Two periods of time

number of people per household is 1 in period 1 and L_2 in period 2. Initial wealth per household is Ko

1 unit saved in period 1 get e^{r2} in period 2

max $\ln(C_{p1}) + e^{-rho}(L_2 \ln(C_{p2})$

after working period 1 each household has

$Ko + w_1$

Saves $Ko + w_{1\,-}\ C_{p1}$

Wealth at beginning of period 2 is

$(Ko + w_1 - C_{p1})e^{r2}$

So after working have

$(Ko + w_1 - C_{p1})e^{r2} + L_2 w$

so

$L_2\ C_{p2} = (Ko + w_1 - C_{p1})e^{r2} + L_2 w_2$

$L_2\ C_{p2}e^{-r2} = (Ko + w_1 - C_{p1}) + L_2 w_2 e^{-r2}$

$L_2\ C_{p2}e^{-r2} + C_{p1} = (Ko + w_1) + L_2 w_2 e^{-r2}$

Almost as simple 3 periods 1 per household period 1, L_2 period 2, and L_3 period 3

So after working period 2 have

$(Ko + w_1 - C_{p1})e^{r2} + L_2 w_2$

Consume $L_2\ C_{p2}$ and save

$(Ko + w_1 - C_{p1})e^{r2} + L_2 w_2 - L_2\ C_{p2}$

for one unit saved in period 2 get e^{r3} in period 3

So wealth before working period 3 is

$((Ko + w_1 - C_{p1})e^{r2} + L_2 w_2 - L_2 C_{p2})e^{r3}$

get $L_3 w_3$ and now consume everything because they can't take it with them so

$L_3 C_{p3} = ((Ko + w_1 - C_{p1})e^{r2} + L_2 w_2 - L_2 C_{p2})e^{r3} + L_3 w_3$

$= ((Ko + w_1 - C_{p1})e^{r2}e^{r3} + L_2 w_2 e^{r3} - L_2 C_{p2}e^{r3} + L_3 w_3$

so

$L_3 C_{p3} + C_{p2}e^{r3} + C_{p1}e^{r2+r3} = ((Ko + w_1)e^{r2+r3} + L_2 w_2 e^{r3} - L_2 C_{p2}e^{r3} + L_3 w_3$

dividing by e^{r2+r3}

$L_3 C_{p3}e^{-(r2+r3)} + C_{p2}e^{-r2} + C_{p1} = ((Ko + w_1) + L_2 w_2 e^{-r2} + L_3 w_3 e^{-(r2+r3)}$

Present value of consumption equals initial wealth plus present value of labor income

$$\sum_{t=1}^{3} e^{-\sum_{s=2}^{t} r_s} L_t C_{pt} = K_0 + \sum_{t=1}^{3} e^{-\sum_{s=2}^{t} r_s} L_t W_t$$

Now to get to the discount factor in Romer, generalize to sum of t from one to T, then take the limit as T goes to infinity

$$\sum_{t=1}^{\infty} e^{-\sum_{s=2}^{t} r_s} L_t C_{pt} = K_0 + \sum_{t=1}^{\infty} e^{-\sum_{s=2}^{t} r_s} L_t W_t$$

OK finally move from discrete time to continuous time so sums become integrals.