Exercise

Mutual funds specializing in small companies gained an average of 8.4% in the first half of 2018. Assume that the distribution of returns in the first half of 2018 for those mutual funds is distributed as a normal random variable with a mean of 8.4% and a standard deviation of 10%. If a sample of 25 mutual funds is selected, find the probability that

1. the sample mean return is greater that 12%
2. the sample mean return is less than 10%

1. First step standardize….

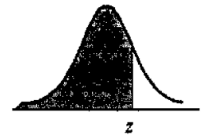
$$P(\bar{X} \geq 12) = P\left(\frac{\bar{X} - 8.4}{10/5} \geq \frac{12 - 8.4}{10/5}\right) = P(Z \geq 1.8) = 0.036$$

2. again…

$$P(\bar{X} \leq 10 = P\left(\frac{\bar{X} - 8.4}{10/5} \leq \frac{10 - 8.4}{10/5}\right) = P(Z \leq 0.8) = 0.7881$$

Distribuzione cumulativa normale standardizzata

$$\Phi(z) = \int_{-\infty}^{z} \varphi(u)\,du = \mathbb{P}[Z \leq z]$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Example

The campaign manager for a political candidate claims that 55% of registered voters favor the candidate over her strongest opponent. Assuming that this claim is true,

what is the probability that in a simple random sample of 300 voters, at least 60% would favor the candidate over her strongest opponent?

p = 0.55, *n* = 300

Standardized sample proportion:

$$z = \frac{0.60 - 0.55}{\sqrt{\frac{0.55(1-0.55)}{300}}} = 1.74$$

$$P(\hat{p} \geq 0.60) = P(z \geq 1.74) = 0.0409$$

Exercise

The waiting time at the post office is a uniform distribution between 5 and 45 minutes

What is the probability a client will wait less than 30 minutes?

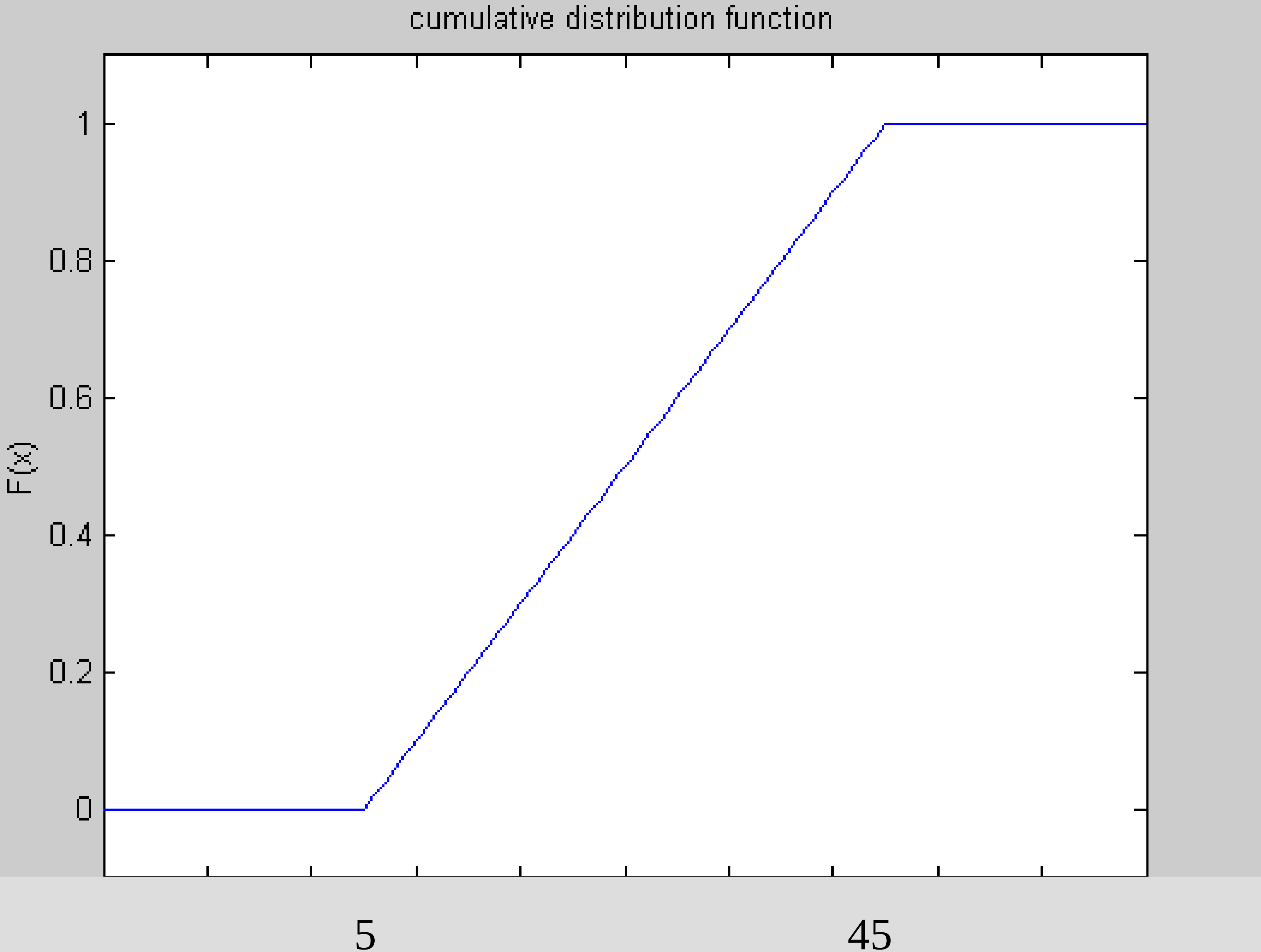
You will go to the post office next week every day from Monday to Friday, what is the probability the maximum time you will be wait is between 30 and 32 minutes?

