1: Consider a consumer who chooses consumption in period t (C_t) for t = 1,2, 3 ... to maximize the Sum from t = 1 to infinity of $-1/((C_t)(1.10)^t)$ subject to the budget constraint that the present discounted value of consumption is equal to initial wealth (K_1) where consumption is discounted at a constant rate r (note this implies that the consumer has no labour income ever)

so K_1 = the sum from 1 to infinity of $C_t/(1+r)^{t-1}$

If initial wealth K_1 = 100000 what is C_1 as a function of r for r>0 ?

extra credit – what can go wrong if r is too far below 0 ?

$u(C)= -1/C = (1/(1-2))C^{1-2}$

$u'(C) = C^{-2}$

$(C_{t+1}^{-2}) = C_t^{-2}(1.1/(1+r))$

$(C_2^{-2}) = C_1^{-2}(1.1/(1+r))$

$(C_3^{-2}) = C_2^{-2}(1.1/(1+r))$

$(C_3^{-2}) = C_1^{-2}(1.1/(1+r))(1.1/(1+r))= C_1^{-2}(1.1/(1+r))^2= C_1^{-2}(1.1/(1+r))^{3-1}$

$C_t^{-2}= C_1^{-2}(1.1/(1+r))^{t-1}$

$C_1^2= C_t^2(1.1/(1+r))^{t-1}$

$C_1^2((1+r)/1.1))^{t-1}= C_t^2$

$C_t= C_1((1+r)/1.1))^{(t-1)/2}$

and Sum $C_t/(1+r)^{t-1}$ = 100000

Sum $C_1((1+r)/1.1))^{(t-1)/2}/(1+r)^{t-1}$ = 100000

C_1 Sum $((1+r)/1.1))^{(t-1)/2}/(1+r)^{t-1}$ = 100000

C_1 Sum $(1/1.1))^{(t-1)/2}(1+r)^{(t-1)/2}/(1+r)^{t-1}$ = 100000

C_1 Sum $(1/1.1))^{(t-1)/2}(1+r)^{(t-1)/2}/(1+r)^{t-1}$ = 100000

C_1 Sum $(1/1.1))^{(t-1)/2}/(1+r)^{(t-1)/2}$ = 100000

C_1 Sum $1/(1.1(1+r))^{(t-1)/2}$ = 100000

C_1 Sum $((1.1(1+r))^{-1/2})^{(t-1)} = 100000$

C_1 $1/(1-(1.1(1+r))^{-1/2}) = 100000$

$C_1 = (1-(1.1(1+r))^{-1/2})100000$

$C_t = (1-(1.1(1+r))^{-1/2})100000((1+r)/1.1))^{(t-1)/2}$

b what if r = -0.999 ?

2: Consider a consumer who chooses C_1 and C_2 to maximize
$C_1^{0.5} + C_2^{0.5}$

Subject to the budget constraint
$C_1+C_2 = W_1 + W_2$

With $W_1 = 2$ and
$W_2 = 18$ with probability 0.5
And = 34/9 with probability 0.5

$C_2 = W_1 – C_1 + W_2$

$C_2 = 2 – C_1 + W_2$

$E (C_1^{0.5} + C_2^{0.5}) = C_1^{0.5} + E ((2 – C_1 + W2)^{0.5})$

$= C_1^{0.5} + 0.5((2 – C_1 + 18))^{0.5}+ 0.5((2 – C_1 + 34/9))^{0.5}$

$0= 0.5 \ C_1^{-0.5 –} 0.5 (0.5((2 – C_1 + 18))^{-0.5}) – 0.5(0.5((2 – C_1 + 34/9))^{-0.5})$

$0= 0.5 \ C_1^{-0.5} - 0.5 (0.5((2 – C_1 + 18))^{-0.5}) – 0.5(0.5((2 – C_1 + 34/9))^{-0.5})$

$0= C_1^{-0.5} - (0.5((2 – C_1 + 18))^{-0.5}) – (0.5((2 – C_1 + 34/9))^{-0.5})$

not 0

$1 - (0.5((19))^{-0.5}) - (0.5((1+ 34/9))^{-0.5}) = 0.5+0.5 - (0.5((19))^{-0.5}) - (0.5((1+ 34/9))^{-0.5}) >0$

so not 1

… so not 2

… so not 3

$0 = ½ - 0.5(16^{-0.5}) - 0.5 (16/9)^{-0.5}$

$½ – 0.5 (¼)- 0.5(9/16)^{0.5} = ½ – 0.5 (¼) – 0.5(3/4) = ½ – 0.5(1/4 + ¾) = ½-0.5 = 0$

a)What is C_1 (hint it is an integer) ?

$C_1 = 4$

b)Now what is C_1 if $W_2 = 96/9$ (so $E(W_2)$ is the same as it was) ?

$C_2 = 2-C_{1+} + 96/9$

$U = C_1^{0.5} + (2-C_1 + 96/9)^{0.5}$

$0 = 0.5\ C_1^{-0.5} - 0.5\ (2-C_1 + 96/9)^{-0.5}$

$(2-C_1 + 96/9)^{-0.5} = C_1^{-0.5}$

$(2-C_1 + 96/9) = C_1^{-}$

$(2 + 96/9) = 2C_1^{-}$

$C_1 = 1+48/9 = 6 + 3/9$

c) Now what is C1 if there is the additional constraint that $C_1 <= W_1$?

$C_1 = 2$