– Business cycle view of bank runs–

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Bank runs and the business cycle/1

• Diamond-Dybvig (1983) is not the only theory on bank runs.

• Deterioration of the quality of the assets in banks' balance sheets due to adverse macro-economic conditions.

• If macro-economic situation gets worse, the value of the assets in a bank's balance sheets diminishes and the risk of insolvency increases.

• Adverse macroeconomic conditions, such as a recession in the economic cycle, can convince consumers/depositors that the intermediary's solvency perspective is getting worse, and this could potentially lead to a bank run.

• These bank runs would not be the result of a coordination failure on an inefficient equilibrium, as in the Diamond Dybvig model, on the contrary they would be easily predictable since their relationship with general economic conditions.

Bank runs and the business cycle/2

- References: Allen and Gale (1998), Bryant (1980)

- Assume that the long-term investment is a constant returns to scale technology with stochastic returns.

Every unit of the good invested at $t = 0$ is transformed into

$$\tilde{R} = \begin{cases} R_H & \text{with probability} \quad \pi_H \\ R_L & \text{with probability} \quad \pi_L \end{cases}$$

- Assume that it is possible to liquidate the long term investment in $t = 1$ obtaining a value $0 < r \leq 1$ per unit invested

- We assume that

$$R_H > R_L > r > 0$$

Bank runs and the business cycle/3

- The intermediary offers a deposit contract such that:
  - every consumer deposits her unitary endowment to the bank at $t = 0$;
  - the bank invests in a portfolio of (x, y) such that $x + y \leq 1$ where x is the share invested long-term and y the share invested short term.
  - the deposit contract promises to the consumer who accepts it, $\hat{c}_1$ in case of withdrawal at $t = 1$ and $\hat{c}_2$ in case of withdrawal at $t = 2$.

- Assume the intermediary cannot observe the consumers' types.

- **Assume** also that the deposit contract is not contingent on the realization of the (risky) long-term investment.

Bank runs and the business cycle/4

• In this new framework, equilibria á la Diamond-Dybvig still exist.

• Diamond-Dybvig (1983) is only a special case relative to this new specification, in which $R_H = R_L$.

• We concentrate only on equilibria which would not exhibit a run in the Diamond-Dybvig approach.

• Hence, we can now study new equilibria in which runs, if they exist, depend on the new assumption on the long-term investment technology.

Bank runs and the business cycle/5

- Assume that the intermediary gets zero profits at equilibrium.

- Consumers will receive the entire value of the assets at $t = 2$.

- Since the value of the long-term investment is uncertain, now, we can consider that the intermediary will propose a contract in which $\hat{c}_2$ is so high to exhaust all assets available in $t = 2$.

- Hence, we only need to determine the value of $\hat{c}_1$.

- Let d be the face value of the deposit contract at $t = 1$, i.e. $d = \hat{c}_1$.

Bank runs and the business cycle/6

• Suppose that the intermediary has chosen a portfolio (x, y) and a face value of the deposit contract equal to d.

• It must be that (x, y, d) satisfy the following feasibility constraints:

$$x + y \leq 1 \tag{1}$$

$$\lambda d \leq y \tag{2}$$

$$(1 - \lambda) c_{2s} = R_s(1 - y) + y - \lambda d \qquad \text{with } s = H, L \tag{3}$$

Bank runs and the business cycle/7

- The deposit contract is incentive compatible if $c_{2s} \geq d$ for every $s = H, L \rightarrow$

$$d \leq R_s(1-y) + y \qquad\qquad s = H, L \tag{4}$$

No *bank runs* at equilibrium

- Since $R_H > R_L$, if

$$d \leq R_L(1-y) + y < R_H(1-y) + y$$

there is an equilibrium in which the patient consumers wait till $t = 2$ to withdraw and the bank is solvent.

Equilibria with *bank runs*

- If instead

$$d > R_s(1 - y) + y$$

incentive compatibility for the late consumers is violated.

Since $R_H > R_L$, it could be that:

1. $R_L(1 - y) + y < d \leq R_H(1 - y) + y \rightarrow$ a run occurs in state L
2. $d > R_H(1 - y) + y > R_L(1 - y) + y \rightarrow$ runs happen in every state

- We only study case 1.: if a run occurs, it will occur in state L.

Equilibria without *bank runs*/1

- Let the constrained maximization problem of the intermediary be given by:

$$\begin{aligned}
&\max_{x,y,d,c_{2s}} \quad && \lambda U(d) + (1-\lambda)[\pi_H U(c_{2H}) + \pi_L U(c_{2L})] \\
&\text{s.t.} && x + y \leq 1 \\
& && \lambda d \leq y \\
& && (1-\lambda)c_{2s} = R_s(1-y) + y - \lambda d \qquad \text{with } s = H, L \\
& && d \leq R_s(1-y) + y \qquad\qquad s = H, L
\end{aligned}$$

- If the IC constraint is not binding at equilibrium, $d \leq R_L(1-y) + y < R_H(1-y) + y$.