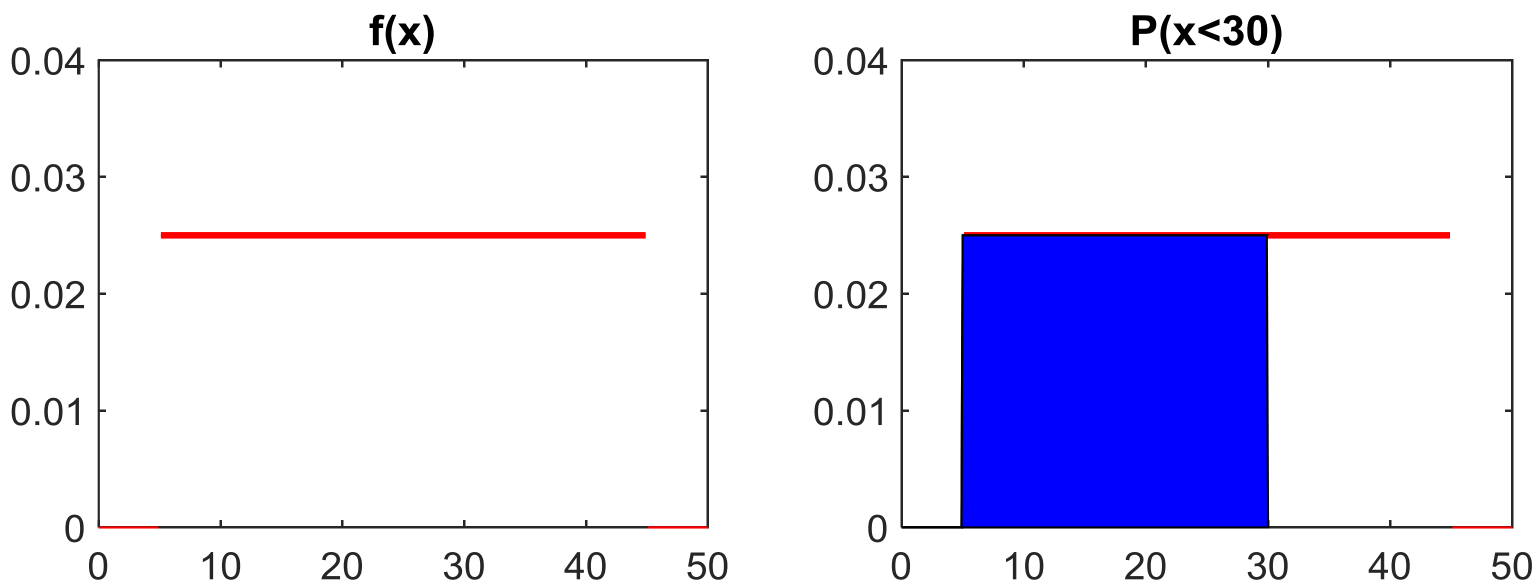
Uniform Distribution

$$f(x) = \frac{1}{b-a} \quad x \in [a,b]$$

$$0 \quad elsewhere$$

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$


