Multivariate time series

Sample exams:

1. Consider the following VAR(p) model of a n-vector time series y_t:

$$\Pi(L)y_t = u_t,$$

   where $\Pi(L)$ is a polynomial matrix of order p, u_t are i.i.d. $N_n(0, \Omega)$ innovations.
   (a) Establish and motivate the conditions of stability of the above VAR in terms of the eigenvalues of the companion matrix.
   (b) Derive the structural VAR (SVAR) representation of series y_t when identification is obtained through long-run restrictions. How many restrictions on the SVAR parameters are required for identification?
   (c) Define the impulse response functions and forecast error variance decompositions and discuss estimation issues.
   Provide a clear outline of all your derivations.

2. Consider the following VAR(p) model of a n-vector time series Y_t:

$$\Pi(L)Y_t = \Phi D_t + \varepsilon_t, \tag{1}$$

   where $\Pi(L)$ is a polynomial matrix of order p, D_t is a vector of deterministic elements, and ε_t are i.i.d. $N_n(0, \Omega)$ innovations.
   (d) Provide the conditions under which series Y_t are CI(1,1), and reparametrize model (1) to make the cointegration relations become apparent. Comment on the issue of non-uniqueness of the cointegrating matrix.
   (e) Under the above conditions, derive the Common Trend (CT) representation assuming that the deterministic term is a restricted trend (i.e. the cointegration relations are I(0) around a deterministic trend)
   (f) Introduce and discuss the Johansen's Trace Statistic and Maximum Eigenvalue Statistic for $\mathrm{H}_0 : r = r_0 < n$. Is H_1 the same for both tests?