On Diamond-Dybvig (1983): A model of liquidity provision

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Theory of Banking

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Bryant (1980) and Diamond and Dybvig (1983)

• Individuals have a preference for liquidity because they are uncertain about the timing of their consumption needs hence they prefer to hold liquid assets.

• A liquid asset is an asset that can be readily converted into consumption without loss of value.

• Depository institutions are interpreted as pools that provide households with insurance against idiosyncratic "liquidity" shocks that affect their consumption needs.

Bryant (1980) and Diamond and Dybvig (1983)

Main Ingredients:

- maturity structure of bank's assets;

- a theory of liquidity preference;

- banks as intermediaries that provide insurance to depositors against unexpected liquidity needs;

- an explanation of bank runs on deposits.

Introduction

• We study a model in which consumers may have uncertain liquidity needs at the time in which they choose the composition of their portfolios.

• Consumers choose between two possible alternatives, a short-term activity and a long-term one.

• Every consumer must balance future liquidity needs, taking into account pros and cons of investing in the two activities.

Introduction

- We consider four reference set-ups:
  - Autarkic Economy
  - Central Planner Economy
  - Asset Market
  - Financial Intermediary

- We will characterize the optimal consumption allocations for each of the scenarios.

- We will then characterize the Pareto efficient allocation, and examine the relationship between each solution and Pareto optimal allocation.

The model/1

- Consider an economy which lasts three periods: $t = 0, 1, 2$

- The economic system is populated by an infinite number of consumers and a unique homogeneous consumption good.

- Each consumer has an initial endowment of 1 unit of the consumption good at $t = 0$ and nothing at $t = 1, 2$.

- Each consumer at $t = 0$ in uncertain on her type of preferences. She could be a *patient* or an *impatient* consumer.

- Uncertainty on individual preferences will realize at time $t = 1$.

- A patient individual obtains utility only from consumption at $t = 2$, while an impatient individual wants to consume only at $t = 1$.

- The (prior) probability that any consumer be impatient is λ

The model/2

There are two possible technologies that consumers can access:

• Short-term investment technology (Storage): one unit invested at $t = 0, 1$ delivers 1 unit after one period,

• Long-term investment technology (Investment): one unit invested at $t = 0$ delivers a real return $R > 1$ after two periods

• Early liquidation of the long-term technology is costly. We assume that it gives zero return.

The model/3

- Given the available technologies, each consumer has to decide how to compose her portfolio using ST and LT activities.

- Denote
  - y: the share of the unitary endowment invested in storage,
  - x: the share of the unitary endowment invested long-term;
  - c_t^τ: consumption choice at time t by an type-τ individual, with $\tau = \{i, p\}$ for an impatient and a patient consumer, respectively.

- Since x and y represent shares of a unit-valued portfolio, it must be verified that

$$x + y \leq 1$$

The individual choice problem: the case of autarky

Individual choice problem: autarky/1

- To examine the consumer's choice problem, we use a backward induction technique.

- Given the portfolio choice (y, x). At time $t = 1$, when each consumer knows her type τ, she will have to decide how much to consume at $t = 1, 2$, i.e. $c_t^\tau(y, x)$ will have to be determined.

- Given the assumptions made on preferences, $c_1^p(y, x) = c_2^i(y, x) = 0$ and

$$c_1^i(y, x) = y$$
$$c_2^p(y, x) = y + Rx$$

Individual choice problem: autarky/1

• In $t = 0$, before knowing her type, the consumer will choose the portfolio composition, i.e. y and x, to maximize her expected utility, given the optimal consumption choices in the following periods $t = 1, 2$.

• The consumer's problem at $t = 0$ can be written as:

$$\begin{aligned} \max_{y,x} \quad & \lambda u(c_1^i(y,x)) + (1-\lambda)u(c_2^p(y,x)) \\ \text{s.t.} \quad & y + x \leqslant 1 \\ & c_1^i(y,x) = y \\ & c_2^p(y,x) = y + Rx \end{aligned}$$

Individual choice problem: autarky/2

• The portfolio constraint is binding. Hence, we can express $x = 1 - y$ and rewrite the consumer's problem as follows:

$$\max_{y \in [0,1]} \lambda u(y) + (1-\lambda) u(y + (1-y)R)$$

• Let $y^* > 0$ be the solution of this problem. If $y^* < 1$, then it must be true that:

$$\lambda u'(y^*) + (1-\lambda) u'(y^* + (1-y^*)R)(1-R) = 0 \qquad (\star)$$

Individual choice problem: autarky/3

• An interior solution to the consumer's maximization problem must satisfy:

$$\lambda u'(y^*) = (1-\lambda)u'(y^* + (1-y^*)R)(R-1) \qquad (\star)$$

• The optimal portfolio composition is such that the marginal benefit of increasing the share of the short-term technology in the consumer's portfolio must be equal to the marginal cost of doing so, which depends on the relative difference of returns between the ST and the LT technologies $(1-R)$.