cumulative distribution function
F(x)
1
0.8
0.6
0.4
0.2
0
5
45


f(x)
P(x<30)
0.04
0.03
0.02
0.01
0
0
10
20
30
40
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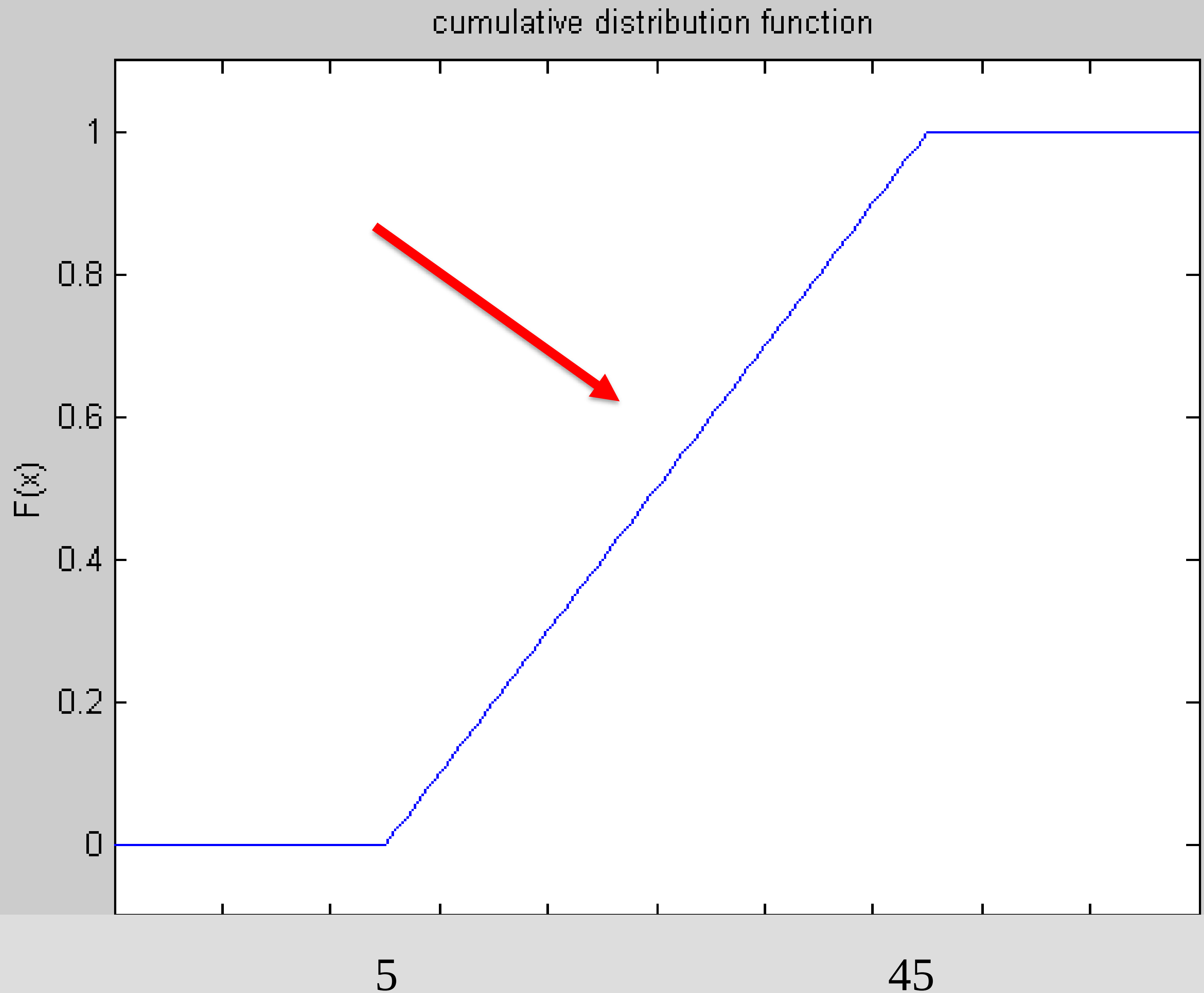