Equilibria without *bank runs*/2

The IC constraint is not binding at equilibrium → bankruptcy is not a problem.

- The constrained maximization problem reduces to:

$$
\begin{aligned}
\max_{x,y,d,c_{2s}} \quad & \lambda U(d) + (1-\lambda)[\pi_H U(c_{2H}) + \pi_L U(c_{2L})] \\
\text{s.t.} \quad & x + y \leq 1 \\
& \lambda d \leq y \\
& (1-\lambda) c_{2s} = R_s(1-y) + y - \lambda d \qquad \text{with } s = H, L
\end{aligned}
$$

- Observe that $x + y = 1$ holds at equilibrium.

Equilibria without *bank runs*/3

- The unknowns of this problem are (y, d), only.
- The constrained maximization problem can be rewritten as:

$$\max_{d,y}$$

$$\lambda U(d) + (1-\lambda)\left[\pi_H U\left(\frac{R_H(1-y)+y-\lambda d}{1-\lambda}\right) + \pi_L U\left(\frac{R_L(1-y)+y-\lambda d}{1-\lambda}\right)\right]$$

s.t. $\quad \lambda d \leq y$

$$y \in [0,1]$$

Equilibria without *bank runs*/4

- Consider only the case in which $0 < y < 1$.
- The FOC with respect to d is given by:

$$U'(d) - [\pi_H U'(c_{2H}) + \pi_L U'(c_{2L})] \geq \mu \quad d \geq 0$$

Recall that $\mu > 0$ if and only if $\lambda d = y; \mu = 0$ if and only if $\lambda d < y$

Equilibria without *bank runs*/5

- The FOC with respect to y is given by:

$$\pi_H U'(c_{2H})(1 - R_H) + \pi_L U'\left(c_{2L}\right)\left(1 - R_L\right) \leq -\mu$$

(with $\mu > 0$ if and only if $\lambda d = y; \mu = 0$ if and only if $\lambda d < y$)

- If $0 < y < 1$, the system of first order condition is

$$U'(d) - [\pi_H U'(c_{2H}) + \pi_L U'(c_{2L})] \geq \mu \qquad\qquad d \geq 0$$

$$\pi_H U'(c_{2H})(1 - R_H) + \pi_L U'(c_{2L})(1 - R_L) \leq -\mu$$

- If $d > 0$ and $0 < y < 1$, the FOC are given by:

$$U'(d) - [\pi_H U'(c_{2H}) + \pi_L U'(c_{2L})] = \mu$$

$$\pi_H U'(c_{2H})(1 - R_H) + \pi_L U'(c_{2L})(1 - R_L) = -\mu$$

($\mu > 0$ if and only if $\lambda d = y; \mu = 0$ if and only if $\lambda d < y$)

• Let (y^*, d^*) be the solution to the optimization problem.

• (y^*, d^*) is an equilibrium if the incentive constraint $d^* \leq R_L(1-y)+y$ is satisfied in state L.

• If the return in the low state is high enough, IC is never binding at equilibrium. Hence, impatient consumers get:

$$c_1^* = d^* = \frac{y^*}{\lambda}$$

and patients get

$$c_{2s}^* = \frac{R_s(1-y^*)}{(1-\lambda)}$$

in every state $s = H, L$.

If the IC is violated

- If (y^*, d^*) does not satisfy the IC constraint, the intermediary can choose
  - a new contract that satisfies the IC with equality, i.e. $d = R_L(1-y) + y$ in state L;
  - a new contract that violates the IC in state L, hence inducing a run on deposits in state L with probability one.

If the IC is satisfied as an equality/1

• When the IC is relevant for the intermediary, the constrained maximization problem of the intermediary can be written as :

$$\max_{x,y,d,c_{2s}} \quad \lambda U(d) + (1-\lambda)[\pi_H U(c_{2H}) + \pi_L U(c_{2L})]$$

s.t.

$$x + y \leq 1$$

$$\lambda d \leq y$$

$$c_{2s} = \frac{R_s(1-y) + y - \lambda d}{(1-\lambda)} \geq d \qquad \text{with } s = H, L$$

• Since $R_H > R_L$, the only relevant IC constraint is $c_{2L} = d = R_L(1-y) + y$.

