THOMSON REUTERS EIKON ADFIN CREDIT CALCULATION GUIDE

DOCUMENT NUMBER 601632.2
1 March 2011

Document History

DOCUMENT NUMBER	DATE	UPDATE
601632.2	1 March 2011	Reformatted "Where" tables
601632.1	11 November 2010	Renamed the calculation guide to Thomson Reuters Eikon Adfin Credit Calculation Guide

TABLE OF CONTENTS

THOMSON REUTERS EIKON ADFIN CREDIT CALCULATION GUIDE

The Adfin Credit Calculation Guide explains the Thomson Reuters Eikon Adfin credit functions, and their formulas to price and analyze Credit Derivatives instruments such as Credit Default Swaps (CDS).

A credit derivative is a derivative security for which payoff depends on the occurrence of a credit event. The aim of any model used in credit derivatives pricing is to express the credit risk linked to this type of instrument.

For instance, a risky bond is a bond whose issuer can default. The most commonly used reference for indicating the probability of default is the rating given to the firm by a rating agency, such as Standard & Poor's or Moody's. The lower the rating, the greater the risk of a bond issued by this firm defaulting before maturity. This risk is referred to as the default probability.

The most common way of pricing a bond is discounting all the cash flows using a zero-coupon curve. In order to take into account the default risk of a risky bond, the principle will be similarly to discount the cash flows using a risky zero-coupon curve. Since this risky zero-coupon curve may not be available, we need to find a model for the default probability, which will allow us to go from the non-risky curve to the risky one.

The model chosen makes it possible to price any instrument and take into account the risk. This requires calculation of the discount factors of bonds, swaps and asset swaps.

To avoid repetition, this book is organized according to calibration models, rather than credit derivative types. The calibration models described are:

- Credit event probability curve from a CDS spread curve or risky zero-coupon curve
- Cox-Ingersoll-Ross (CIR) coefficients from a CDS spread curve or risky zero-coupon curve
- Jarrow-Lando-Turnbull (JLT) method to reflect transitions between ratings and market information

From the calibration model, the Net Present Value and CDS Spread calculations are detailed using practical examples.

CREDIT DERIVATIVES OVERVIEW

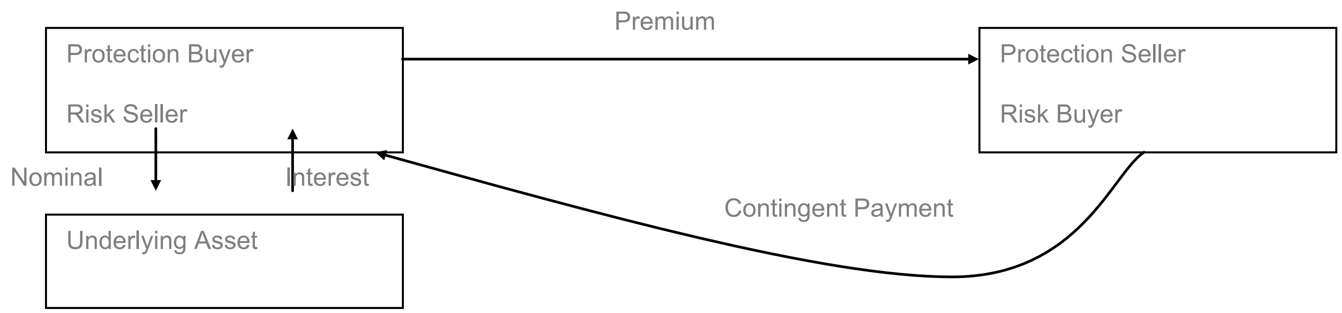
Terminology

Credit Event	A default event, which can take different forms: bankruptcy, ratings downgrade, restructuring of debt, failure to meet a payment obligation...
Recovery rate	The percentage of a claim that is recoverable in the event of counterparty to a transaction defaulting. Notation: R.
CDS	Stands for Credit Default Swap.
CLN	Stands for Credit Linked Notes

The type of contracts a buyer and seller can agree upon in case of a credit event are:

- Credit Default Swaps (CDS)
- Credit Link Notes (CLN)

CREDIT DEFAULT SWAPS (CDS)



In a credit default swap, the protection seller agrees to pay the contingent payment if the default has happened. If there is no default before the maturity of the credit default swap, the protection seller pays nothing.

Three kinds of contingent payment exist:

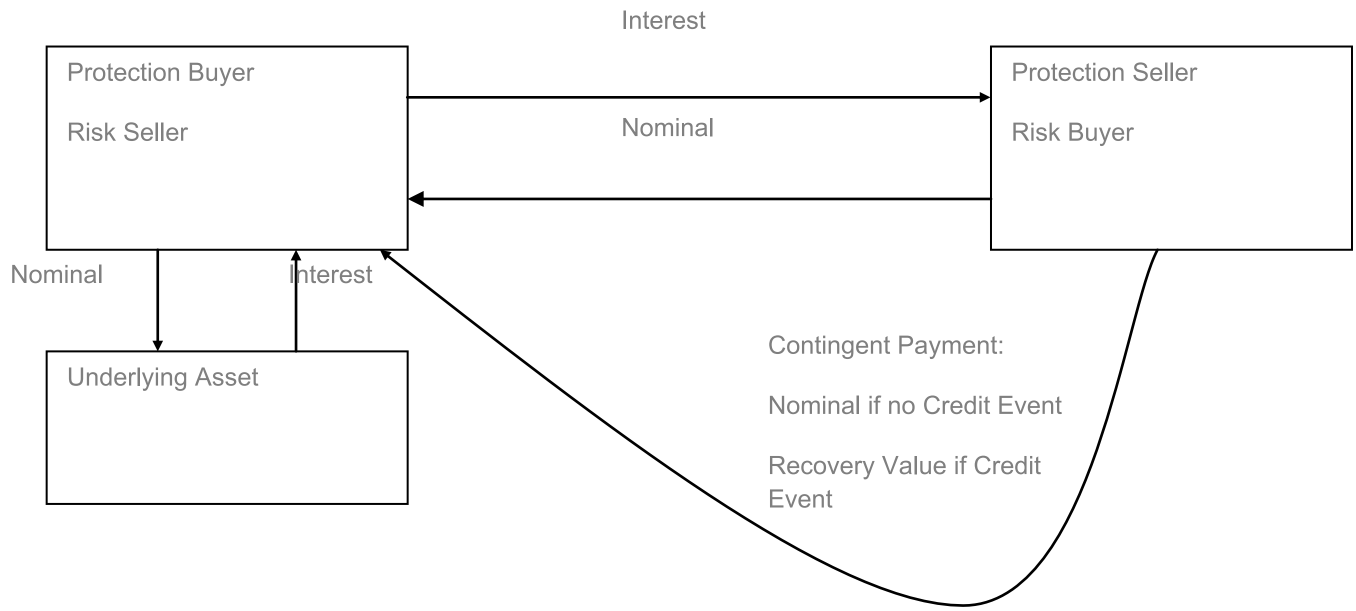
- Cash settlement: the notional minus the market value of the reference asset after the default.
- Binary: a pre-agreed percentage of the notional amount
- Physical delivery: delivery of the reference asset.

Extensions

- CDS callable: the risk seller has the right to cancel the CDS. The premium given to the risk buyer will be higher than for a straight CDS.
- CDS Quanto: the currency of the premium is different from the one of the underlying flows.

CREDIT LINK NOTES (CLN)

A credit-linked note is a combination of a default swap with a bond issued by the protection buyer.



RISK MODEL CALIBRATION

This section describes the calibration of the different risk models, in order to set the default probability term structure. The calibrations are:

- Calibration of the Credit Event Probability Curve
- Calibration Examples in Thomson Reuters Eikon Excel
- Calibration with Jarrow-Lando-Turnbull (JLT) Method

CALIBRATION OF THE CREDIT EVENT PROBABILITY CURVE

Calibration of the Credit Event Probability Curve from a CDS Spread Curve

The idea is to compute the default probabilities given that an n-sized CDS spread curve, such as the net present value for each of these CDS, is equal to zero.

