Classy Application 2: The Term Structure of Interest Rates

the pattern of interest rates for bonds of different maturities decompose long term rates =f() of current and expected future short term rates.

n-period bonds in period t pay a lump sum in t+n.

one dollar invested at period t returns $(1 + R(t, n))^n$ dollars in period t+n.

$R(t, n)$ as n varies is the "yield curve" at period t.

Forward Rates

Forward rates are agreed to in period t for one-period bonds purchased in period t+j-1 and redeemed in period t+j(j=1,...) with return $F(t,j)$

Consider selling a two-period bond and buying a three-period bond.

This is the same as buying a forward commitment for a one-period bond purchased in period two. Suppose both bonds have face value $1. In two periods you will pay out $(1+R(t,2))^2$ dollars and in the following period you will receive $(1+R(t,3))^3$ dollars.

Implications

$$F(t,3) = (1 + R(t,3))^3 (1 + R(t,2))^{-2} - 1$$

Why?

The first few are related by

$$1 + R(t,1) = 1 + F(t,1)$$

$$(1 + R(t,2))^2 = (1 + R(t,1))(1 + F(t,2))$$

$$(1 + R(t,3))^3 = (1 + R(t,1))(1 + F(t,2))(1 + F(t,3))$$

A Simplification for Empirical Work

With simple interest or small rates (logs & Taylor approximation)

$$R(t,1) = F(t,1)$$

$$R(t,2) = (R(t,1) + F(t,2))/2$$

$$R(t,3) = (R(t,1) + F(t,2) + F(t,3))/3$$

Alternatively

$$F(t,j) = jR(t,t+j) - (j-1)R(t,t+j-1)$$

Meiselman (1962)

Speculators indifferent to risk will enter securities markets, forcing forward rates to expected future spot rates.

Rational expectations implies these are the expectations of the spot rates

$$F(t, j) = E(R(t + j - 1, 1)|R(t, 1), R(t - 1, 1), ...)$$

Under the assumption that conditional expectations are linear we can write

$$\begin{aligned} E_{t+1}(R, (t + j, 1)) &= E_t(R(t + j, 1)) \\ &\quad +(R(t + 1, 1) - E_t(R(t + 1, 1))) \end{aligned}$$

Empirical Specification

$$\begin{aligned} E_t(R(t+1,1)) &= E(R(t+1,1)|R(t,1),R(t-1,1),... \\ &= F(t,1) \end{aligned}$$

Upon substituting

$$F(t,j) - F(t-1,j+1) = (R(t,1) - F(t-1,2))$$

Meiselman estimated equations of the form

$$F(t,j) - F(t-1,j+1) = \alpha_j + \beta_j(R(t,1) - F(t-1,2))$$

What are the coefficients?
"Error Learning Model" (Sargent)

Results

Many practical difficulties – bonds pay coupons, etc. Meiselman addressed these carefully and systematically.

For given t, a plot was made of the yield to maturity versus the term of maturity. Observations are securities.

The constant terms are small and insignificantly different from zero.

First empirical application of rational expectations???