Macroeconometrics

Lecture 1

Introduction

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Syllabus

1. Introduction
   - General review of time series models.
2. Filtering theory Part 1
   - Examples of Linear filters
   - Gain and phase (e.g. Band-pass filters)
3. Filtering theory Part 2
   - State-space form
   - Kalman filter and Smoother
4. Estimation
   - Maximum likelihood
   - Bayesian estimation
5. TVP VAR

Introduction(1)

- One of the key issues economist have faced in characterising the dynamic behaviour of macroeconomic variables, such as output, unemployment and inflation, is separating trends from cycles.
- The decomposition of economic time series has a long tradition, dating back to the 19th century.
- Along with providing a description of the salient features of a series, the distinction of what is permanent and what is transitory in economic dynamics bears relevant implications for monetary and fiscal policy.

Introduction(2)

- The underlying idea is that trends and cycles can be ascribed to different economic mechanisms and an understanding of their determinants helps to define policy targets and instruments.
- This course focusses on structural time series analysis
- The term **structural time series** refers to a class of **parametric models** that are specified directly in terms of unobserved components which capture essential features of the series, such as trends, cycles and seasonality.

Introduction(3)

- The approach is amenable to the analysis of macroeconomic time series, where latent variables such as trends and cycles, and more specialised notions, such as the output gap, core inflation and the natural rate of unemployment, need to be measured.
- The signal extraction problems relating to latent variables, such as the output gap, core inflation and the NAIRU, can be consistently formulated within a model based framework, and in particular within the class of unobserved components time series models, formalising the fundamental economic relationships with observable macroeconomic aggregates.

Introduction: univariate Methods and Models

- The course focuses on time series methods and models for signal extraction.
- In univariate analysis, the cycle can be identified as the stationary or transitory component in a measure of aggregate economic activity, such as gross domestic product (GDP).
- Estimating the cycle thus amounts to *detrending* the series.
- We shall confine our attention to the e.g. log-additive decomposition of real output, y_t, into potential output, μ_t, and the cycle (aka the output gap), ψ_t:

$$y_t = \mu_t + \psi_t.$$

- In the model–based approach a parametric representation for the components is needed; furthermore, the specification of the model is completed by assumptions on the covariance among the various components.
- The cycle is extracted by performing (possibly linear) operations on the observed time series. The corresponding signal extraction filter is the cycle estimator.
- We set off by considering the properties of such an estimator.

Stationary processes and their second order properties

Def: Stationarity. A random process y_t is (covariance/weakly) stationary if $\forall t$:

$$\begin{aligned} \mathrm{E}(y_t) &= \mu < \infty \\ \mathrm{E}(y_t - \mu)^2 &= \gamma(0) \quad < \infty \\ \mathrm{E}[(y_t - \mu)(y_{t-h} - \mu)] &= \gamma(h) \end{aligned}$$

The **autocovariance function**, $\gamma(h)$, is symmetric: $\gamma(h) = \gamma(-h)$.
Autocorrelation function (ACF):

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)}$$

Properties:
i) $\rho(0) = 1$; ii) $|\rho(h)| < 1$; iii) $\rho(h) = \rho(-h)$.

Definition of AR process

- An autoregressive model is an ARDL model without DL part, i.e. without regressors.
- Autoregressive process of order p, AR(p) in brief:

$$y_t = \alpha_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t, \quad (1)$$

  where ϵ_t is WN$(0, \sigma^2)$.
- We can also include dummy variables (e.g. to capture a deterministic seasonal component).
- With the lag polynomial notations, the AR(p) is written

$$\phi_p(L) y_t = \underbrace{\alpha_0}_{\phi_p(1)\mu} + \epsilon_t \quad (2)$$

  where $\phi_p(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p$.

Stationarity of AR process

- The AR(p) process is CS if $\phi_p(L)$ is stable.
- AR(1): the stability condition is $|\phi_1| < 1$, so that after substitutions

$$\begin{aligned} y_t &= \alpha_0 \sum_{i=0}^{\infty} \phi_1^i + \sum_{i=0}^{\infty} \phi_1^i \epsilon_{t-i} = \alpha_0 \sum_{i=0}^{\infty} \phi_1^i + \sum_{i=0}^{\infty} \phi_1^i L^i \epsilon_t \\ &= \frac{\alpha_0}{1-\phi_1} + \frac{\epsilon_t}{1-\phi_1 L}. \end{aligned}$$

Stationarity corresponds to the fact that the impact multipliers (ϕ_1^i) of shocks tend to 0 quickly enough.