- The calibration is done with a classical bootstrapping method:
- We assume that the default probability at time 0 is 0.
- Given the spread for a 1Y maturity CDS, we compute the default probability at time 1Y.
- Given the spread for a nY maturity CDS, we compute the default probability at time nY.

If there are gaps in the CDS curve, we use the interpolation method linked to the Risk Model in order to calculate the convenient default probabilities.

Example in Thomson Reuters Eikon Excel

CreditStructure RISKMODEL:CURVE ND:DIS NBDAYS:146 RECOVERY:0.3 INSTTYPE:CDS

RateStructure RM:YC ZCTYPE:DF IM LIN

CDS Spread Curve

23-Sep-02	6M	308.7	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	1Y	312	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	2Y	321.8	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	3Y	320.1	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	4Y	350.9	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	5Y	362.7	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	7Y	428	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	10Y	461.8	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0

Risk-Free Zero Coupon Curve

19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%
19-Sep-06	86.05%
19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

Syntax

```
=AdCreditStructure(Risk-Free Zero Coupon Curve, CDS Spread Curve,"RISKMODEL:CURVE ND:DIS
NBDAYS:146 RECOVERY:0.3 INSTTYPE:CDS","RM:YC ZCTYPE:DF IM:LIN")
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CURVE	the credit event probability curve
NBDAYS:i	the number of days per discrimination interval for pricing of American CDS
RECOVERY:XX	the recovery rate value in percentage
INSTTYPE:CDS	the model calibration by using a credit default swap curve

Result returned by the function

`=AdCreditStructure()` returns:

MATURITY	DEFAULT PROBA
19-Sep-2002	0
24-Mar-2003	2.261138%
23-Sep-2003	4.459299%
23-Sep-2004	8.940447%
23-Sep-2005	13.015425%

MATURITY	DEFAULT PROBA
25-Sep-2006	18.667467%
24-Sep-2007	23.469816%
23-Sep-2009	36.963514%
24-Sep-2012	51.761091%

Manual Calculation

In this part an explicit example focused on the first CDS is used to explain the calculation. To retrieve the whole default probability curve by bootstrapping, the same method is used for all CDS.

Default Probability Curve

19-Sep-2002	0
24-Mar-2003	P(t<T2) (unknown DP) =1%

Where P(t<T) is the probability that the credit event occurs at time t when T is the maturity date.

We know that on the 19th September 2002, the default value is 0. Then we compute the default probability at period 6M by solving the following problem:

We assume that the unknown default probability (P(t<T2)) on the 24th March 2003 is 1%. Note that to use the Excel solver a value of P(t<T2) has to be given as input.

The value of the P(t<T2) is the one which enables to return a Net Present Value of the CDS equal to 0. We use the `AdCdsNpv()` function for our calculations. For more information on how to calculate the Net Present Value of a Credit Default Swap refer to CDS Pricing and Evaluation .

Syntax

```
=AdCdsNpv("19SEP02","23SEP02","6M",308.7, Risk-Free Zero Coupon Curve,Default proba
curve,"CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0","RISKMODEL:CURVE RECOVERY:0.3
NBDAYS:146 ND:DIS","RM:YC ZCTYPE:DF"
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CURVE	the credit event probability curve
NBDAYS:i	the number of days per discrimination interval for pricing of American CDS
RECOVERY:XX	the recovery rate value in percentage
CDSTYPE:AMERCDS	an American CDS

Result returned by the function

=`AdCdsNpv()` returns

Npv	-0.008596728

To find P(t<T2) we will solve the formula NPV=0 using the Excel solver for instance, we have:

Default Probability Curve

DATE	PROBABILITY
19-Sep-2002	0
24-Mar-2003	2.261138%

To calculate the next default probability, we use the part of the default probability curve already built. The date of the unknown default probability corresponds to the maturity date of the CDS.

Calibration of the Credit Event Probability Curve from a Risky ZC Curve

We use the direct formula linking a risk-free zero-coupon price B(0,T), a risky zero-coupon price B'(0,T) and the recovery rate R:

$$DefaultP(T) = \frac{B(0,T) - B'(0,T)}{(1-R) \times B(0,T)}$$

Example in Thomson Reuters Eikon Excel

Risk-Free Zero Coupon Curve

DATE	PROBABILITY
19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%
19-Sep-06	86.05%
19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

Risky Zero Coupon Curve

DATE	PROBABILITY
19-Sep-02	100.00%
19-Sep-03	93.77%

DATE	PROBABILITY
19-Sep-04	87.77%
19-Sep-05	81.76%
19-Sep-06	74.93%
19-Sep-07	68.85%
19-Sep-08	61.87%
19-Sep-09	55.34%
19-Sep-10	50.15%
19-Sep-11	45.35%
19-Sep-12	40.92%

Syntax

```
=AdCreditStructure(Risk-Free Zero Coupon Curve,Risky Zero Coupon Curve,"RISKMODEL:CURVE ND:DIS
RECOVERY:0.3 INSTTYPE:DF”,”RM:YC ZCTYPE:DF IM:LIN"
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage
INSTTYPE:DF	the model calibration by using a credit zero-coupon curve

Result returned by the function

=`AdCreditStructure()` returns:

MATURITY	DEFAULT PROBABILITY
19-Sep-2002	
19-Sep-2003	4.371520%
19-Sep-2004	8.855097%
19-Sep-2005	12.891637%
19-Sep-2006	18.461028%
19-Sep-2007	23.244905%
19-Sep-2008	29.990696%
19-Sep-2009	36.683166%

MATURITY	DEFAULT PROBABILITY
19-Sep-2010	41.595154%
19-Sep-2011	46.478514%
19-Sep-2012	51.374916%

Manual calculation

According to the formula, we have on the 19th September 2003:

$$DefaultP(T) = \frac{B(0,T) - B'(0,T)}{(1-R) \times B(0,T)} = \frac{96.73\% - 93.77\%}{(1-30\%) \times 96.73\%} = 4.371520\%$$

To retrieve the whole default probability curve, the same method is used for all maturity dates.

Calculation of Default Probabilities from the Credit Event Probability Curve

Formula

$$P = \frac{P(t_2) - P(t_1)}{1 - P(t_1)}$$

WHERE	DENOTES
P	probability to default between t1 and t2
P(t1)	interpolation from the input default probability curve at date t1
P(t2)	interpolation from the input default probability curve at date t2

Example in Thomson Reuters Eikon Excel

The default probability curve used in the following example is the one calculated from the CDS spread curve in the previous section.

Maturity Array:

01-Jan-03
01-Jul-03
01-Jan-04
01-Jul-04
01-Jan-05
01-Jul-05
01-Jan-06
01-Jul-06
01-Jan-07

01-Jul-07
01-Jan-08

DEFAULT PROBA CURVE	DFI *
19-Sep-02	0
24-Mar-03	2.2611%
23-Sep-03	4.4593%
23-Sep-04	8.9404%
23-Sep-05	13.0154%
25-Sep-06	18.6675%
24-Sep-07	23.4698%
23-Sep-09	36.9635%
24-Sep-12	51.7611%

WHERE	EQUALS
Period Start Date:	01JAN03
CreditStructure:	RISKMODEL:CURVE ND:DIS
AdMode:	LAY:H

Syntax

```
=AdDefaultProba("01JAN03",Maturity_Array,Default proba curve,"RISKMODEL:CURVE ND:DIS","LAY:H"
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CURVE	the credit event probability curve

Result returned by the function

=`AdDefaultProba()` returns:

MATURITY	DEFAULT PROBABILITIES
01-Jan-03	0.00%
01-Jul-03	2.2140%
01-Jan-04	4.4760%
01-Jul-04	6.7328%

MATURITY	DEFAULT PROBABILITIES
01-Jan-05	8.9052%
01-Jul-05	10.9518%
01-Jan-06	13.4614%
01-Jul-06	16.2846%
01-Jan-07	18.9355%
01-Jul-07	21.3541%
01-Jan-08	24.3433%

Manual Calculation

Example on the first default probability (01JUL03):

t1: 01JAN03

t2: 01JUL03

P(01JAN03)= 1.264% (This can be calculated using the `AdInterp()` function)

P(01JUL03)= 3.450%

Then using the formula:

$$P = \frac{P(t_2) - P(t_1)}{1 - P(t_1)} = \frac{3.450\% - 1.264\%}{1 - 1.264\%} = 2.2140\%$$

To retrieve the whole default probability curve, use the same formula for all maturity dates.

