher profits than the original one, which is absurd. We can prove in the same way that $C'(q_2) > \theta_2$ is impossible (just change the sign of ε).

It is an easy and useful exercise to obtain graphical proofs of these five points. The optimal pair of contracts appears to be located as shown in figure 2.3. (q_1, t_1) is on the zero utility indifference line of the Agent of type 1, and (q_2, t_2) is the tangency point between an isoprofit curve of the seller and the indifference line of the Agent of type 2 that goes through (q_1, t_1).

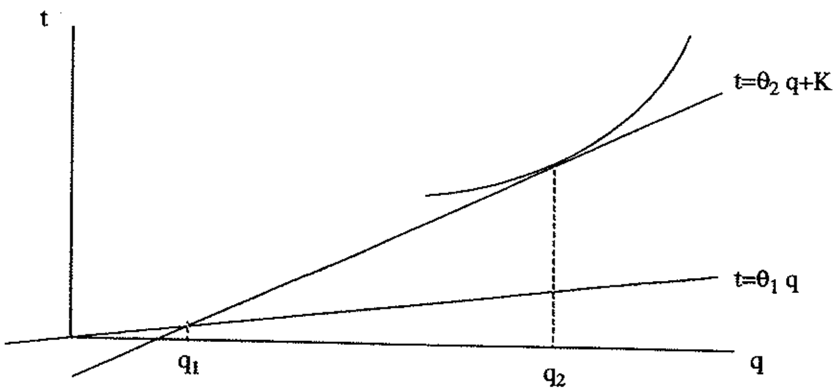

Figure 2.3
The second-best optimum

To fully characterize the optimal pair of contracts, we just have to let (q_1, t_1) in figure 2.3 slide on the line $t_1 = \theta_1 q_1$. Formally the optimum is obtained by replacing q_2 with q_2^* and expressing the values of t_1 and t_2 as functions of q_1, using

$$\begin{cases} t_1 = \theta_1 q_1 \\ t_2 - t_1 = \theta_2(q_2 - q_1) \end{cases}$$

This gives

$$\begin{cases} q_2 = q_2^* \\ t_1 = \theta_1 q_1 \\ t_2 = \theta_1 q_1 + \theta_2(q_2^* - q_1) \end{cases}$$

We can substitute these values in the expression of the Principal's profit and solve

$$\max_{q_1} \left(\pi(\theta_1 q_1 - C(q_1)) - (1 - \pi)(\theta_2 - \theta_1)q_1\right)$$

Note that the objective of this program consists of two terms. The first term is proportional to the social surplus[10] on type 1 and the second represents the effect on incentive constraints on the seller's objective. Dividing by π, we see that the Principal should maximize

$$(\theta_1 q_1 - C(q_1)) - \frac{1 - \pi}{\pi}(\theta_2 - \theta_1)q_1$$

which we can call the *virtual surplus*. We will see a similar formula in section 2.3. The difference between the social surplus and the virtual surplus comes from the fact that when the Principal increases q_1, he makes the type 1 package more alluring to type 2. To prevent type 2

10. The social surplus is the sum of the objectives of the Principal and the type 1 Agent. We do not have to worry about the social surplus derived from selling to Agent 2, since we know that we impl...

from choosing the contract designated for type 1, he must therefore reduce t_2, which decreases his own profits.

We finally get

$$C'(q_1) = \theta_1 - \frac{1-\pi}{\pi}(\theta_2 - \theta_1) < \theta_1$$

so that $q_1 < q_1^*$: the quality sold to the frugal consumers is subefficient.[11]

The optimal mechanism has five properties that are common to all discrete-type models and can usually be taken for granted, thus making the resolution of the model much easier:

- The highest type gets an efficient allocation.
- Each type but the lowest is indifferent between his contract and that of the immediately lower type.
- All types but the lowest type get a positive surplus: their *informational rent*, which increases with their type.
- All types but the highest type get a subefficient allocation.
- The lowest type gets zero surplus.

Informational rent is a central concept in adverse selection models. The Agent of type 2 gets it because he can always pretend his type is 1, consume quality q_1, pay the price t_1, and thus get utility

$$\theta_2 q_1 - t_1$$

which is positive. However, type 1 cannot gain anything by pretending to be type 2, since this nets him utility

$$\theta_1 q_2 - t_2$$

11. If the number of frugal consumers π is low, the formula will give a negative $C'(q_1)$. Then it is optimal for the seller to propose a single contract designed for the sophisticated consumers. A more general treatment should take this possibility into account from the start. Here this *exclusion* phenomenon can be prevented by assuming that π is high enough. We will see in section 3.2.6 that this is not possible when

which is negative. For n types of consumers $\theta_1 < \dots < \theta_n$, each type $\theta_2, \dots, \theta_n$ can get informational rent, and this rent will increase from θ_2 to θ_n. Only the lowest type, θ_1, will receive no rent.

Remark By the taxation principle, there is a nonlinear tariff that is equivalent to the optimal mechanism. It is simply

$$\begin{cases} t = t_1 & \text{if } q = q_1 \\ t = t_2 & \text{if } q = q_2 \\ t = \infty & \text{otherwise} \end{cases}$$

So the seller needs only to propose the two qualities that will segment the market.[12]

2.3 The Standard Model

The model we study in this section sums up reasonably well the general features of standard adverse selection models. It introduces a Principal and an Agent who exchange a vector of goods q and a monetary transfer p. The Agent has a characteristic θ that constitutes his private information. The utilities of both parties are given by

$$\begin{cases} W(q,t) & \text{for the Principal} \\ U(q,t,\theta) & \text{for the Agent of type } \theta \end{cases}$$

Note that we do not make the Principal's utility function depend on the type θ of the Agent. This is because the model involves "private values" as opposed to "common values." This distinction will be used again in chapter 3. When the contract is signed, the Agent knows his

12. Such an extremely nonlinear tariff is less reasonable when the variable q is a quantity index, as it is in the price discrimination problem studied by Maskin-Riley (1984). Then it is sometimes possible to implement the optimum mechanism by using a menu of linear tariffs. Rogerson (1987) proves that a necessary and sufficient condition is that the optimal nonlinear schedule $t = T(q)$ be convex