Individual choice problem: autarky/4

- Let us represent the individual problem directly in terms of consumption, c_1^i and c_2^p.
- Recall that

$$c_1^i = y$$

$$c_2^p = y + (1 - y)R$$

- Consider two particular consumption vectors:

$$(\bar{c}_1^i, \bar{c}_2^p) = (0, R)$$

which represents the case of a portfolio with only LT investment; and

$$(\tilde{c}_1^i, \tilde{c}_2^p) = (1, 1)$$

which represents the case of investing only in ST technology, rolling over every period.

Individual choice problem: autarky/5

- Take the consumption vectors: $(\bar{c}_1^i, \bar{c}_2^p) = (0, R) \qquad \& \qquad (\tilde{c}_1^i, \tilde{c}_2^p) = (1, 1)$

and compute the expression of the line passing through these points. We obtain:

$$c_2^p - R = (1 - R)c_1^i$$

which we rewrite as:

$$(R - 1)c_1^i + c_2^p = R$$

This is the individual budget constraint.

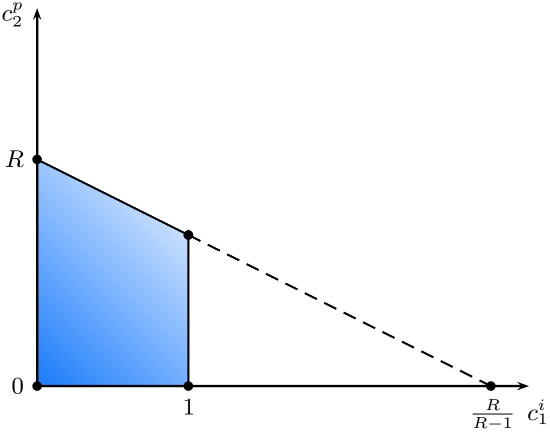
Individual choice problem: autarky/6

• However, not all allocations on the budget constraint are feasible in this problem, since there is an additional constraint to be satisfied: $c_1^i \leqslant 1$.

• The set of feasible allocations is then described by:

$$(R-1)c_1^i + c_2^p = R \quad \& \quad c_1^i \leqslant 1$$

Individual choice problem: autarky/7



Individual choice problem: autarky/8

- Let us reconsider now the consumer's problem at $t = 0$:

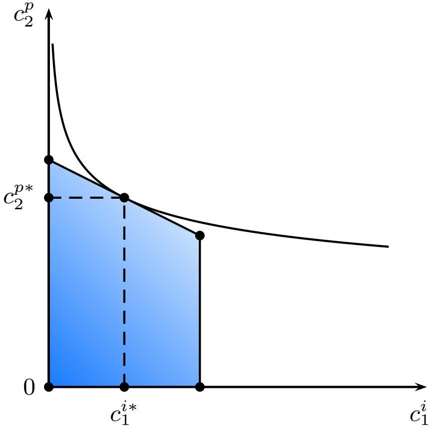
$$
\begin{aligned}
\max_{c_1^i, c_2^p} \quad & \lambda u(c_1^i) + (1-\lambda) u(c_2^p) \\
\text{s.t.} \quad & (R-1)c_1^i + c_2^p = R \\
& c_1^i \leqslant 1
\end{aligned}
$$

- Let (c_1^{i*}, c_2^{p*}) be a candidate solution for that problem with $0 < c_1^{i*} < 1$.

- As usual, at the solution the marginal rate of substitution is equal to the absolute value of the slope of the budget constraint, i.e.:

$$\frac{\lambda u'(c_1^{i*})}{(1-\lambda) u'(c_2^{p*})} = (R-1)$$

Individual choice with no trade/9



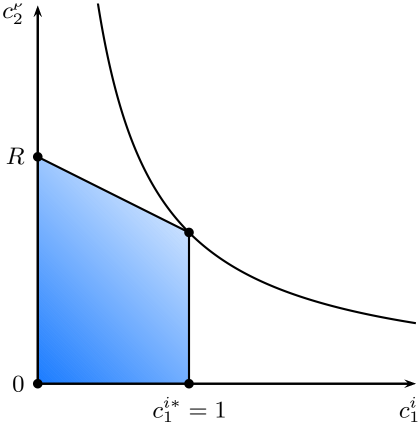
Individual choice problem: autarky/10

- Assume that $u(c) = \log c$, it is easy to show that in this case:

$$c_1^{i*} = \frac{\lambda R}{(R-1)} \quad \& \quad c_2^{p*} = (1-\lambda)R$$

- If $\lambda < (R-1)/R$, the problem exhibits an interior solution of the form, $\left(c_1^{i*}, c_2^{p*}\right)$.
- If $\lambda > (R-1)/R$, we have a corner solution in which $c_1^{i*} = 1$ and $c_2^{p*} = 1$.

Individual choice problem: autarky/11



The central planner problem

Efficient allocation/1

• Consider now the case of a (benevolent) central planner, who wants to find the optimal mix of ST and LT investment to maximize the expected utility of the representative consumer.

• The planner chooses the amount of per capita investment in the ST technology y and in the LT technology x.

• The planner also chooses the per capita level of consumption for both types of consumers at $t = 1, 2$.

• Let us assume that the planner can observe the consumers' types.

• The solution of the planner's problem is the Pareto (or first best) efficient one.

Efficient allocation/2

- The planners' problem can be characterized by the following set of feasibility constraints:

$$
\begin{aligned}
x + y &\leqslant 1 && \text{(A)} \\
\lambda c_1 &\leqslant y && \text{(B)} \\
(1-\lambda)c_2 &\leqslant (y - \lambda c_1) + xR && \text{(C)}
\end{aligned}
$$

- At the planner's optimum, constraint (A) is binding.

- We can hence rewrite:

$$
\begin{aligned}
\lambda c_1 + (1-\lambda)c_2 &\leqslant R + y(1-R) && \text{(A}'\text{)} \\
\lambda c_1 &\leqslant y && \text{(B}'\text{)}
\end{aligned}
$$

Efficient allocation/3

$$\lambda c_1 \leqslant y \qquad (\mathrm{B}')$$

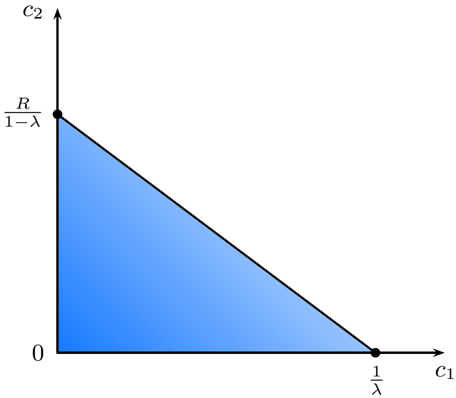
- For $c_1 > 0$ and constraint (B') to be satisfied, it must be $y > 0$. Constraint (B') is also binding.

- Hence, we can finally rewrite the planner's feasibility constraint as:

$$\lambda R c_1 + (1-\lambda) c_2 \leqslant R \qquad (\mathrm{A}^*)$$

- This identifies all the consumption points below a straight line with slope $-\frac{\lambda R}{1-\lambda}$ in the consumption space (c_1, c_2).

Efficient allocation/4



Efficient allocation/5

- The planners' problem therefore can be rewritten as follows:

$$\max_{c_1,c_2} \quad \lambda u(c_1) + (1-\lambda)u(c_2)$$

$$\text{s.t.} \quad \lambda R c_1 + (1-\lambda)c_2 \leqslant R$$

- An interior solution $(c_1^*, c_2^*) > 0$ is characterized by the necessary condition:

$$\frac{\lambda u'\left(c_1^*\right)}{\left(1-\lambda\right)u'\left(c_2^*\right)} = \frac{\lambda R}{\left(1-\lambda\right)},$$

which simplifies to:

$$\frac{u'\left(c_1^*\right)}{u'\left(c_2^*\right)} = R$$

Efficient allocation/6

- The optimal consumption allocation satisfies :