cumulative distribution function
F(x)
0
0.2
0.4
0.6
0.8
1
5
45

$$P(X \le 30) = \int_5^{30} \frac{1}{40}\,dx = \left.\frac{x}{40}\right|_5^{30} = \frac{25}{40} = 0.625$$

$$f_{X_{(n)}}(x) = nf(x)\big(F(x)\big)^{n-1}$$

$$P(30 < X_{(n)} \le 32) = \int_{30}^{32} f_{X_{(n)}}(x)dx = \int_{30}^{32} 5\,\frac{1}{40}\left(\frac{x-5}{40}\right)^4 dx = \left.\left(\frac{x-5}{40}\right)^5\right|_{30}^{32} =$$

$$\left(\frac{32-5}{40}\right)^5 - \left(\frac{30-5}{40}\right)^5 = 0.1401 - 0.0954 = 0.0447$$

Exercise

The waiting time at the post office is a uniform distribution between 5 and 45 minutes

What is the probability a client will wait less than 30 minutes?

You will go to the post office next week every day from Monday to Friday, what is the probability the minimum time you will be wait is between 10 and 12 minutes?

Minimum of Uniform

$$f_{X_{(1)}}(x) = n\big(1 - F(x)\big)^{n-1} f(x)$$

$$f_{X_{(1)}}(x) = 5\left(\frac{45 - x}{40}\right)^4 \frac{1}{40}$$

$$P\big(10 < X_{(1)} \leq 12\big) = \int_{10}^{12} 5\left(\frac{45 - x}{40}\right)^4 \frac{1}{40}\,dx = -|_{10}^{12}\left(\frac{45 - x}{40}\right)^5$$

$\left(\frac{45-10}{40}\right)^5$-$\left(\frac{45-12}{40}\right)^5$=$\left(\frac{35}{40}\right)^5$-$\left(\frac{33}{40}\right)^5$=0.1307

Exercise

The waiting time at the post office is distributed as an exponential with mean 20 minutes.

What is the probability a client will wait less than 30 minutes?

You will go to the post office next week every day from Monday to Friday, what is the probability the minimum time you will be wait is between 10 and 12 minutes?

Exponential

$$f(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}$$

$$\mathrm{F}(x) = 1 - e^{-\frac{x}{\lambda}}$$

$$P(X \leq 30) = 1 - e^{-\frac{30}{20}} = 1 - e^{-\frac{3}{2}} = 0.7769$$

$$f_{X_{(1)}}(x) = n\big(1 - F(x)\big)^{n-1} f(x)$$

$$f_{X_{(1)}}(x) = 5\left(e^{-\frac{x}{\lambda}}\right)^{4} \frac{1}{\lambda} e^{-\frac{x}{\lambda}} = \frac{5}{\lambda} e^{-\frac{5x}{\lambda}}$$