Calibration of Cox Ingersoll Ross Coefficients

The diffusion concerns the default intensity (notation: ht), which is defined as follows:

$$Probability(default\_between\_t\_and\_t + dt) = h_t \times dt$$

The default intensity diffusion process is given by the CIR (Cox Ingersoll Ross) equation:

$$dh_t = \lambda \times (h_\infty - h_t) \times dt + \sigma \times \sqrt{h_t} \times dW_t \quad (1)$$

WHERE	DENOTES
h∞	long term intensity
Λ	convergence speed
σ	volatility coefficient
Wt	Brownian motion under the neutral risk probability

The interest of this model is to ensure positive values for ht.

If we consider a deterministic case, the equation (2) becomes:

$$dh_t = \lambda \times (h_\infty - h_t) \times dt$$

Using equation (1), we can calculate the survival probability at time t:

$$Q(\tau > T) = E\left[\exp\left(-\int_t^T h_s \times ds\right)\right] \quad (3)$$

Using equation (2), the two probabilities useful for credit derivatives pricing have the following form:

Survival probability:

$$Q(\tau > t) = \exp\left\{-(h_0 - h_\infty) \times \frac{1 - e^{-\lambda t}}{\lambda} - h_\infty \times t\right\} \quad (4)$$

Default probability:

$$Q(\tau < t) = 1 - \exp\left\{-(h_0 - h_\infty) \times \frac{1 - e^{-\lambda t}}{\lambda} - h_\infty \times t\right\} \quad (5)$$

WHERE	DENOTES
h0	initial intensity
h∞	long-term intensity
t	the current time
T	the maturity of the instrumental, which is unknown
τ	the time of the default

Parameters

h0, h∞ and λ can be calibrated thanks to a CDS premium curve; the aim being the minimization of the distance between prices given by the model and prices given by the market.

The credit derivatives market is not liquid enough to enable the calibration of σ. Moreover this parameter does not have much influence on credit derivatives prices; for this reason, the non-stochastic version of this model is implemented (sigma is considered null).

Levenberg-Marquardt Method

Since Cox-Ingersoll-Ross is a parametric model and there is no simple formula to link the calibration inputs and outputs, we need a method to approximate the three coefficients describing this model.

Adfin Analytics allows you, via `AdCdsSpread()`, to calculate the three intensity and convergence speed parameters of the Cox, Ingersoll, and Ross model, using either an algorithm that links the spread to the maturity or the following approximated formula (in order to improve the performance):

$$P = I \times (1 - R)$$

$$I = h_\infty + (h_0 - h_\infty) \times \frac{1 - e^{-\lambda T}}{\lambda T}$$

WHERE	DENOTES
I	average default intensity
P	spread of the credit default swap
R	recovery rate
T	maturity of the credit default swap

In order to do so, we chose the non-linear square method of Levenberg-Marquardt. (Refer to the book Numerical Recipes, listed in the Bibliography on page 38).

CALIBRATION EXAMPLES IN THOMSON REUTERS EIKON EXCEL

Calibration of Cox-Ingersoll-Ross Coefficients from a CDS Spread Curve

CreditStructure	RISKMODEL:CIR APPROX:YES NBDAYS:146 INSTTYPE:CDS RECOVERY:0.3
RateStructure	RM:YC ZCTYPE:DF IM:LIN

CDS Spread Curve

23-Sep-02	6M	308.7	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	1Y	312	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	2Y	321.8	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	3Y	320.1	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	4Y	350.9	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	5Y	362.7	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	7Y	428	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0
23-Sep-02	10Y	461.8	CLDR:EMU_FI DMC:M CDSTYPE:AMERCDS LFIXED FRQ:4 CCM:MMA0

Risk-Free Zero Coupon Curve

19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%
19-Sep-06	86.05%

19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

Syntax

```
=AdCreditStructure(Risk-Free Zero Coupon Curve,Risky Zero Coupon Curve,"RISKMODEL:CIR APPROX:YES
INSTTYPE:DF RECOVERY:0.3", "RM:YC ZCTYPE:DF IM:LIN","LAY:H")
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CIR	the Cox-Ingersoll-Ross model
RECOVERY:XX	the recovery rate value in percentage
INSTTYPE:DF	the model calibration by using a credit zero-coupon curve

Result returned by the function

=`AdCreditStructure()` returns:

Default intensity start value	0.032410
Default intensity long term value	0.141134
Default intensity convergence speed	0.100831

Calculation of Default Probabilities from Cox-Ingersoll-Ross Coefficients

According to the previous part, we know that:

Default probability equals to:

$$Q(\tau < t) = 1 - \exp\left\{-(h_0 - h_\infty) \times \frac{1 - e^{-\lambda t}}{\lambda} - h_\infty \times t\right\}$$

WHERE	DENOTES
h_∞	initial intensity
t	the current time

WHERE	DENOTES
T	the maturity of the instrumental, which is unknown
τ	the time of the default

The default probability curve used in the following example is the one calculated from the CDS spread curve in the previous section.

Maturity Array:

01-Jan-03
01-Jul-03
01-Jan-04
01-Jul-04
01-Jan-05
01-Jul-05
01-Jan-06
01-Jul-06
01-Jan-07
01-Jul-07
01-Jan-08

CIR Array

0.040089
0.101171
0.109750

Period Start Date	01JAN03
CreditStructure	RISKMODEL:CIR ND:DIS
AdMode	LAY:H

Syntax

```
=AdDefaultProba("01JAN03",Maturity_Array,CIR_Array,"RISKMODEL:CIR ND:DIS","LAY:H")
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CIR	the Cox-Ingersoll-Ross model

Result returned by the function

=`AdDefaultProba()` returns:

MATURITY	DEFAULT PROBABILITY
01-Jan-03	0.00%
01-Jul-03	2.0476%
01-Jan-04	4.2396%
01-Jul-04	6.4874%
01-Jan-05	8.8492%
01-Jul-05	11.2329%
01-Jan-06	13.7032%
01-Jul-06	16.1665%
01-Jan-07	18.6922%
01-Jul-07	21.1870%
01-Jan-08	23.7234%

Manual Calculation

Example on the first default probability (01JUL03):

So we have

T	0.495890*
h0	0.040089
h∞	0.101171
λ	0.109750

*Can be calculated using the `DfCountYears()` function.

Then using the formula:

$$Q(\tau < t) = 1 - \exp\left\{-(h_0 - h_\infty) \times \frac{1 - e^{(-\lambda \times t)}}{\lambda} - h_\infty \times t\right\} = 1 - \exp\left\{-(0.040089 - 0.101171) \times \frac{1 - e^{(-0.109750 \times 0.495890)}}{0.109750} - 0.101171 \times 0.495890\right\} = 2.0476\%$$

To retrieve the whole default probability curve, use the same formula for all maturity dates.

CALIBRATION WITH JARROW-LANDO-TURNBULL (JLT) METHOD

Markov Model

The Jarrow, Lando and Turnbull model is based on the assumption that the bankruptcy process follows a discrete state space Markov chain in credit ratings.

The stability of this model has been enhanced by Kijima and Komoribayashi.