Stationarity of AR process

- AR(p) process:

$$\begin{aligned} \phi_p(L)y_t &= \alpha_0 + \epsilon_t && (3) \\ \Rightarrow y_t &= \frac{\alpha_0}{\phi_p(L)} + \frac{\epsilon_t}{\phi_p(L)} = \frac{\alpha_0}{\phi_p(1)} + \frac{\epsilon_t}{\phi_p(L)} && (4) \\ &= \mu + \psi(L)\epsilon_t, && (5) \end{aligned}$$

  where $\psi(L) = \sum_{i=0}^{\infty} \psi_i L^i = 1/\phi_p(L)$.
- The impact multipliers of shocks are the ψ_i coefficients.
- From the last equality, $\mathrm{E}(y_t) = \mu < \infty$ (by stability). Also, we can write the model as

$$\phi_p(L)(y_t - \mu) = \epsilon_t, \quad (6)$$

  instead of (2).

Autocorrelations of AR process (Jule-Walker), see Hamilton (1994)

- AR(1): $\rho_j = \phi_1^j$, for $j = 0, 1, 2, \ldots \Leftrightarrow \rho_j = \phi_1 \rho_{j-1}$ for $j \geq 1$. (Left as an exercise)
- AR(p) process: it can be shown that:

  1) $\rho_1, \rho_2, \ldots, \rho_p$ can be obtained from $\phi_1, \phi_2, \ldots, \phi_p$ in a unique way.

  2) $\rho_j = \phi_1 \rho_{j-1} + \phi_2 \rho_{j-2} + \ldots + \phi_p \rho_{j-p}$ for $j \geq p$.

  3) By stability, $\rho_j \leftrightarrow 0$ as $j \leftrightarrow \infty$ (the decay is monotone or by oscillations).
- AR(2): $\rho_1 = \frac{\phi_1}{1-\phi_2}$, $\rho_2 = \phi_2 + \frac{\phi_1^2}{1-\phi_2}$,

$$\rho_j = \phi_1 \rho_{j-1} + \phi_2 \rho_{j-2} \text{ for } j \geq 2.$$

Partial autocorrelation coefficients

- AR(1) process: $\text{Corr}(y_t, y_{t-2}) = \phi_1^2 \neq 0$ even though y_{t-2} does not appear in the model. There is a "transmission" effect:
  $\text{Corr}(y_t, y_{t-2}) = \text{Corr}(y_t, y_{t-1}) \times \text{Corr}(y_{t-1}, y_{t-2})$.
  The autocorrelation coefficient ρ_j includes these transmission effects.
- The **PARTIAL AUTOCORRELATION COEFFICIENT (PAC)** between y_t and y_{t-s}, denoted by a_{ss}, eliminates this effect of intervening variables $y_{t-1}, \ldots, y_{t-s+1}$.
  NB: $a_{11} = \rho_1$ (no intervening variable).
- The PAC a_{ss} for $s \geq 1$ are defined as the regression coefficients of y_{t-s} in

$$y_t = \delta + a_{s1} y_{t-1} + a_{s2} y_{t-2} + \ldots + a_{ss} y_{t-s} + u_t, \tag{7}$$

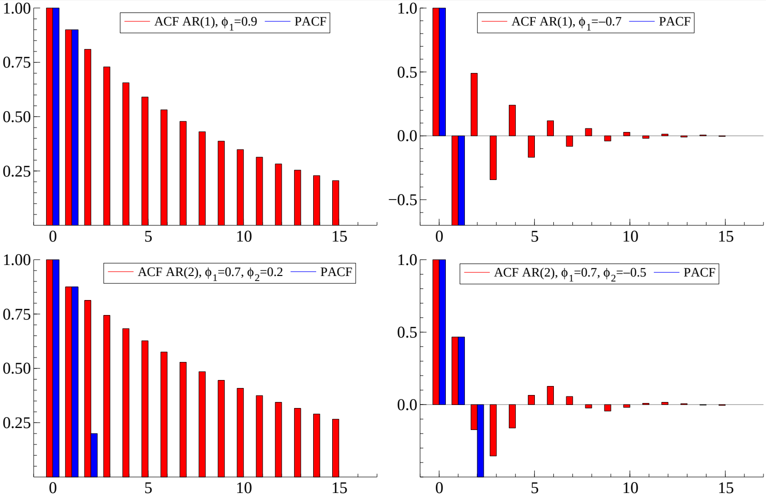
where u_t is an error term with zero mean.