If the IC is satisfied as an equality/2

- Having determined $d = R_L(1-y) + y$ in the constrained maximization problem of the intermediary we only have to determine the optimal y:

$$\max_y \quad [\lambda + (1-\lambda)\pi_L]U\left(d\right) + (1-\lambda)\pi_H U\left(\frac{R_H(1-y) + y - \lambda d}{(1-\lambda)}\right)$$

$$\text{s.t.} \qquad 0 < y < 1$$

$$\lambda d \leq y$$

$$d = R_L(1-y) + y$$

- Let (y^{**}, d^{**}) be the solution of this optimization problem.
- Denote U^{**} the maximized expected utility of the consumer.

If the IC is satisfied as an equality/3

- IC binding and the constraint $\lambda d \leq y$ imply that:

$$\lambda(R_L(1-y)+y) \leq y \qquad \rightarrow \qquad y \geq \frac{\lambda R_L}{\lambda R_L + (1-\lambda)}$$

- Hence, this equilibrium can be supported for specific values of R_L.

If the IC is satisfied as an equality/4

If the Incentive Compatibility constraint is binding, the solution to this optimization problem has the following features:

- Impatient consumers always get:

$$c_1^{**} = d^{**} = R_L(1 - y^{**}) + y^{**}$$

- Patient consumers get the same consumption level of the impatients if state L realizes, otherwise they get higher consumption, c_{2H}:

$$c_{2L}^{**} = c_1^{**}$$

$$c_{2H}^{**} = \frac{R_H(1 - y^{**})}{(1 - \lambda)}.$$

If the IC is violated/1

- Keep assuming that (y^*, d^*) does not satisfy the IC constraint.

- Let the intermediary consider a new contract that violates the IC in state L, i.e. $c_{2L} < d$ or alternatively $d > R_L(1-y) + y$.

- When offering this contract the intermediary knows that if state L realizes every consumer (patient and impatient) will withdraw at $t = 1$.

- The total value of every portfolio of individual assets at $t = 1$, in case of early liquidation of the long-term investments, is $rx + y$ with $r \leq 1$, hence $rx + y \leq 1$.

- Hence, the intermediary will be insolvent.

- If all consumers withdraw in $t = 1$, they can at most get

$$c_{1L} = c_{2L} = r(1-y) + y$$

If the IC is violated/2

• The equilibrium $(\tilde{y}, \tilde{d})$ in which a run occurs in state L with probability one is characterized by:

1. Patient and impatient consumers withdrawing at $t = 1$ in case L realises, getting

$$\tilde{c}_{1L} = \tilde{c}_{2L} = r(1 - \tilde{y}) + \tilde{y}$$

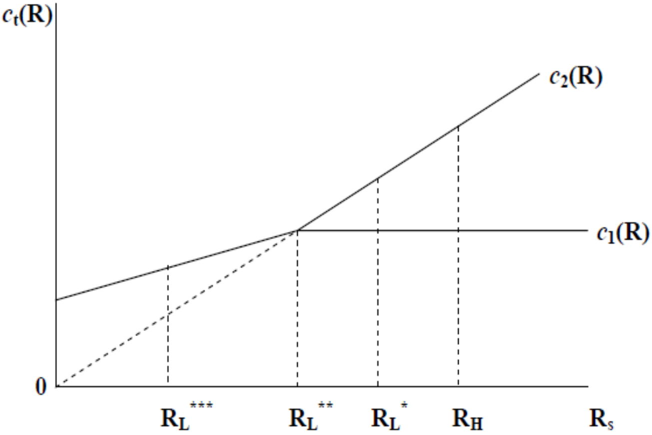
2. In case the H realizes, impatients withdraw in $t = 1$ and patients wait till $t = 2$

$$\tilde{c}_{1H} = \tilde{d} \qquad \text{and} \qquad \tilde{c}_{2H} = \frac{R_H(1 - \tilde{y})}{(1 - \lambda)}.$$

3. IC is violated and the intermediary defaults $\quad \rightarrow \quad \tilde{d} > R_L(1 - \tilde{y}) + \tilde{y}$;

4. Default is preferred to solvency $\quad \rightarrow \quad \tilde{U} > U^{**}$.

Figure 6: Consumption as a function of R



Conclusions/1

• Three possible equilibria may emerge here depending on the value of production in state L:

1. R_L is high enough to guarantee that the IC is never binding;
2. R_L takes some intermediate values which allow to support a binding IC constraint in state L, default is possible;
3. R_L is so low that IC is violated, the intermediary is liquidity constrained at $t = 1$ and a run on deposits happens with probability one.

Conclusions/2

- These equilibria are such that:

  1. the incentive constraint is never biding and bankruptcy never occurs;
  2. bankruptcy is a possibility but the intermediary finds it optimal to choose a deposit contract and a portfolio of assets so that IC is satisfied and patient consumers are indifferent between anticipating their withdrawals or waiting till maturity;
  3. the costs of distorting the deposit contract and the portfolio are so high for the intermediary that she finds it optimal to have a bankruptcy realized in some situations.