QUASI-CONCAVE FUNCTIONS

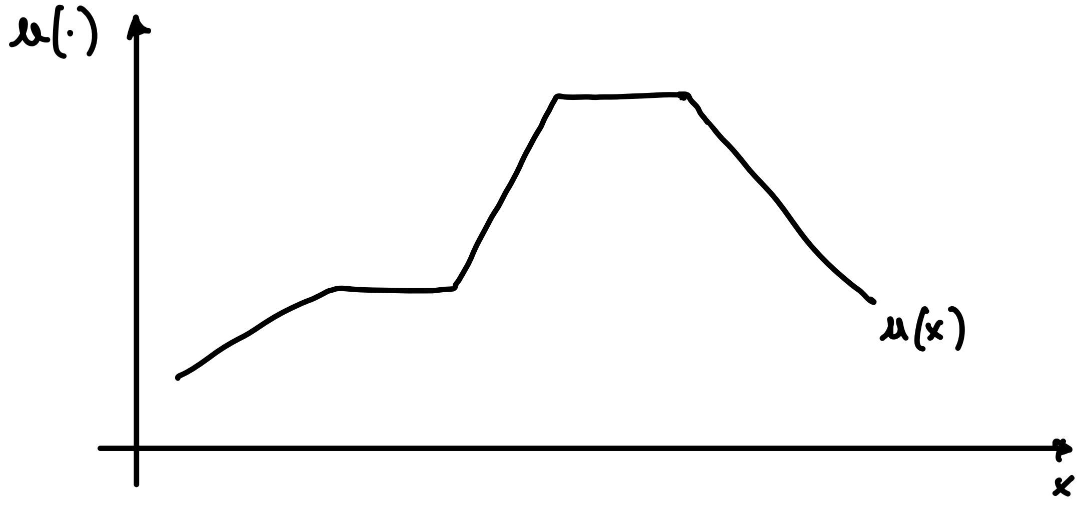
Consider $X = \mathbb{R}_+$, $L=1$

the utility function $u : X \to \mathbb{R}$ is QUASI-CONCAVE if :

$$\forall \; x, y \in \mathbb{R}_+ \quad \text{and} \quad \forall \; \alpha \in [0,1]$$

$$u(\alpha x + (1-\alpha) y) \geq \min \{ u(x), u(y) \}$$

For example



this is not concave, but it's quasi-concave.

$u : X \to \mathbb{R}$ is CONCAVE if

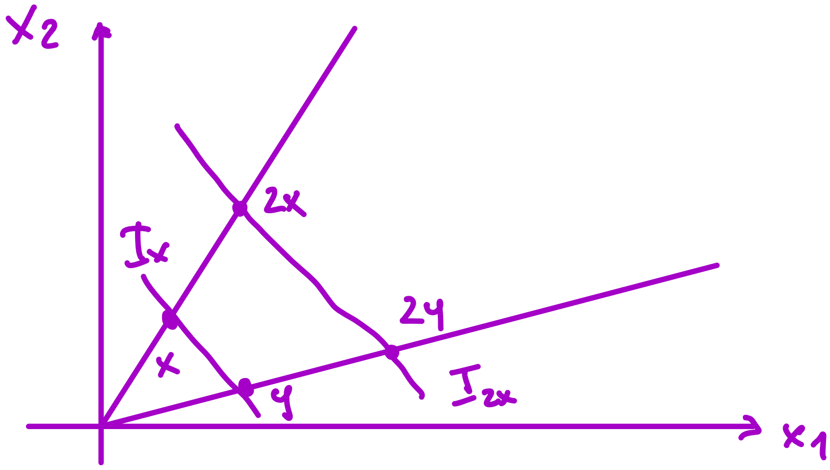
$\forall \; x, y \in \mathbb{R}_+$ and $\alpha \in [0,1]$

$$u(\alpha x + (1-\alpha) y) \geq \alpha u(x) + (1-\alpha) u(y)$$

HOMOTHETIC PREFERENCES & UTILITY

A monotone preference relation $\succsim$ on $\mathbb{R}^L_+$ is HOMOTHETIC if all indifference sets are related by proportional expansions along rays, that is

if $x \sim y \Rightarrow$ then $\alpha x \sim \alpha y \quad \forall \alpha \geq 0$



A continuous $\succsim$ is homothetic iff. it admits a $u(x)$ that is homogeneous of degree one : $u(\alpha x) = \alpha u(x) \quad \forall \alpha > 0$

QUASI-LINEAR $\succsim$ AND UTILITY

the preference order $\succsim$ on

$$(-\infty, +\infty) \cup \mathbb{R}_+^{L-1}$$

is quasi-linear w.r.t. commodity 1 (numeraire) if

1) all indifference sets are parallel displacements along the axis of commodity 1, that is

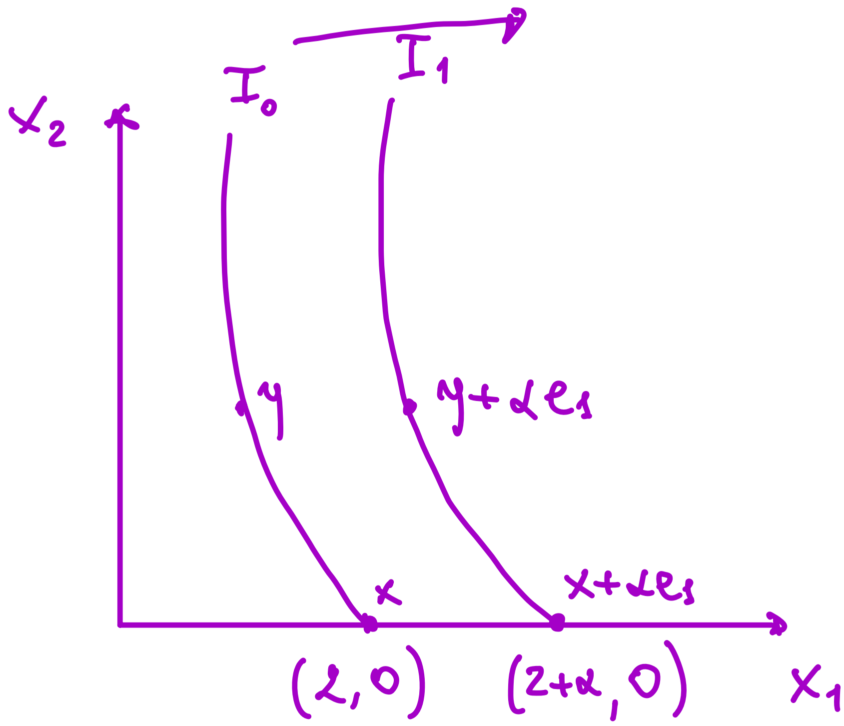
if $x \sim y$ then $(x + \alpha e_1) \sim (y + \alpha e_1)$

with $e_1 = (1, 0, 0, \dots, 0)$ and $\forall \alpha \in \mathbb{R}$

2) commodity 1 is desirable

$x + \alpha e_1 \succ x \quad \forall \alpha > 0 \quad \forall x \in X$

QUASI-LINEAR $\succsim$



A continuous $\succsim$ on $(-\infty,+\infty) \cup \mathbb{R}^{L-1}_+$ is quasi-linear w.r.t. commodity 1 iff it admits a utility function $u(x)$ s.t.

$$u(x_1, x_2, \dots, x_L) = x_1 + \phi(x_2, \dots, x_L)$$

Solve ex. 3.C.5 a) of MWG.