Problem Set 2

Theory of Banking - Academic Year 2016-17

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Exercise 1

Consider an agency relationship in which the principal contracts the agent, whose effort determines the outcomes. The principal owns a stochastic production technologies that yields a monetary outcome $x \in \{60.000, 30.000\}$ euros. Assume that uncertainty is represented by three states of nature $\{\epsilon_1, \epsilon_2, \epsilon_3\}$. The agent can choose between two effort levels, and her decision affects the probability distribution over the states. The results are shown in Table 1.

		states of nature		
		ϵ_1	ϵ_2	ϵ_3
efforts	e=6	60.000	60.000	30.000
	e=4	30.000	60.000	30.000

Table 1: Effort Levels and States of Nature

The principal and the agent both believe that the probability of each state is 1/3. The objective functions of the principal and the agent are, respectively:

$$B(x, w) = x - w$$

$$U(w, e) = \sqrt{w} - C(e)$$

where x is the monetary outcome of the relationship, w is the monetary wage that the agent receives and $C(e) = e^2$ is the cost of effort. Assume that the agent only accepts the contract if he obtains an expected utility level of at least 114 (his reservation utility level).

(a) What is the attitude of each player towards risk?

(b) What is the wage that a principal proposes to an agent that exerts effort $e = 6$? Determine the agent's and the principal's utility at this contract.

(c) What is the wage that a principal proposes to an agent that exerts effort $e = 4$? Determine the agent's and the principal's utility at this contract.

(d) What would the optimal level of effort and the optimal wage be in a situation in which the principal can contract on agent's effort and on wage?

Exercise

Consider an insurance economy, with two states of nature ($s \in \{s_1, s_2\}$, with s_1 = no accident and s_2 = accident) and complete and perfect information. Let the consumer be risk-averse and the insurer be risk-neutral. The consumer is endowed with $W > 0$ units of the consumption good, but with a certain probability he will have an accident and suffer a loss $0 < L < W$.
The agent can be of two types:

i) *good type* (type G), the probability of an accident is π_G;

ii) *bad type* (type B), the probability of an accident is π_B.

Assume that $\pi_G < \pi_B$. Consider the Cartesian space defined over the contingent resources available in case of no accident (x_1) -x axis- and in case of accident (x_2) -y axis-.lLet the agents' utility function be $\sqrt{x}$.

1) Write down the expected utility function for each type of consumer.

2) Characterize the indifference curves of each consumer' type in the Cartesian space (x_1, x_2). Are they increasing? Are they convex?

3) Compare the marginal rate of substitution (in absolute value) along the indifference curves for the different types, what can you conclude?

Consider now a perfectly competitive insurance market. Insurance is provided to the consumer by means of contracts which specify a premium, α, that the consumer pays when signing the contract, and a total reimbursement β that the consumer receives in case of accident.

4) Write down the expected profits of the insurance company for each type of consumer.

5) Write down the contingent consumptions (x_1, x_2) for the consumer in case he decides to buy insurance.

6) Characterize the insurer's iso-profit lines in the Cartesian space (x_1, x_2). Are they increasing? Are they convex?

7) Represent the iso-profit function that corresponds to insurer's zero expected profits (ZPL) in the Cartesian space (x_1, x_2), for every type of consumer. Show that these iso-profit functions pass through the endowment point $(W, W - L)$ and have a slope equal to $-(1 - \pi_i)/\pi_i$ with $i = G, B$.

8) Show that if the riskiness of the consumer is public information, every consumer's type achieves full insurance and the insurance premium is increasing with the consumer's riskiness.