Partial autocorrelations of AR processes

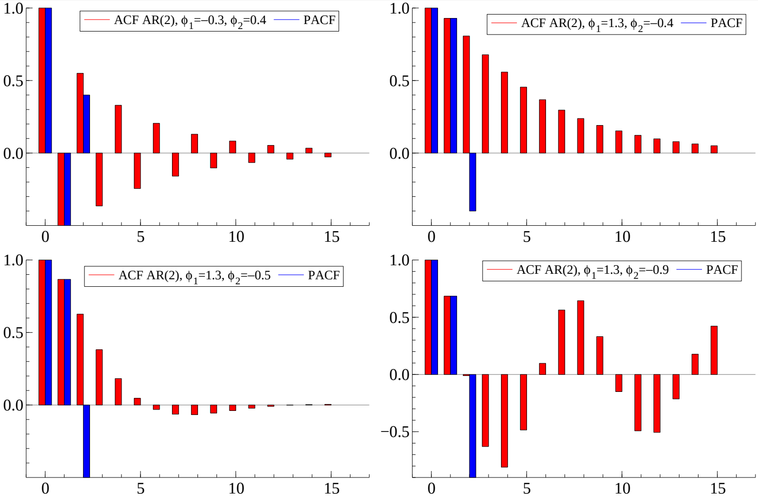
- AR(1): $a_{11} = \rho_1 = \phi_1$, $a_{ss} = 0$ for $s \geq 2$.
- AR(p) process:
  - $a_{ss} = 0$ for all $s \geq p+1$: this is another distinctive property of AR processes (cutoff of the PACF at $p+1$).

  - For $s \leq p$, one can compute the PAC from the autocorrelations ρ_j (thus indirectly from the parameters of the process).
  In particular, $a_{pp} = \phi_p$.
- In an AR(2):
  $a_{22} = \rho_2 - \phi_1\rho_1 = \phi_2$: $\phi_1\rho_1$ is the effect of y_{t-1} in
  $y_t = \delta + a_{21}y_{t-1} + a_{22}y_{t-2} + u_t$.
- Each AR process has its distinctive ACF and PACF pair.
- To estimate consistently a_{ss}, one can take the OLS estimate of a_{ss} in (7).

ACF/PACF of AR processes



ACF/PACF of AR processes

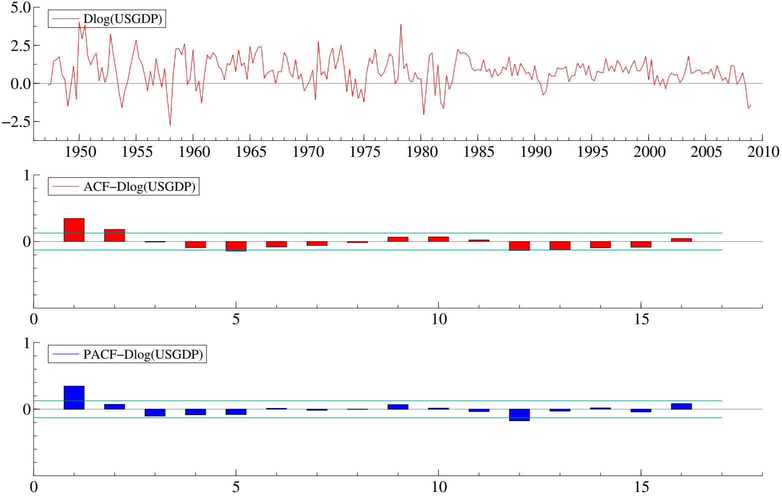


Choice of p?

