Problem Set 4

Theory of Banking - Academic Year 2016-17

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Exercise 1: Collateralizable Debt Contracts

Consider an entrepreneur who can choose between two investment projects, a and b, which need I to be activated. The outcome of the every project $i = a, b$ is risky,

$$\tilde{X}_i = \begin{cases} X_i & \text{with probability } p_i \\ 0 & \text{with probability } 1 - p_i. \end{cases}$$

Assume that both project are valuable, and project a is less risky than b:

$$p_a X_a > p_b X_b > I, \quad 1 > p_a > p_b > 0, \quad X_b > X_a.$$

The entrepreneur has no initial endowment, hence must borrow the amount I, but owns some $W > 0$ which can be used as collateral for the loan. Borrower and lender are risk neutral, and limited liability holds, in that the entrepreneur's utility cannot be negative. Let the bank offer a collateralizable debt contract (I, R, C), in which I is fixed, R represents the gross interest rate of the debt contract, C the amount of collateral which the bank can seize in case of failure.

1) Compute the expected payoff of the entrepreneur and the expected profit of the bank.

2) Define the bank optimal contract and the entrepreneur's project choice under symmetric information (the bank observes which project is actually implemented).

3) Write down the profit maximisation problem of that bank in case of asymmetric information. Explain the role of every constraint.

4) Examine the Incentive Compatibility constraint, and find the interest rate threshold as a function of the collateral. Explain what is the effect of having introduced collateral on the incentives of the entrepreneur.

5) Define the relevant range of collateral, which are admissible for the entrepreneur, given his initial collateralizable wealth and the limited liability constraint.

6) Explain what are the possible (optimal) contractual offers of the bank, as a function of the results obtained in 4) and 5). Is there any case in which credit rationing may occur?

Exercise 2: Credit rationing

Consider an economy populated by 100 entrepreneurs, each entrepreneur can undertake two type of investment projects, but has no initial resources. To start the project, an entrepreneur can only rely on external financing, through which he should raise $I = 1$.

The first project (project a) yields, if successful, a return $R^a = 2$, and in case of failure 0, the probability of success is $p^a = 0.9$. The second project (project b) yields, if successful, a return $R^b = 2.5$, and in case of failure 0 and the probability of success is $p^b = 0.45$.

Each lender can raise funds at a net interest rate equal to r_d and can only offer a standard debt contract that prescribes a loan equal to 1 and a repayment $R = (1 + r)$ with $r \in \mathbb{R}_+$. Moreover, no collateral can be required ($C = 0$). The banking sector is competitive (meaning that banks have to break even).

Entrepreneurs and lenders are risk neutral. The entrepreneur is protected by limited liability. The supply of deposits is given by $D = A + 20r_d$.

Moreover assume that we can apply the Law of Large Numbers to approximate the share of successful projects by p^a, in case firms find optimal to undertake project a, and p^b if the optimal investment choice is b.

1. Which of the two projects has the highest net present value?

2. Assume that the project choice is contractible. Banks are going to implement the efficient project and $A = 91.2$.

   2a) Write down the expected profit function of the banks per unit invested.

   2b) Determine the equilibrium deposit rate (r_d) that clears the deposit market, i.e. such that supply equals demand.

   2c) Determine the interest rate (r) charged by banks in equilibrium, in such a competitive loan market (recall that banks have to break even).

   2d) Compute the firms' expected profits, Π_F^e.

   2e) Compute the depositors' and the bank's expected profits, Π_D^e and Π_B^e. Finally, compute the social welfare $SW = \Pi_F^e + \Pi_D^e + \Pi_B^e$.

3. Assume now that the project choice is not contractible, then:

   3a) Write down the expected profits of the firm as a function of the project's choice.

   3b) Define the firms' optimal choice over projects as function of the loan rate, r.

   3c) What is the expected gross return of the bank per unit invested, as function of r? Characterize this relationship in the Cartesian space, in which r is on the horizontal axis.

4. Keep assuming that the project choice is not contractible. Characterize the equilibrium in the deposit market.

   4a) Characterize the market-clearing deposit rate as a function of A.

   4b) Which value of A leads to credit rationing?

5. Let $A = 91.2$. Again, derive the market-clearing deposit rate. Which will be in this case the market-clearing loan rate? Which loan rate will a bank charge? What is the aggregate volume of loans?

6. Calculate the expected profits of those firms who receive a loan and compare them to the results from 2d).

7. Calculate the social welfare SW, illustrate its composition and compare with the result obtained in 2e).