Problem Set 3

Theory of Banking - Academic Year 2016-17

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Exercise 1. *Consider an economy á la Diamond and Dybvig (1983), that lasts for three periods ($t = 0, 1, 2$) and with a unique good. The economy is populated by a continuum of ex-ante identical agents (of mass 1) and each agent is endowed, at time $t = 0$, with one unit of the good. There are two types of agents: patient and impatient agents. Indeed each agent will know, at $t = 1$, whether he has to consume at $t = 1$ (impatient agent) or at $t = 2$ (patient agent). A non random proportion π_1 of the agents will be impatient and the complementary proportion, $\pi_2 = 1 - \pi_1$, will be patient. The agent's type is not observable.*
The agents' utility function is:

$$\begin{cases} \sqrt{C_1} & \text{for impatient agents} \\ \rho\sqrt{C_2} & \text{for patient agents} \end{cases}$$

Moreover, in the economy, any agent has access to both a storage technology and a long-run technology:

- *the storage technology yields a zero net interest per unit of good stored;*

- *otherwise any agent can invest, at $t = 0$, in a long-run technology. This technology yields $R > 1$ units of the good, at $t = 2$, per unit invested, whereas if the investment is prematurely liquidated at $t = 1$ yields only $L < 1$ unit of the good per unit invested.*

(a) *Write down the constrained maximization problem of the consumer who lives in autarky.*

(b) *Characterize the consumption allocation that guarantees the highest utility to the consumer in autarky.*

(c) *Compute the (first best) efficient allocation and explain how it depends on ρR.*

(d) *Assume $\rho = 1$, can the optimal allocation be achieved in a market economy as the one described in class? (The asset traded in the economy is a zero coupon bond, that yields one unit of the $t = 2$ consumption good.)*

(e) *Is such a market allocation optimal? Explain.*

(f) *Is there any value of ρ such that the optimal allocation can be achieved in a market economy? Explain.*

Exercise 2. *In the same framework of the exercise above, assume now $\rho R > 1$.*

1) *Can the optimal allocation be implemented by a financial intermediary by means of a deposit contract? Explain how the optimal contract will work. Characterize the bank's balance sheet.*

2) *Now consider an economy in which at $t = 0$ the financial intermediary is financed entirely by consumers with all their endowments and each consumer receives a share of the bank. At $t = 0$, the financial intermediary announces the dividend per share that will be distributed at $t = 1$. At $t = 1$ the financial intermediary distributes a dividend to every shareholder, $d > 0$ (after each consumer has known his type and before he has to consume). At $t = 2$ the financial intermediary is liquidated and all its cash-flow is distributed to shareholders on a pro-rata basis. At $t = 1$, after dividends have been distributed and before agents have to consume, a (secondary) market for dividends opens in which consumers can buy or sell dividends of the bank.*
Can the optimal allocation be achieved in such an equity economy for the intermediary? Explain the result.

3) *Assume $\rho R < 1$, then:*

   3.1) *Determine the optimal deposit contract for a financial intermediary.*