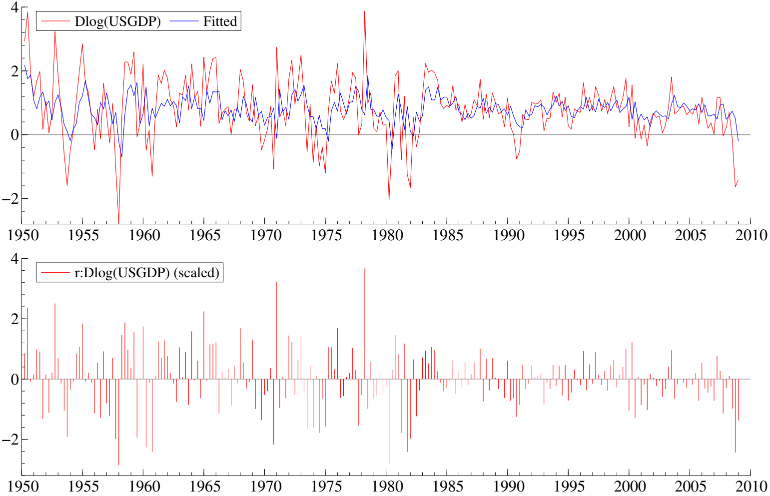
General to specific strategy:

- Choose a high enough p to start (based on data frequency, and ACF/PACF of data compared to typical ones of AR processes).
- Estimate and test that the starting model has no significant autocorrelation remaining in the residuals.
  - if yes: this defines the general unrestricted model (GUM), hence go to step 3.
  - if no: restart at step 1 with a higher p (or another model).
- Simplify the procedure drops insignificant lags, each time testing for lack of autocorrelation in residuals, and the validity of the imposed restrictions (with respect to the GUM). Stop with the most simple acceptable model.

Dlog(USGDP): ACF and PACF



Fit and residuals of AR(1,12)



Definition of MA process

- A moving average process of order q, MA(q) in brief, is defined by

$$y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_q \epsilon_{t-q}, \tag{8}$$

  where $\epsilon_t \sim \mathrm{WN}(0, \sigma^2)$.
- We can also include dummy variables (e.g. to capture a deterministic seasonal component).
- With the lag polynomial notations, the MA(q) process is written

$$\begin{aligned} y_t &= \mu + \theta_q(L)\epsilon_t, \quad \textit{with} & (9) \\ \theta_q(L) &= \theta_0 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q, \quad \textit{with} & (10) \\ \theta_0 &= 1. & (11) \end{aligned}$$

Stationarity of MA process

- The mean $\mathrm{E}(y_t) = \mu$, the $\mathrm{Var}(y_t) = \sigma^2(1+\theta_1^2)$, $\rho_1 = \frac{\theta_1}{1+\theta_1^2}$, $\rho_j = 0$ for $j \geq 2$. The process is CS without restricting θ_1.
- A MA(q) is CS without any restriction on the lag polynomial $\theta_q(L)$ (NO need for stability), with

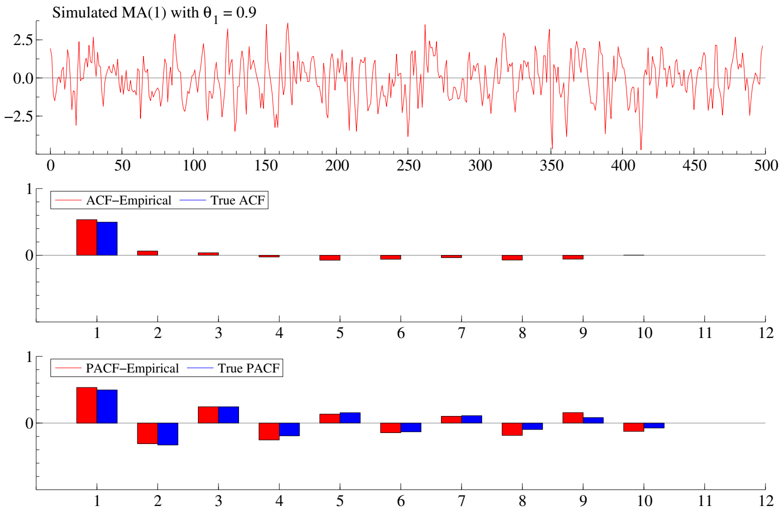
$$\begin{aligned}
\mathrm{E}(y_t) &= \mu && (12)\\
\mathrm{Var}(y_t) &= \sigma^2 \sum_{i=0}^{q} \theta_i^2 && (13)\\
\mathrm{Cov}(y_t, y_{t-j}) &= \begin{cases} \sigma^2 \sum_{i=0}^{q-j} \theta_i \theta_{i+j} & \textit{if} \quad j \leq q \\ 0 & \textit{if} \quad j > q \end{cases} && (14)
\end{aligned}$$