$$u'\left(c_1^*\right) = Ru'\left(c_2^*\right)$$

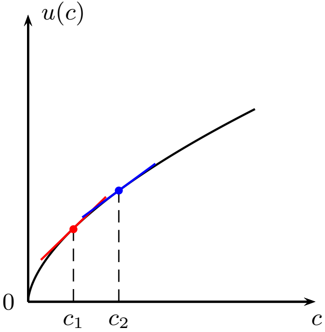
- Since $R > 1$,

$$u'(c_1^*) = u'(c_2^*)R > u'(c_2^*),$$

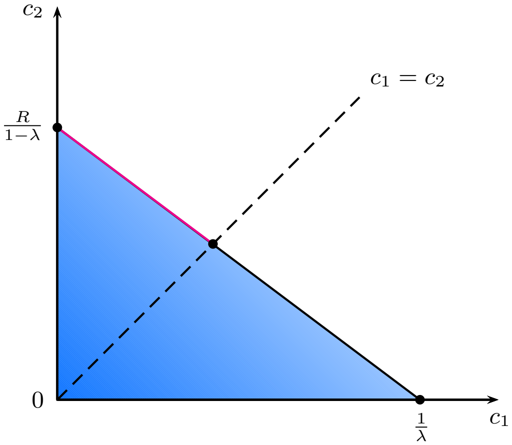
hence, under assumption of a concave utility function, $c_1^* < c_2^*$.

- To get the efficient allocation (c_1^*, c_2^*), in $t = 0$ the central planner will choose optimal portfolio shares $y^* = \lambda c_1^*$ and $x^* = 1 - y^*$.

Concave utility function



Efficient allocation/7



Incentive compatibility and efficiency/8

• Observe that in this framework the optimal allocation is incentive compatible.

• Early consumers have no opportunity to mis-represent themselves as late consumers, since they only value consumption at $t = 1$ they would be worse off if they had to wait until $t = 2$ to consume c_2.

• Late consumers could have an incentive to mis-represent their type, pretending to be impatient players. They could demand to consume c_1 at $t = 1$, then store the value and consume it at $t = 2$ using storage.

• The allocation for a patient consumer is incentive compatible if $c_1^* \leq c_2^*$, which is satisfied by the first-best solution.

• Hence, the first-best allocation is also second-best efficient.

An economy with an asset market

Asset market/1

• Let us study now how the presence of an asset market may change the consumer's decisions.

• Having observed the realization of her preferences, assume that every consumer has access to an asset/financial market at $t = 1$.

• The asset allows to trade the good at $t = 1$ against a promise to receive some quantity of the consumption good at $t = 2$, by either buying or selling it.

• Let $P > 0$ denote the price at $t = 1$ for the asset, which by convention yields one unit of consumption good at $t = 2$.

• Assume that every consumer takes this price as given (price taker).

Asset market/2

• Consider a portfolio of assets $(y, x) \neq (0, 0)$ chosen at $t = 0$, with y representing the share of per capita investment in the ST technology, and x the share invested in the LT technology.

• At $t = 1$ the impatient consumer will want to sell the amount x, she invested in the LT technology in $t = 0$ and will be able to obtain the consumption level:

$$c_1^i = y + Px$$

• If $x > 0$, then $c_1^i > y$.

Asset market/3

• If $x > 0$, then $c_1^i > y$.

• Relative to the autarkic case, if there is an asset market at $t = 1$ the impatient consumers can gain by selling the share of their portfolios they invested in the LT technology at $t = 0$.

Asset market/4

• At $t = 1$ the patient consumers will want to buy long-term assets selling the amount y invested in the short term technology at $t = 0$.

• If y is the amount of short-term investment in the portfolio of a patient consumer, the quantity of asset that she will be able to buy is given by y/P. The consumption level which a patient consumer can attain at $t = 2$ is thus equal to:

$$c_2^p = \left(\frac{y}{P} + x\right) R$$

• Observe that a patient consumer will hold a portfolio constituted only by LT assets, at $t = 1$, if $\frac{R}{P} \geq 1 \rightarrow \; P \leqslant R$.

Asset market/5

- We can rewrite c_2^p as follows:

$$c_2^p = (y + Px)\,\frac{R}{P}\,,$$

and therefore, :

$$c_2^p = \left(\frac{R}{P}\right) c_1^i \tag{$\star$}$$

Asset market/6

• At $t = 0$ the consumer will choose the optimal composition of her portfolio of assets to maximize her expected utility:

$$\max_{y,x} \quad \lambda u(c_1^i) + (1-\lambda)u(c_2^p)$$

$$\text{s.t.} \quad y + x \leqslant 1$$

with the condition

$$c_2^p = \left(\frac{R}{P}\right) c_1^i \tag{$\star$}$$

Maximizing the expected utility is equivalent to solve the following problem:

$$\max_{y,x} y + Px \quad \text{s.t.} \quad y + x \leqslant 1$$

Asset market/7

- Which price P is compatible with the equilibrium on the asset market?

- Consider $P > 1$: long-term asset dominates the short-term investment.

