Macroeconometrics II - Introduction to state space models

Group Assignment: modelling and forecasting cyclical time series

Due date: 1 week before the final exam

1. Task

The project deals with time series analysis and forecasting of the US quarterly Gross Domestic Product (GDP). Let y_t denote the logarithm of GDP.

1. Estimate a trend plus cycle model, $y_t = \mu_t + \psi_t$, where the trend is specified as follows:

$$
\begin{aligned}
\mu_{t+1} &= \mu_t + \beta_t + \eta_t, & \eta_t &\sim \text{NID}(0, \sigma_\eta^2), \\
\beta_{t+1} &= \beta_t + \zeta_t, & \zeta_t &\sim \text{NID}(0, \sigma_\zeta^2),
\end{aligned}
$$

   and the cycle is
$$
\begin{aligned}
\psi_{t+1} &= \rho \cos \lambda_c \psi_t + \rho \sin \lambda_c \psi_t^* + \kappa_t, \\
\psi_{t+1}^* &= -\rho \sin \lambda_c \psi_t + \rho \cos \lambda_c \psi_t^* + \kappa_t^*,
\end{aligned}
$$
   where we take $\kappa_t \sim \text{NID}(0, \sigma_\kappa^2)$, $\kappa_t^* \sim \text{NID}(0, \sigma_\kappa^2)$, and all the disturbances are mutually uncorrelated.

2. Compute the likelihood ratio test of the hypothesis $H_0 : \lambda_c = \pi/16$ (the cycle period is 8 years).

3. Plot and discuss the real time and the smoothed interval estimates of the cycle and the underlying trend growth (β_t). It is often found that trend growth is 'procyclical'. Do you find evidence for a procyclical trend growth?

4. Compare the cycle smoothed estimates with the Hodrick and Prescott cycle.

5. Fit an autoregressive process with Markov-Switching in the mean model (MS-AR) and comment on the results.

6. For the trend-cycle model and the MS-AR model run a recursive forecasting experiment according to which, starting from the first quarter of 2010 you estimate the model, forecast GDP, GDP quarterly and yearly growth one step ahead and compare the forecast accuracy (using the mean square forecast error and other sensible measures of forecasting accuracy) and continue adding an observation until the end of the sample is reached.

2. Data

The US GDP data can be dowloaded from the FRED Economic data website at

https://research.stlouisfed.org/fred2/series/GDPC96/

3. Notes on marking

This assignment is worth 40% of the final grade. The project will be marked out of 40.

- Five (5) marks will be based on the computing abilities maturated throughout the course. The scripts and computer programmes written for the assignment will be reviewed and should be sent by e-mail.

- Thirty-five (35) points will be awarded based on the written report. You should focus on the description of the series main stylized facts, and on systematic model fitting and evaluation. The ability to address the main task with both originality and critical judgement will be assessed.

- This is a group assignment. The maximum group size is 2.