Markov Chain

Let Q(K,K) be the transition matrix defining the Markov process, where the (K – 1)th line corresponds to the lower credit rating class, and the Kth line represents the bankruptcy state: it is also called the absorbing state of this process, since the firm's rating cannot change after bankruptcy. The element of the matrix Q, $q_{i,j}$, is the probability for a bond which rating is i to become rated j within a time step.

$$\begin{bmatrix} q_{1,1} & q_{1,2} & \dots & q_{1,K} \\ \dots & \dots & \dots & \dots \\ q_{K-1,1} & \dots & \dots & q_{K-1,K} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Approximations of the $q_{i,j}$ can be found in Moody's or Standard & Poor's reports, available at http://www.reuters.com/credit.

Here is an example for bonds maturing in one year. The matrix line stands for the rating after transition, and the matrix column stands for the base rating.

Transition Matrix

The aim is now to compute the transition matrix for going from step t to step t+1. The matrix $Q'_{t,t+1}$ is written as follows :

$$\begin{bmatrix} q'_{1,1}(t,t+1) & q'_{1,2}(t,t+1) & \dots & q'_{1,K}(t,t+1) \\ \dots & \dots & \dots & \dots \\ q'_{K-1,1}(t,t+1) & \dots & \dots & q'_{K-1,K}(t,t+1) \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

	Aaa	Aa	A	Baa	Ba	B
Aa	0.0108	0.887	0.0955	0.0034	0.0015	0.0015
A	0.0006	0.0288	0.9021	0.0592	0.0074	0.0018
Baa	0.0005	0.0034	0.0707	0.8524	0.0605	0.0101
Ba	0.0003	0.0008	0.0056	0.0568	0.8357	0.0808
B	0.0001	0.0004	0.0017	0.0065	0.0659	0.827
Caa	0	0	0.0066	0.0105	0.0305	0.0611
D	0	0	0	0	0	0

Jarrow and al.(1997) assume that the q'i,j(t,t+1) can be calculated using:

$$q'_{i,j}(t,t+1) = \pi_i(t) \times q_{i,j} \quad (6)$$

Where $\pi_i(t)$ is a deterministic function of time that ensures that:

$$\left.\begin{array}{l} q'_{i,j}(t,t+1) \geq 0, \forall i,j,i \neq j \\ \sum_{j=1}^{K} q'_{i,j}(t,t+1) \leq 1 \end{array}\right\}$$

Using a matrix notation this can be summarized by:

$$Q'_{t,t+1} - I_k = \pi(t) \times (Q - I_k) \quad (7)$$

Where $\pi(t)$ is a diagonal matrix with the $\pi_i(t)$ on the diagonal and Ik is a null matrix with 1 in the last column (k).

Evaluation

The aim of this part is to evaluate ($\pi_1(t)$,… $\pi_{K-1}(t)$ for $t \in [0, \tau - 1]$)

Let $v^i(t,T)$ be the price of a risky zero-coupon of maturity T issued by a firm in credit class I at time t. We know that:

$$v^i(t,T) = p(t,T) \times \left(R + (1-R) \times Q'^i_t(\tau > T)\right)$$

Where p(t,T) is a default free zero coupon, R the recovery rate and $Q'^i_t(\tau > T)$ the probability that the firm will not default before maturity.

If we assume that we have for each credit class and for each maturity a default free zero coupon and a risky zero coupon, Kijima and al. give formulas to find the values of the $\pi_i(t)$.

For t=0:

$$\pi_i(0) = \frac{1}{1 - q_{j,k}} \times \frac{v^i(0,1) - R \times p(0,1)}{p(0,1) \times (1-R)} \quad \text{for } i=1..K-1 \ (9)$$

Assuming that $Q'^{-1}_{0,t}$ exists, for $t = 1..\tau$:

$$\pi_i(t) = \sum_{j=1}^{K} q'^{-1}_{i,j}(0,t) \times \frac{v^i(0,t+1) - R \times p(0,t+1)}{p(0,t+1) \times (1-R) \times (1-q_{ik})}$$

Default Probability

The survival probability can now be expressed as follows:

$$Q'^{i}_{t}(\tau > T) = \sum_{j-1}^{K-1} q'_{i,j}(t,T) = 1 - q'_{i,K}(t,T)$$

Knowing this probability, the credit derivatives prices can be calculated.

Example in Thomson Reuters Eikon Excel

We want to calibrate a downgrade probability curve from a transition matrix using Jarrow, Lando and Turnbull model for a single A bond that remains single A at the credit event date (RATING:1 to CREDITEVENT:1).

Transition matrix for Rating:i along Y-axis and Credit event:j along X-axis

TRANSITION MATRIX	A	B	C	D
A	0.95	0.03	0.01	0.01
B	0.1	0.7	0.1	0.1
C	0.1	0.2	0.4	0.3
D	0	0	0	1

Risk Free Curve

17-Jul-02	100.00%
17-Jul-03	89.90%
17-Jul-04	86.00%

	RISKY CURVE A	RISKY CURVE B	RISKY CURVE C
17-Jul-02	100.00%	100.00%	100.00%
17-Jul-03	89.00%	84.50%	74.00%
17-Jul-04	85.00%	74.00%	63.00%

Recovery rate:	50%
RateStructure:	RM:YC ZCTYPE:DF
CreditStructure:	RISKMODEL:CURVE INSTTYPE:DF RECOVERY:0.5 RATING:1 CREDITEVENT:1

In this example the return value of the `AdJLTCreditStructure()` function is retrieved and then the formula to recalculate the result is applied manually.

Syntax

```
=AdJLTCreditStructure(RiskFree Curve,Risky Curve, Transition Matrix, "RISKMODEL:CURVE RECOVERY:0.5
RATING:1 CREDITEVENT:1","RM:YC ZCTYPE:DF")
```

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage
RATING:i	the issuing firm rating expressed as the column number in the transition matrix
CREDITEVENT:i	the rating which corresponds to the credit event

Result returned by the function

=`AdJLTCreditStructure()` returns:

17-Jul-02	100.00%
17-Jul-03	94.04%
17-Jul-04	91.17%

Manual Calculation

In this part, an explicit example focused on all transition matrix and probability date is used to explain the calculation.

$$I_k = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad Q = \begin{pmatrix} 0.95 & 0.3 & 001 & 0.01 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.4 & 0.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

WHERE	DENOTES
Ik	A null matrix with 1 in the last column
Q	The transition matrix

T=0: Q'(0,1) calculation: Downgrade probability matrix from RATING:i to CREDITEVENT:j from the 17JUL02 to the 17JUL03

$$Q'_{0,1} - I_k = \pi(0) \times (Q - I_k)$$

According to the equation (9):

$$\pi_1(0) = \frac{1}{1-q_{1,4}} \times \frac{v^1(0,1) - R \times p(0,1)}{p(0,1) \times (1-R)} = \frac{1}{1-0.01} \times \frac{89\% - 50\% \times 89.9\%}{89.9\% \times (1-50\%)} = 0.989877$$

$$\pi_2(0) = \frac{1}{1-q_{2,4}} \times \frac{v^2(0,1) - R \times p(0,1)}{p(0,1) \times (1-R)} = \frac{1}{1-0.1} \times \frac{84.5\% - 50\% \times 89.9\%}{84.5\% \times (1-50\%)} = 0.977629$$

$$\pi_3(0) = \frac{1}{1-q_{3,4}} \times \frac{v^3(0,1) - R \times p(0,1)}{p(0,1) \times (1-R)} = \frac{1}{1-0.1} \times \frac{74\% - 50\% \times 89.9\%}{74\% \times (1-50\%)} = 0.923248$$

$$\pi_4(0) = 1$$

With the matrix $L(0) = \begin{pmatrix} 0.989877 \\ 0.977629 \\ 0.923248 \\ 1 \end{pmatrix}$ and therefore $\pi(0) = \begin{pmatrix} 0.989877 & 0 & 0 & 0 \\ 0 & 0.977629 & 0 & 0 \\ 0 & 0 & 0.923248 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

