Problem Set 1

Theory of Banking - Academic Year 2016-17

Maria Bachelet
maria.jua.bachelet@gmail.com

February 24, 2017

Exercise 1. *An individual consumer has an income stream* (Y_0, Y_1) *and can borrow and lend at the interest rate* i*. For each of the following data points, determine whether the consumption stream* (C_0, C_1) *satisfies the intertemporal budget constraint.*

	(C_0, C_1)	(Y_0, Y_1)	$(1+i)$
case (a)	(18, 11)	(15, 15)	1.1
case (b)	(10, 25)	(15, 15)	1.8

Table 1: Exercise 1 - Consumption and income streams.

Draw a graph to illustrate your answer in each case.

Exercise 2. *Mr. Jones has an income stream* $(Y_0, Y_1) = (100, 50)$ *and can borrow and lend at the interest rate* $i = 0.11$*. His preferences are represented by the additively separable utility function:*

$$U(C_0, C_1) = \frac{C_0^{1-\eta}}{1-\eta} + 0.9\frac{C_1^{1-\eta}}{1-\eta},$$

where the risk-aversion is given by the parameter $\eta = 2$*.*

(a) Define the marginal utility of consumption at $t = 0$ *and* $t = 1$*, respectively. Define the MRS between* C_0 *and* C_1 *and explain its meaning.*

(b) Write down the consumer's intertemporal budget constraint and the first order condition that must be satisfied by the optimal consumption stream.

(c) Use the first order condition and the consumer's inter-temporal budget constraint to find the consumption stream $(C_0^\star, C_1^\star)$ *that maximizes utility.*

(d) How much will the consumer save at $t = 0$*? How much will his savings be worth at* $t = 1$*?*

(e) Check that he can afford the optimal consumption $C_1^\star$ *in* $t = 1$*.*

Exercise 3. *Consider a portfolio choice in a two-period economy with one risky and one riskless assets. The safe asset gives back the initial investment (it yields one unit of the good at date 1 for each unit invested at date 0). The risky asset returns* $R_H = 1.15$ *with probability* $\pi = \frac{3}{5}$ *and* $R_L = 0.8$ *with probability* $(1 - \pi)$ *per unit invested. Consider a consumer who owns initial wealth* $W_0 = 100$ *with preferences represented by* $log(C)$.

1. *Denote the fraction of wealth invested the risky asset by* θ. *Write down the contingent consumptions in the high and low state, respectively* C_H *and* C_L.
2. *Identify the set of feasible consumptions profiles* (C_H, C_L) *and represent it in the cartesian space* (C_H, C_L). *Compute the slope of the frontier of this set. Interpret it.*
3. *Write down the consumer's constrained optimization problem. Identify its indifference curves and compute the MRS (Marginal Rate of Substitution) between* C_H *and* C_L.
4. *Find the optimal portfolio choice* θ^*.