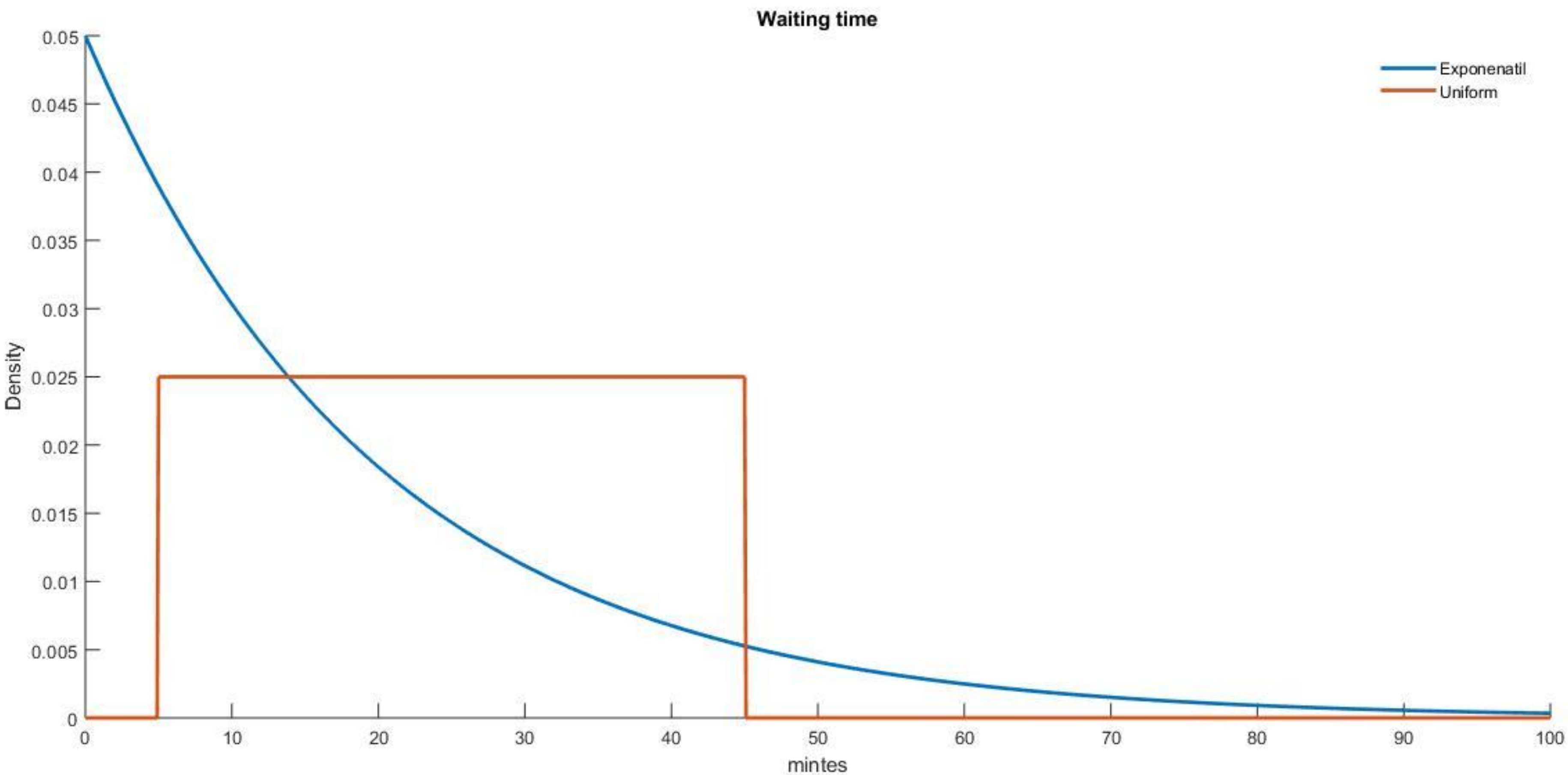
If $X_1,\ldots, X_n$, are independent exponential with $\lambda_1,\ldots, \lambda_n$, then $\min\{X_1,\ldots, X_n,\}$ is exponential with $\lambda=\lambda_1+\ldots+\lambda_n$

Minimum of Exponential

$$f_{X_{(1)}}(x) = \frac{5}{\lambda} e^{-\frac{5x}{\lambda}}$$

$$P\left(10 < X_{(1)} \leq 12\right) = \int_{10}^{12} \frac{5}{\lambda} e^{-\frac{5x}{\lambda}} dx$$