- All consumers in $t = 0$ will choose a specialized portfolio with only long-term activities $(x = 1, y = 0)$.

- In $t = 1$ the impatient consumers would like to sell their x. However, there is no demand for those securities because the patient consumers do not have any short-term asset to trade.

- Hence, the price will fall to $P = 0$, a contradiction.

• Hence, an equilibrium on the asset market can only feature $P \leq 1$.

• Consider the case $P < 1$: short-term investment dominates the long-term asset.

• All consumers will choose in $t = 0$ a portfolio specialized in short-term activities $(x = 0, y = 1)$.

• In $t = 1$ the patient consumers would like to buy the asset to increase the value of their long-term investment. But, there is no supply for these assets, since the impatient consumers do not have any long-term asset in their portfolio.

• Hence, the price will rise and hit the bound $P = R$, a contradiction.

• There cannot exist an equilibrium in the asset market if $P \neq 1$.

Asset market/9

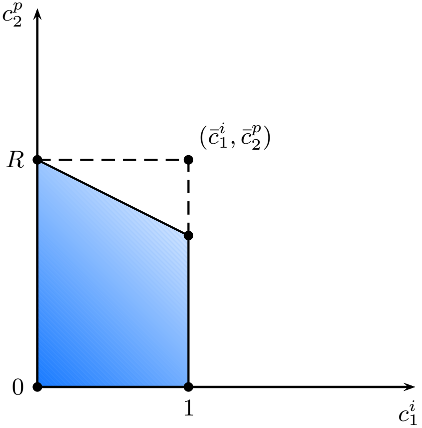
- For the asset market to be in equilibrium, it must be that $P = 1$.
- If this is the case, then

$$\bar{c}_1^i = x + y = 1 \quad \& \quad \bar{c}_2^p = (x + y)R = R$$

- The portfolio choice of the agent is immaterial.
- The shares of the portfolio which support this consumption allocation can be:

$$\bar{y} = \lambda \quad \& \quad \bar{x} = (1 - \lambda)$$

Asset market/10



Efficiency of the asset market allocation/1

- Is the consumption allocation that the consumers obtain with an asset market efficient?

- Recall that the optimal consumption allocation with an asset market is $(\bar{c}_1, \bar{c}_2) = (1, R)$, supported by a portfolio $(\bar{y}, \bar{x}) = (\lambda, 1 - \lambda)$ chosen at $t = 0$.

- First: we have to check that this consumption allocation is feasible, i.e. satisfies the set of constraints of the planner's problem.

- Since $\lambda \bar{c}_1 = \bar{y}$, the constraint (B') is satisfied.

- It is also true that the constraint (A') is satisfied.

- Since $\bar{c}_2 > \bar{c}_1$, the consumption bundle is feasible.

In general, it is not efficient, though.

Efficiency of the asset market allocation/2

• The optimal consumption bundle with asset markets satisfies the relationship:

$$\bar{c}_2 = \bar{c}_1 R$$

which does not depend on consumer's preferences.

• The efficient (Pareto optimal) bundle satisfies the following condition:

$$\frac{u'\left(c_1^*\right)}{u'\left(c_2^*\right)} = R$$

in which marginal utilities appear. These two conditions are not the same, in general. Hence, such an asset market is not efficient.

• The efficiency of the asset market allocation depends on the preferences of the consumer.

The solution with financial intermediation

Financial intermediaries and liquidity/1

• Let us see how the presence of a financial intermediary can accomplish with the service of liquidity provision.

• Here, financial intermediaries only propose deposit contracts to consumers.

• Let us consider a contract such that, the consumer deposits her unitary endowment at the bank at $t = 0$. The bank commits to deliver a predetermined amount $\hat{c}_1$ if the consumer withdraws her deposit at $t = 1$ (i.e. is impatient) and a predetermined $\hat{c}_2$ if the consumer waits and access the bank at $t = 2$ (i.e. shows up as a patient).

• In exchange for the promise to deliver consumption according to the realized consumer's preferences, the bank collects the initial endowments and invests the resources of the economy in the short-term and in the long-term technologies, appropriately.

Financial intermediaries and liquidity/2

• If the expected profits of the bank are bound to zero, the problem of the bank is exactly the same as that of the central planner.

• It follows that the bank will offer to the consumer a contract such that

$$\hat{c}_1 = c_1^*$$

$$\hat{c}_2 = c_2^*$$

• The consumption allocation that the bank can guarantee to the consumer is efficient.

• The consumers will accept the contract and will hence obtain an efficient insurance against their uncertain demand for liquidity.