Macroeconomics September 2021

Please solve two exercises (not 3 not 1, 2)

1: Consider a consumer who chooses consumption in period t (C_t) for t = 1,2, 3 ... to maximize the Sum from t = 1 to infinity of $-1/((C_t)(1.10)^t)$ subject to the budget constraint that the present discounted value of consumption is equal to initial wealth (K_1) where consumption is discounted at a constant rate r (note this implies that the consumer has no labour income ever)

so K_1 = the sum from 1 to infinity of $C_t/(1+r)^{t-1}$

If initial wealth $K_1 = 100000$ what is C_1 as a function of r for $r>0$?

extra credit – what can go wrong if r is too far below 0 ?

2) Consider the model of investment with no financial market imperfections presented by Romer.
a)Write down the equations for Kdot and Qdot -- the time derivatives of K and Q (you are not obliged to re-derive them)
b) draw a graph of Q on K showing the Qdot = 0 curve and the Kdot=0 curve (that is draw the phase diagram)
c) K* is the steady state level of capital. Write down the equation for K* (that is find the point where Kdot=0 and Qdot= 0)
d) Imagine that initial K = K* . Assume there are no taxes and at first agents all believe there will never be taxes, but then shockingly at t1 the state introduces a tax τ on profits so from the point of view of the firm $\pi(K)$ is replaced by $(1-\tau)\pi(K)$. All agents assume that τ will then remain the same forever . Look at the equations for Qdot and Kdot. Which one is changed by the tax ?
e) Illustrate what happens after t1 .

3: Consider a Solow growth model with no depreciation or population growth. The rate of technological progress g = 0.02..

1) $Y = 0.1K^{0.5}(AL)^{0.5}$

a)Ramsey Cass Koopmans Find the steady state capital to effective labour ratio if consumers act to maximize the presented discounted value of the square root of consumption (so theta = 0,5) with a discount rate rho of 0.02
b)Draw a phase diagram of c and k illustrating the convergence to the steady state you found in a.