$$= {}_{10}^{12}\Big| -e^{-\frac{5x}{\lambda}} = e^{-\frac{5\times 10}{20}} - e^{-\frac{5\times 12}{20}} = 0.0323$$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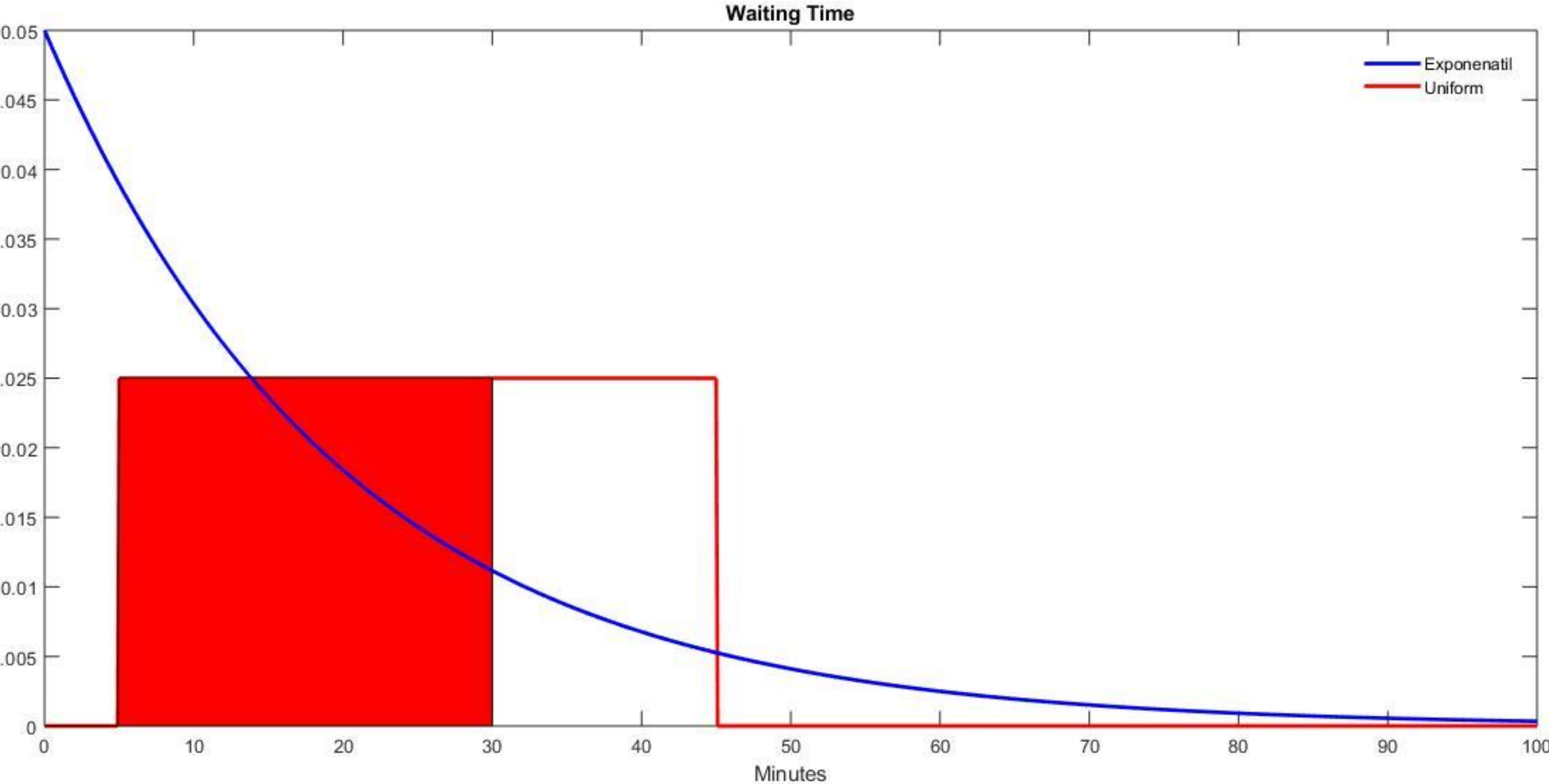
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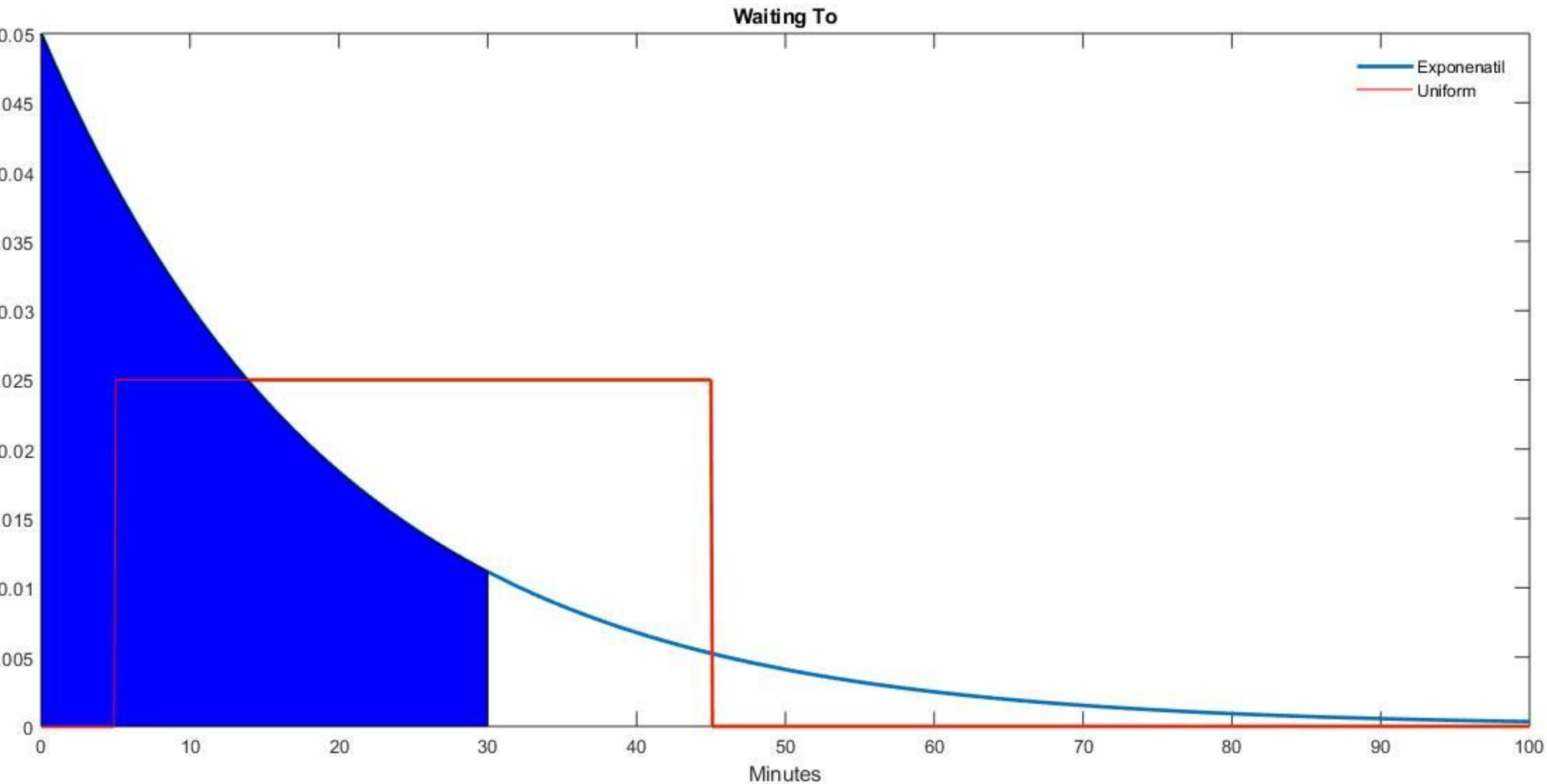
Minutes


Waiting To
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Minutes

Dartmouth College would like to have 1050 freshmen. This college cannot accommodate more than 1060. Assume that each applicant accepts with probability 0.6 and that the acceptances can be modeled by Bernoulli trials. If the college accepts 1700, what is the probability that it will have too many acceptances?

If it accepts 1700 students, the expected number of students who matriculate is $.6 \cdot 1700 = 1020$. The standard deviation for the number that accept is $\sqrt{1700 \cdot .6 \cdot .4} \approx 20$.

Thus we want to estimate the probability

$$P\left(\sum_{i=1}^{1700} X_i > 1060\right) = P\left(\frac{\sum_{i=1}^{1700} X_i - 1700 \times 0.6}{\sqrt{1700 \times 0.6 \times 0.4}} > \frac{1060 - 1700 \times 0.6}{\sqrt{1700 \times 0.6 \times 0.4}}\right)$$

$$P(Z > 1.98) = 0.024$$