ACF and PACF of MA process

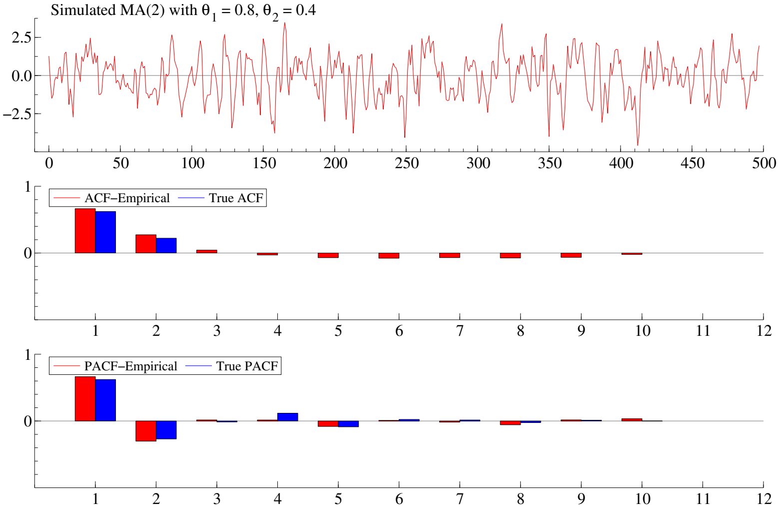
- The ACF of a MA(q):
  1) $\rho_1, \rho_2, \ldots, \rho_q$ can be obtained from $\theta_1, \theta_2, \ldots, \theta_q$ in a unique way.

  2) for $j > q$: $\rho_j = 0$ (cutoff of the ACF at $j + 1$).
- The PACF has no cutoff: $a_{ss} \leftrightarrow 0$ as $s \leftrightarrow \infty$ (monotone decay or by oscillations).
- Each MA process has its distinctive ACF and PACF pair.
- These ACF/PACF shapes are typical of MA processes. They mirror the shapes of the PACF/ACF of AR processes.
- Notice also that an AR(p) can be written as a MA with q infinite, see (5): in the AR(1) case, the MA coefficients are ϕ_1^i.

ACF/PACF of MA(1) process



ACF/PACF of MA(2) process



AR or MA?

- Do not fit a MA if the data ACF suggest there is no cutoff in the ACF.
- In the MA(1) process, ρ_1 cannot be smaller than -0.5 ($\theta_1 = -1$) or larger than 0.5 ($\theta_1 = +1$). In a stationary AR(1) process, ρ_1 ($= \phi_1$) can take any value between -1 and $+1$.
- By increasing q, we can increase the range of ρ_1 but not fully. For example for $q = 2$, ρ_1 is bounded between -0.66 ($\theta_1 = -1, \theta_2 = 1$) and 0.66 ($\theta_1 = \theta_2 = 1$).
- Do not fit a MA if the data first autocorrelation is high.

MA modelling Dlog(USGDP)

- The ACF suggests maybe to include lag 12 $\leftrightarrow$ try a MA(12). No evidence for significant AC is found in the residuals of this model.
- We simplify by elimination of all lags except lags 1, 2, 5: the LR test for the 9 restrictions gives

  $\chi^2(9) = 9.73534$ [0.3723] (ML estimation),

  and this final MA model has no apparent residual AC:

  Portmanteau(15): $\chi^2(3) = 3.6665$ [0.2998].

Fit and residuals of MA(1,2,5)