And $Q - I_k = \begin{pmatrix} 0.95 & 0.3 & 001 & -0.99 \\ 0.1 & 0.7 & 0.1 & -0.9 \\ 0.1 & 0.2 & 0.4 & -0.7 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Then we have:

$$\pi(0).(Q - I_k) = \begin{pmatrix} 0.95 & 0.3 & 001 & -0.99 \\ 0.1 & 0.7 & 0.1 & -0.9 \\ 0.1 & 0.2 & 0.4 & -0.7 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 0.989877 & 0 & 0 & 0 \\ 0 & 0.977629 & 0 & 0 \\ 0 & 0 & 0.923248 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.940383 & 0.029696 & 0.009899 & -0.97998 \\ 0.097763 & 0.684341 & 0.097763 & -0.87987 \\ 0.092325 & 0.18465 & 0.369299 & -0.64627 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Q'(0,1) = \pi(0).(Q - I_k) + I_k = \begin{pmatrix} 0.95 & 0.3 & 001 & -0.99 \\ 0.1 & 0.7 & 0.1 & -0.9 \\ 0.1 & 0.2 & 0.4 & -0.7 \\ 0 & 0 & 0 & 0 \end{pmatrix} . \begin{pmatrix} 0.989877 & 0 & 0 & 0 \\ 0 & 0.977629 & 0 & 0 \\ 0 & 0 & 0.923248 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.940383 & 0.029696 & 0.009899 & -0.97998 \\ 0.097763 & 0.684341 & 0.097763 & -0.87987 \\ 0.092325 & 0.18465 & 0.369299 & -0.64627 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.940383 & 0.029696 & 0.009899 & 0.020022 \\ 0.097763 & 0.684341 & 0.097763 & 0.120133 \\ 0.092325 & 0.18465 & 0.369299 & 0.353726 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T=1: Q'(1,2) calculation: Downgrade probability matrix from RATING:i to CREDITEVENT:j from the 17JUL03 to the 17JUL04

$$Q'_{1,2} - I_k = \pi(1) \times (Q - I_k)$$

According to the equation (9):

$$\frac{v^1(0,2)-R\times p(0,2)}{p(0,2)\times(1-R)}=\frac{85\%-50\%\times 86\%}{86\%\times(1-50\%)}=0.976744$$

$$\frac{v^2(0,2)-R\times p(0,2)}{p(0,2)\times(1-R)}=\frac{74\%-50\%\times 86\%}{86\%\times(1-50\%)}=0.72093$$

$$\frac{v^3(0,2)-R\times p(0,2)}{p(0,2)\times(1-R)}=\frac{63\%-50\%\times 86\%}{86\%\times(1-50\%)}=0.465116$$

$$L'(1)=\begin{pmatrix}0.976744\\0.72093\\0.465116\\1\end{pmatrix}$$

With

$$Q^{-1}=\begin{pmatrix}1.069459 & -0.04165 & -0.01764 & -0.01017\\ -0.1234 & 1.578471 & -0.41455 & -0.04052\\ -0.20567 & -0.77882 & 2.919518 & -0.93503\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$L(1)=\left[Q^{-1}.L'(1)\right]\begin{pmatrix}\frac{1}{0.01}\\ \frac{1}{0.1}\\ \frac{1}{0.3}\\ 1\end{pmatrix}=\begin{pmatrix}1.069459 & -0.04165 & -0.01764 & -0.01017\\ -0.1234 & 1.578471 & -0.41455 & -0.04052\\ -0.20567 & -0.77882 & 2.919518 & -0.93503\\ 0 & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}0.976744\\0.72093\\0.465116\\1\end{pmatrix}.\begin{pmatrix}\frac{1}{0.01}\\ \frac{1}{0.1}\\ \frac{1}{0.3}\\ 1\end{pmatrix}=\begin{pmatrix}1.006357\\0.824622\\0.595556\\0\end{pmatrix}.\begin{pmatrix}\frac{1}{0.01}\\ \frac{1}{0.1}\\ \frac{1}{0.3}\\ 1\end{pmatrix}=\begin{pmatrix}1.016522\\0.916247\\0.850794\\1\end{pmatrix}$$

$$\pi(1)=\begin{pmatrix}1.016522 & 0 & 0 & 0\\ 0 & 0.916247 & 0 & 0\\ 0 & 0 & 0.850794 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

and therefore,

$$Q-I_k=\begin{pmatrix}0.95 & 0.3 & 001 & -0.99\\ 0.1 & 0.7 & 0.1 & -0.9\\ 0.1 & 0.2 & 0.4 & -0.7\\ 0 & 0 & 0 & 0\end{pmatrix}$$

And

Then we have:

$$\pi(1).(Q-I_k)=\begin{pmatrix}0.95 & 0.3 & 001 & -0.99\\ 0.1 & 0.7 & 0.1 & -0.9\\ 0.1 & 0.2 & 0.4 & -0.7\\ 0 & 0 & 0 & 0\end{pmatrix}.\begin{pmatrix}1.016522 & 0 & 0 & 0\\ 0 & 0.916247 & 0 & 0\\ 0 & 0 & 0.850794 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0.965696 & 0.030496 & 0.010165 & -1.00636\\ 0.091625 & 0.641373 & 0.091625 & -0.82462\\ 0.085079 & 0.170159 & 0.340318 & -0.59556\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$$Q'(1,2)=\pi(1).(Q-I_k)+I_k=\begin{pmatrix}0.95 & 0.3 & 001 & -0.99\\ 0.1 & 0.7 & 0.1 & -0.9\\ 0.1 & 0.2 & 0.4 & -0.7\\ 0 & 0 & 0 & 0\end{pmatrix}.\begin{pmatrix}1.016522 & 0 & 0 & 0\\ 0 & 0.916247 & 0 & 0\\ 0 & 0 & 0.850794 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}+\begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}0.965696 & 0.030496 & 0.010165 & -1.00636\\ 0.091625 & 0.641373 & 0.091625 & -0.82462\\ 0.085079 & 0.170159 & 0.340318 & -0.59556\\ 0 & 0 & 0 & 0\end{pmatrix}+\begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}0.965696 & 0.030496 & 0.010165 & -0.00636\\ 0.091625 & 0.641373 & 0.091625 & 0.175378\\ 0.085079 & 0.170159 & 0.340318 & 0.40444\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Therefore the downgrade probability matrix from RATING:i to CREDITEVENT:j from the 17JUL02 to the 17JUL04 is:

$$Q'(0,2)=Q'(1,2).Q'(0,1)=\begin{pmatrix}0.965696 & 0.030496 & 0.010165 & -0.00636\\ 0.091625 & 0.641373 & 0.091625 & 0.175378\\ 0.085079 & 0.170159 & 0.340318 & 0.40444\\ 0 & 0 & 0 & 1\end{pmatrix}.\begin{pmatrix}0.940383 & 0.029696 & 0.009899 & 0.020022\\ 0.097763 & 0.684341 & 0.97763 & 0.120133\\ 0.092325 & 0.18465 & 0.369299 & 0.353726\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}0.911687 & 0.049408 & 0.015649 & 0.023256\\ 0.165429 & 0.458534 & 0.096967 & 0.27907\\ 0.137496 & 0.184084 & 0.143536 & 0.534884\\ 0 & 0 & 0 & 1\end{pmatrix}$$

In our example, we need the downgrade probability table (here given to two decimal places) from RATING:1 to CREDITEVENT:1. We can retrieve these values from the matrices:

17-Jul-02	100.00%*
17-Jul-03	94.04%**
17-Jul-04	91.17%***

* On 17th July 2002 the downgrade probability from RATING:1 to CREDITEVENT:1 is equal to 1.

** T=0

*** T=1

CDS PRICING AND EVALUATION

CDS pricing and evaluation is based on the default probability term structure returned by risk model calibration. Refer to Risk Model Calibration for a description of risk models.

NOTATION	IS THE
$P(\tau \le T_i)$	Probability that the credit event occurs at time t when T is the maturity date.
P	the CDS premium.
R	recovery rate.
CP	1 – Recovery = Contingent Payment.
DF(Ti)	discount factor at time Ti.
n	the number of cash flows of the CDS if no credit event.
t	the time of the default event.
Settle	CDS settlement date.
Maturity	CDS maturity date.
NbDay	time step.
Fx	outright if the CDS is cross-currency (fx=1 otherwise).

