Univariate Time Series

Questions

Consider the model

$$y_t = \mu_0 + \mu_1 t + u_t, \tag{1}$$

where

$$u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \varepsilon_t, \tag{2}$$

$\varepsilon_t \sim WN$, and $\phi \in (-1.1]$.

1. Rewrite the above model as an autoregressive process around a deterministic trend.
2. Derive the conditions under which the process is I(1).
3. Describe in details how you would perform the ADF test for the null hypothesis that the process is I(1) *without* drift.

Provide a clear outline of your derivation.

Solutions

1. Multiply both side of Equation (1) by

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2$$

and get

$$\begin{aligned}\phi(L)y_t &= \phi(1)\mu_0 + [\phi(1)L + \Delta\phi^\dagger(L)]\mu_1 t + \varepsilon_t \\ &= \underbrace{\phi(1)\mu_0 - \phi(1)\mu_1 + \phi^\dagger(1)\mu_1}_{\alpha} + \underbrace{\phi(1)\mu_1}_{\beta} t + \varepsilon_t\end{aligned}$$

where

$$\phi(L) = \phi(1)L + \Delta \underbrace{[1 + \phi_2 L]}_{\phi^\dagger(L) = 1 - \phi_1^\dagger L}$$

(see p. 49 of the slides).

2. The process is I(1) when a root of the polynomial $\phi(L)$ lies outside the unit circle and the other one is equal to 1, i.e. $\phi(1) = 0$. Notice that in this case $\beta = 0$ and $\alpha = \phi^\dagger(1)\mu_1$.

3. When the null hypothesis is that y_t is I(1) *without* drift, no linear trend must be included in the RHS of the auxiliary ADF regression equation, which then reads

$$\Delta y_t = -\rho\mu_0 + \rho y_{t-1} - \phi_2 \Delta y_{t-1} + \varepsilon_t, \quad \rho = (\phi - 1),$$

Notice that under the null $\rho = 0$ the constant term cancels out.