Consumption exercises.

1 Derive offer curves

a) Max over c1 and c2 $\ln(c1) + \ln(c2)/(1+d)$ s.t $c1 + c2/(1+r) = w1$

$C2 = (1+r)(w1-c1)$

Max over c1 $\ln(c1) + \ln((1+r)(w1-c1))/(1+d)$

FOC $1/c1 - (1/ [(1+r)(w1-c1)])((1+r)/(1+d))=0$

FOC $1/c1 - ((1+r)/(1+d))/ [(1+r)(w1-c1)]=0$
$= 1/c1 - (1/(1+d))/(w1-c1))$

$1/c1 = (1/(w1-c1)) (1/(1+d))$

$(w1-c1) = c1(1/(1+d))$
$W1 = c1((1+d)/(1+d)+1/(1+d)) = c1 ((2+d)/(1+d))$
$C1 = w1((1+d)/(2+d))$

b) Max $\ln(c1) + \ln(c2)/(1+d)$ s.t $c1 + c2/(1+r) = w1 + w2/(1+r)$
$C2 = (1+r)(w1-c1)+w2$

Substitute
Max $\ln(c1) + \ln((1+r)(w1-c1)+w2)/(1+d)$

Foc
$1/c1 -((1+r)/(1+d))/((1+r)(w1-c1) +w2)) = 0$

$1/c1 = ((1+r)/(1+d))/((1+r)(w1-c1) +w2))$
$((1+r)(w1-c1)+w2)/c1 = (1+r)/(1+d)$
$((1+r)(w1-c1)+w2) = [(1+r)/(1+d)]c1$
$((1+r)w1+w2) = (1+r)c1 + ((1+r)/(1+d))c1$

$W1 + w2/(1+r) = (1+1/(1+d))c1= ((2+d)/(1+d))c1$

$C1 = ((1+d)/(2+d)) (w1 + w2/(1+r))$

$C2 = [(1+r)/(1+d)]C1 = [(1+r)/(1+d)] ((1+d)/(2+d)) (w1 + w2/(1+r))$

c) Max $c1^{0.5} + c2^{0.5}/(1+d)$ s.t $c1 + c2/(1+r) = w1$

$C2 = (1+r)(w1-c1)$

Max $c1^{0.5} + ((1+r)(w1-c1))^{0.5}/(1+d)$

Max $c1^{0.5} + (w1-c1)^{0.5}(1+r)^{0.5}/(1+d)$

FOC

$0.5/c1^{0.5} - 0.5((1+r)^{0.5}/(1+d))/((w1-c1))^{0.5}) = 0$

$1/c1^{0.5} = ((1+r)^{0.5}/(1+d))/(w1-c1)^{0.5}$

$(w1-c1)^{0.5} = (1+r)^{0.5}/(1+d))\ c1^{0.5}$

$W1-c1 = (1+r)/(1+d)^2 c1$

$W1 = (1+(1+r)/(1+d)^2)c1$

$C1 = w1/(1+(1+r)/(1+d)^2)$

d) Max $-c1^{-1} + -c2^{-1}/(1+d)$ s.t $c1 + c2/(1+r) = w1$

$C2 = (1+r)(w1-c1)$

Max $-c1^{-1} - (w1-c1)^{-1}(1+r)^{-1}/(1+d)$

Foc $1/c1^2 - [1/(w1-c1)^2]\ (1+r)^{-1}/(1+d)=0$

$(w1-c1)^2 = c1^2(1+r)^{-1}/(1+d)$

$W1-c1 = c1\ (1+r)^{-0.5}/(1+d)^{0.5})$

$C1 = W1/[1+ (1+r)^{-0.5}/(1+d)^{0.5}]$

2 Simple infinite horizon

a) max $\Sigma\ \ln(c_t)/(1+d)^{(t-1)}$

s.t. $\Sigma\ c_t/(1+r)^{t-1} = W1$

FOC that is Euler equation

$1/c_t = (1+r)/(1+d)\ /c_{t+1}$

$c_{t+1}= (1+r)/(1+d)c_t$

$c_t = c_1[\ (1+r)/(1+d)]^{t-1}$

$\Sigma\ c_1[\ (1+r)/(1+d)]^{t-1}/(1+r)^{t-1} = W1$

$\Sigma\ c_1[\ 1/(1+d)]^{t-1} = W1$

$C_1\Sigma\ [\ 1/(1+d)]^{t-1} = W1$

C1(1/(1-1/(1+d))=w1

C1 = w1(1-1/(1+d))

b) max $\Sigma\ c_t^{0.5}/(1+d)^{(t-1)}$

s.t. $\Sigma\ c_t/(1+r)^{t-1} = W1$

given $(1+r)/(1+d)^2 < 1$

FOC that is Euler equation

$0.5/c_t^{0.5} = (1+r)/(1+d)\ 0.5/c_{t+1}^{0.5}$

$c_{t+1}^{0.5}= [(1+r)/(1+d)]c_t^{0.5}$

$c_{t+1}= [(1+r)/(1+d)]^2c_t$

$c_t = c_1[((1+r)/(1+d))^2]^{t-1}$

$\Sigma\ c_1[\ ((1+r)/(1+d))^2]^{t-1}/(1+r)^{t-1} = W1$

$\Sigma\ c_1[\ (1+r)^{2(t-1)}/(1+d))^{2(t-1)}/(1+r)^{t-1}] = W1$

$\Sigma\ c_1[\ (1+r)^{(t-1)}/(1+d))^{2(t-1)}] = W1$

$C_1\Sigma\ [\ (1+r)^{(t-1)}/(1+d))^{2(t-1)}] = W1$

If and ONLY if $(1+r)/(1+d)^2 < 1$

$C_1(\ 1/(1- (1+r)/(1+d)^2)) =w1$

$C_1 = w1((1- (1+r)/(1+d)^2))$