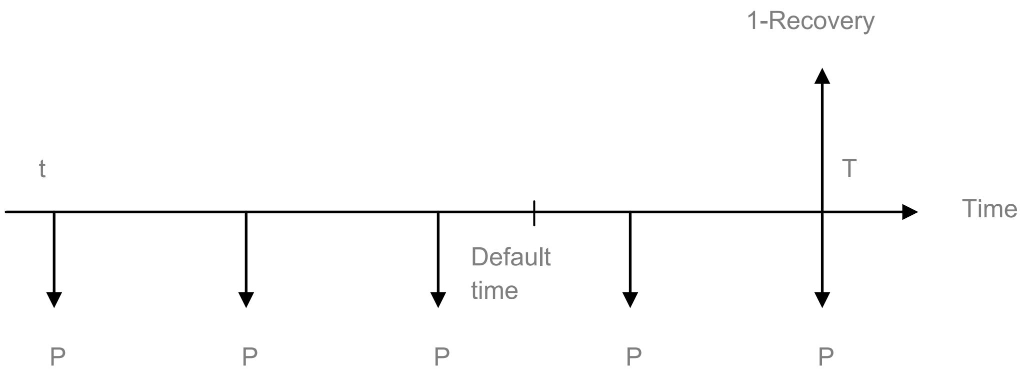
The different types of pricing are:

- European CDS Pricing
- American CDS Pricing
- CDS Big Bang Impact on Pricing with Adfin
- Credit Linked Note Pricing
- Spread Calculation

EUROPEAN CDS PRICING

Analytical Formulas

For a European CDS, the contingent payment is paid at maturity:



Fixed Leg Calculation

$$NPV_{Fixed} = \sum_{i=1}^{n} CF_i \times DF(T_i) \times P(\tau \geq T_i) \times fx$$

WHERE	DENOTES
Fx	Outright if the CDS is cross-currency (fx=1 otherwise)
$P(\tau \geq T_i)$	The probability to default after the time Ti of the ith cash flows payment
DF(Ti)	Discount factor at time Ti.

Floating Leg Calculation

$$NPV_{Float} = (1-R) \times DF(T_i) \times P(\tau \leq T)$$

WHERE	DENOTES
Fx	Outright if the CDS is cross-currency (fx=1 otherwise)
$P(\tau \geq T_i)$	The probability to default after the time Ti of the ith cash flows payment.
DF(Ti)	Discount factor at time Ti.

CDS Net Present Value Calculation

PAID LEG	NPV FORMULA
Fixed	$NPV = NPV_{Float} - NPV_{Fixed}$
Floating	$NPV = NPV_{Fixed} - NPV_{Float}$

Example of a CDS Pricing from Cox-Ingersoll-Ross Coefficients

Example in Thomson Reuters Eikon Excel

Consider a Credit Default Swap, which expires on 15th September 2004. The settlement date is on 23rd September 2001, it starts on 15th September 2002, the spread is 200 basis points, and the recovery rate is 30%.

Settle: 23SEP01, Start Date: 15SEP02, Maturity Date: 15SEP04, Spread=200, R=30%,

CdsStructure	CDSTYPE:EURCDS CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4
CreditStructure	RISKMODEL:CIR RECOVERY:0.3 ND:DIS
RateStructure	RM:YC ZCTYPE:DF

CIR Array

0.040089
0.101171
0.109750

19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%
19-Sep-06	86.05%
19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

In this example the return value of the `AdCdsNpv()` function is retrieved and then the formulas to recalculate the result is applied manually.

Syntax

```
=AdCdsNpv("23SEP01","15SEP02","15SEP04",200,Risk-Free DF curve,CIR_Array,"CDSTYPE:EURCDS CLDR:NULL LFLOAT LFIXED FRQ:4","RISKMODEL:CIR RECOVERY:0.3 ND:DIS","RM:YC ZCTYPE:DF")
```

CreditStructure

KEYWORD	SPECIFIES
CDSTYPE:EURCDS	a European CDS
RISKMODEL:CIR	the Cox-Ingersoll-Ross model
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage

Result returned by the function

`=AdCdsNpv()` returns:

NPV	0.021275

Manual Calculation

Fixed leg

$DATE_i$	CF_i	$P(T \leq T_i)$	$P(T \geq T_i) = 1 - P(T \leq T_i)$	$DF(T_i)$	$CFIXP(T \geq T_i) X\ DF(T_i)$
15-Dec-02	50.00	0.010149	0.989851	0.992206	49.106779
15-Mar-03	50.00	0.020476	0.979524	0.984143	48.199547
15-Jun-03	50.00	0.031308	0.968692	0.975901	47.267337
15-Sep-03	50.00	0.042396	0.957604	0.967658	46.331666
15-Dec-03	50.00	0.053594	0.946406	0.959789	45.417475
15-Mar-04	50.00	0.064874	0.935126	0.951932	44.508782
15-Jun-04	50.00	0.076597	0.923403	0.943989	43.584109
15-Sep-04	50.00	0.088492	0.911508	0.936045	42.660623

In the table:

TO CALCULATE	USE	EXAMPLE
$P(\tau \leq T_i)$	`AdDefaultProba()`	`=AdDefaultProba("15SEP02",Datei,CIR Array, "LAY:H")`
Discount Factors	`AdRate()`	For example on the first cash flow: `DF2 = AdRate("23SEP01", "15DEC02", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.992206`

TO CALCULATE	USE	EXAMPLE
	`AdInterp()`	Interpolation on the Yield Curve at the date (15DEC02) of the first cash flow for example: `AdInterp("15DEC02";ZCDate;ZCRate;"IM:LIN") = 0.992206`

Hence,

$$NPV_{Fixed} = \sum_{i=1}^{8} CF_i \times DF(T_i) \times P(\tau \geq T_i) = 0.036708$$

Floating leg

DATE	CONTINGENT PAYMENT	DF	P(T≤T)
15-Sep-04	0.70	0.936045	0.088425

In the table

TO CALCULATE	USE	EXAMPLE
Discount Factor at time T	`AdRate()`	`DF2=AdRate("23SEP01", "15SEP04", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.936045`
	`AdInterp()`	Interpolation on the Yield Curve at the maturity date (15SEP04): `=AdInterp("15SEP04";ZCDate;ZCRate;"IM:LIN") = 0.936045`

Hence:

$$NPV_{Float} = (1-R) \times DF(T_i) \times P(\tau \leq T) = (1-30\%) \times 0.936045 \times 0.088492 = 0.057983$$

Therefore the Net Present Value of the CDS is:

$$NPV = NPV_{Float} - NPV_{Fixed} = 0.057983 - 0.036708 = 0.021275$$

Example of CDS Pricing from a Default Probability Curve

Example in Thomson Reuters Eikon Excel

Consider a Credit Default Swap, which expires on 15th September 2004. The settlement date is on 23rd September 2001, it starts on 15th September 2002, the spread is 200 basis points, and the recovery rate is 30%.

Settle: 23SEP01, Start Date: 15SEP02, Maturity Date: 15SEP04, Spread=200, R=30%,

CdsStructure CDSTYPE:EURCDS CLDR:NULL LFLOAT LFIXED FRQ:4

CreditStructure RISKMODEL:CURVE RECOVERY:0.3 ND:DIS

Ratestructure RM:YC ZCTYPE:DF

Risk-Free DF Curve

19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%
19-Sep-06	86.05%
19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

Probaility Curve

19-Sep-02	0%
24-Mar-03	2.26%
23-Sep-03	4.46%
23-Sep-04	8.94%
23-Sep-05	13.02%
25-Sep-06	18.67%
24-Sep-07	23.47%
23-Sep-09	36.96%
24-Sep-12	51.76%

In this example the return value of the `AdCdsNpv()` function is retrieved and then the formula to recalculate the result is applied manually.

Syntax

```
=AdCdsNpv("23SEP01","15SEP02","15SEP04",200,Risk-Free DF curve,Proba Curve,"CDSTYPE:EURCDS
CLDR:NULL LFLOAT LFIXED FRQ:4","RISKMODEL:CURVE RECOVERY:0.3 ND:DIS","RM:YC ZCTYPE:DF")
```

CreditStructure

KEYWORD	SPECIFIES
CDSTYPE:EURCDS	a European CDS
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage

Result returned by the function

=`AdCdsNpv()` returns:

NPV	0.021263

Manual Calculation

Fixed leg

$DATE_I$	CF_I	$P(T \le T_I)$	$P(T \ge T_I)=1 - P(T \le TI)$	$DF(T_I)$	$CFIXP(T \ge T_I)X\ DF(T_I)$
15-Dec-02	50.00	0.010576	0.989424	0.992206	49.106779
15-Mar-03	50.00	0.021517	0.978483	0.984143	48.199547
15-Jun-03	50.00	0.032581	0.967419	0.992206	49.085595
15-Sep-03	50.00	0.043632	0.956368	0.984143	48.148333
15-Dec-03	50.00	0.054755	0.945245	0.975901	47.205228
15-Mar-04	50.00	0.065897	0.934103	0.967658	46.271872
15-Jun-04	50.00	0.077161	0.922839	0.959789	45.361758
15-Sep-04	50.00	0.088425	0.911575	0.951932	44.460122

In the table

TO CALCULATE	USE	EXAMPLE
$P(\tau \le T_i)$	`AdDefaultProba()`	`=AdDefaultProba("15SEP02",Datei,CIR Array, "LAY:H")`
Discount Factors	`AdRate()`	For example on the first cash flow: `DF2 = AdRate("23SEP01", "15DEC02", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.992206`

TO CALCULATE	USE	EXAMPLE
	`AdInterp()`	Interpolation on the Yield Curve at the date (15DEC02) of the first cash flow for example: `AdInterp("15DEC02";ZCDate;ZCRate;"IM:LIN") = 0.992206`

Hence,

$$NPV_{Fixed} = \sum_{i=1}^{8} CF_i \times DF(T_i) \times P(\tau \geq T_i) = 0.036708$$

Floating leg

$DATE_i$	CF_i	$P(T \leq T_I)$	$P(T \geq T_i) = 1 - P(T \leq TI)$
15-Sep-04	0.70	0.936045	0.088425

In the table:

TO CALCULATE	USE	EXAMPLE
Discount Factor at time T	`AdRate()`	`DF2=AdRate("23SEP01", "15SEP04", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.936045`
	`AdInterp()`	Interpolation on the Yield Curve at the maturity date (15SEP04): `=AdInterp("15SEP04";ZCDate;ZCRate;"IM:LIN") = 0.936045`

Hence,

$$NPV_{Float} = (1-R) \times DF(T_i) \times P(\tau \leq T) = (1-30\%) \times 0.936045 \times 0.088492 = 0.057983$$

Therefore the Net Present Value of the CDS is:

$$NPV = NPV_{Float} - NPV_{Fixed} = 0.057983 - 0.036708 = 0.021275$$

Example of CDS Pricing from a Default Probability Curve

Example in Thomson Reuters Eikon Excel

Consider a Credit Default Swap, which expires on 15th September 2004. The settlement date is on 23rd September 2001, it starts on 15th September 2002, the spread is 200 basis points, and the recovery rate is 30%.

Settle: 23SEP01, Start Date: 15SEP02, Maturity Date: 15SEP04, Spread=200, R=30%,

CdsStructure	CDSTYPE:EURCDS CLDR:NULL LFLOAT LFIXED FRQ:4
CreditStructure	RISKMODEL:CURVE RECOVERY:0.3 ND:DIS
Ratestructure	RM:YC ZCTYPE:DF

Risk-Free DF Curve

19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%
19-Sep-06	86.05%
19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

Probability Curve

19-Sep-02	0%
24-Mar-03	2.26%
23-Sep-03	4.46%
23-Sep-04	8.94%
23-Sep-05	13.02%
25-Sep-06	18.67%
24-Sep-07	23.47%
23-Sep-09	36.96%
24-Sep-12	51.76%

In this example the return value of the `AdCdsNpv()` function is retrieved and then the formula to recalculate the result is applied manually.

Syntax

```
=AdCdsNpv("23SEP01","15SEP02","15SEP04",200,Risk-Free DF curve,Proba Curve,"CDSTYPE:EURCDS
CLDR:NULL LFLOAT LFIXED FRQ:4","RISKMODEL:CURVE RECOVERY:0.3 ND:DIS","RM:YC ZCTYPE:DF")
```

CreditStructure

KEYWORD	SPECIFIES
CDSTYPE:EURCDS	a European CDS
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage

Result returned by the function

=`AdCdsNpv()` returns:

NPV	0.021263

Manual Calculation

Fixed leg

$DATE_I$	CF_I	$P(T \leq_{TI})$	$P(T \geq T_I)=1 - P(T \leq TI)$	$DF(T_I)$	$CFIXP(T \geq T_I) X DF(T_I)$
15-Dec-02	50.00	0.010576	0.989424	0.992206	49.085595
15-Mar-03	50.00	0.021517	0.978483	0.984143	48.148333
15-Jun-03	50.00	0.032581	0.967419	0.975901	47.205228
15-Sep-03	50.00	0.043632	0.956368	0.967658	46.271872
15-Dec-03	50.00	0.054755	0.945245	0.959789	45.361758
15-Mar-04	50.00	0.065897	0.934103	0.951932	44.460122
15-Jun-04	50.00	0.077161	0.922839	0.943989	43.557476
15-Sep-04	50.00	0.088425	0.911575	0.936045	42.663778

In the table:

TO CALCULATE	USE	EXAMPLE
$P(\tau \leq T_i)$	`AdDefaultProba()`	`=AdDefaultProba("15SEP02",Datei, CIR Array, "LAY:H")`
Discount Factor at time T	`AdRate()`	For example on the first cash flow: `DF2 = AdRate("23SEP01", "15DEC02", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.992206`
	`AdInterp()`	Interpolation on the Yield Curve at the date (15DEC02) of the first cash flow for example: `AdInterp("15DEC02";ZCDate;ZCRate;"IM:LIN") = 0.992206`

Hence,

$$NPV_{Fixed} = \sum_{i=1}^{8} CF_i \times DF(T_i) \times P(\tau \geq T_i) = 0.036708$$

Floating leg

DATE	CONTINGENT PAYMENT	DF	P(T≤T)
15-Sep-04	0.70	0.936045	0.088425

In the table:

TO CALCULATE	USE	EXAMPLE
Discount Factor at time T	`AdRate()`	`DF2=AdRate("23SEP01", "15SEP04", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.936045`
	`AdInterp()`	Interpolation on the Yield Curve at the maturity date (15SEP04): `=AdInterp("15SEP04";ZCDate;ZCRate;"IM:LIN") = 0.936045`

Hence:

$$NPV_{Float} = (1-R) \times DF(T_i) \times P(\tau \leq T) = (1-30\%) \times 0.936045 \times 0.088425 = 0.057939$$

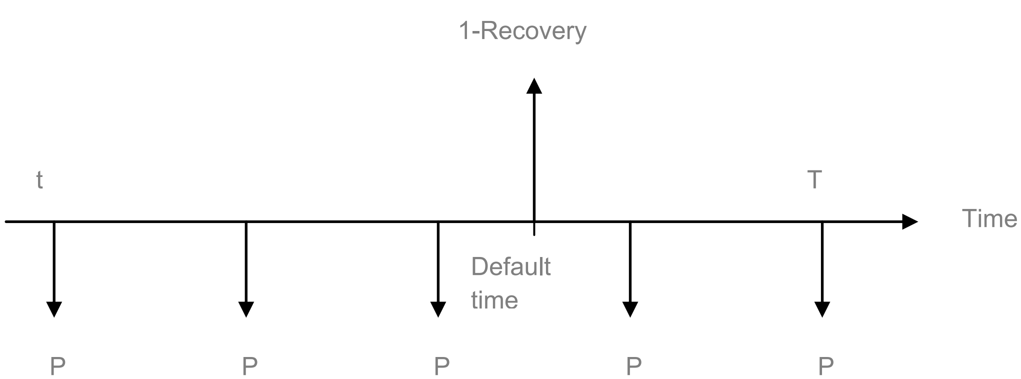
Therefore the Net Present Value of the CDS is:

$$NPV = NPV_{Float} - NPV_{Fixed} = 0.057939 - 0.036675 = 0.021263$$

AMERICAN CDS PRICING

Analytical Formulas

For an American CDS, the contingent payment is paid at the time of the default event:



Fixed leg calculation

$$NPV_{Fixed} = \sum_{i=1}^{n} CF_i \times DF(T_i) \times P(\tau \geq T_i) \times fx$$

WHERE	DENOTES
Fx	Outright if the CDS is cross-currency (fx=1 otherwise)

$P(\tau \geq T_i)$ which is the probability to default after the time Ti of the ith cash flows payment.

DF(T_i)= discount factor at time Ti.

Floating leg calculation

$$n = \mathrm{int}\left(\frac{Maturity - StartDate}{NbDays}\right)$$

$$dt = \frac{Maturity - StartDate}{n}$$

$$NodeDate_i = StartDate + i \times dt$$ for i=1,…,n

$$NPV_{Float} = (1-R) \times \sum_{j=1}^{m} DF(T_j) \times \left[P(\tau \geq T_{j-1}) - P(\tau > T_j)\right]$$

WHERE	DENOTES
$P(\tau \geq T_{j-1})$	The probability to default after the time Tj-1 of the (j-1)th discretization step.
$P(\tau \geq T_j)$	The probability to default after the time Tj of the jth discretization step.
$DF(T_j)$	Discount factor at time T_j.

CDS Net Present Value calculation

PAID LEG	NPV FORMULA
Fixed	$NPV = NPV_{Float} - NPV_{Fixed}$
Floating	$NPV = NPV_{Fixed} - NPV_{Float}$

Example of CDS pricing from Cox-Ingersoll-Ross Coefficients

Example in Thomson Reuters Eikon Excel

Consider a Credit Default Swap, which expires on 15th September 2004. The settlement date is on 23rd September 2001, it starts on 15th September 2002, the spread is 200 basis points, and the recovery rate is 30%.

So we have:

Settle	23SEP01
Start Date	15SEP02
Maturity Date	15SEP04
Spread	200
R	30%
NbDays	146

CdsStructure	CDSTYPE:AMERCDS CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4
CreditStructure	RISKMODEL:CIR RECOVERY:0.3 NBDAYS:146 ND:DIS
Ratestructure	RM:YC ZCTYPE:DF

RISK-FREE DF CURVE	
23-Sep-01	100.00%
23-Sep-02	96.73%
23-Sep-03	93.57%
23-Sep-04	89.87%
23-Sep-05	86.05%
23-Sep-06	82.23%
23-Sep-07	78.31%
23-Sep-08	74.46%
23-Sep-09	70.75%
23-Sep-10	67.22%
23-Sep-11	63.90%

CIR ARRAY

0.040089
0.101171
0.109750

In this example the return value of the `AdCdsNpv()` function is retrieved and then the formula to recalculate the result is applied manually.

Syntax

```
=AdCdsNpv("23SEP01","15SEP02","15SEP04",200,Risk-Free DF curve,CIR_Array,"CDSTYPE:AMERCDS
CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4","RISKMODEL:CIR RECOVERY:0.3 NBDAYS:146 ND:DIS","RM:YC
ZCTYPE:DF")
```

CdsStructure

KEYWORD	SPECIFIES
CDSTYPE:AMERCDS	an American CDS
AOD:YES	that the accrued is paid at the credit event date

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CIR	the Cox-Ingersoll-Ross model
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage

Result returned by the function

=`AdCdsNpv()` returns:

NPV	**0.021263**

Manual Calculation

Fixed leg

DATE$_i$	CF$_i$	$P(T \le T_i)$	$P(T \ge T_i) = 1 - P(T \le T_i)$	$DF(T_i)$	$CF_i X P(T \ge T_i) X DF(T_i)$
15-Dec-02	50.00	0.010576	0.989851	0.992206	49.106779
15-Mar-03	50.00	0.021517	0.979524	0.984143	48.199547
15-Jun-03	50.00	0.032581	0.968692	0.975901	47.267337
15-Sep-03	50.00	0.043632	0.957604	0.967658	46.331666
15-Dec-03	50.00	0.054755	0.946406	0.959789	45.417475
15-Mar-04	50.00	0.065897	0.935126	0.951932	44.508782
15-Jun-04	50.00	0.077161	0.922839	0.943989	43.557476
15-Sep-04	50.00	0.088425	0.911575	0.936045	42.663778

In the table:

TO CALCULATE	USE	EXAMPLE
$P(\tau \le T_i)$	`AdDefaultProba()`	`=AdDefaultProba("15SEP02",Datei,CIR Array, "LAY:H")`
Discount Factors	`AdRate()`	For example on the first cash flow: `DF2 = AdRate("23SEP01", "15DEC02", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.992206`

TO CALCULATE	USE	EXAMPLE
	`AdInterp()`	Interpolation on the Yield Curve at the date (15DEC02) of the first cash flow for example: `AdInterp("15DEC02";ZCDate;ZCRate;"IM:LIN") = 0.992206`

Hence,

$$NPV_{Fixed} = \sum_{i=1}^{8} CF_i \times DF(T_i) \times P(\tau \geq T_i) = 0.036708$$

Floating leg

DATE	CONTINGENT PAYMENT	DF	P(T≤T)
15-Sep-04	0.70	0.936045	0.088425

In the table:

TO CALCULATE	USE	EXAMPLE
Discount Factor at time T	`AdRate()`	`DF2=AdRate("23SEP01", "15SEP04", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.936045`
	`AdInterp()`	Interpolation on the Yield Curve at the maturity date (15SEP04): `=AdInterp("15SEP04";ZCDate;ZCRate;"IM:LIN") = 0.936045`

Floating leg

Cash flows dates calculation for the floating leg are:

Cash flows dates calculation for the floating leg are:

$$n = int(\frac{Maturity - StartDate}{NbDays}) = int(\frac{15SEP04 - 15SEP02}{146}) = 5$$

$$dt = \frac{15SEP04 - 15SEP02}{5} = 146$$

$$NodeDate_i = StartDate + i \times dt = 15SEP02 + i \times 146$$ for i=1,…,5

Hence we have the table:

DATEJ	CONTINGENT PAYMENT	$DF(T_J)$	$P(T \leq T_J)$	$P(T \geq T_J)=1 - P(T \leq T_J)$	$DF(T_J)X[P(T \geq T_{J-1}) - P(T \geq T_J)]$
15-Sep-02	0.7	1	0	1	
08-Feb-03	0.7	0.987278	0.016428	0.983572	0.016219
04-Jul-03	0.7	0.974198	0.033578	0.966422	0.016708
27-Nov-03	0.7	0.961343	0.051362	0.948638	0.017096
21-Apr-04	0.7	0.948737	0.069567	0.930433	0.017272
15-Sep-04	0.7	0.936045	0.088492	0.911508	0.017715

In the table:

TO CALCULATE	USE	EXAMPLE
Discount Factor at time T	`AdRate()`	`DF2=AdRate("23SEP01", "08FEB03", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.987278`
	`AdInterp()`	Interpolation on the Yield Curve at the at the date (08FEB03): `=AdInterp("08FEB03";ZCDate;ZCRate;"IM:LIN") = 0.987278`

Hence,

$$NPV_{Float} = (1-R)\times\sum_{j=1}^{m} DF(T_j)\times\left[P(\tau \geq T_{j-1}) - P(\tau > T_j)\right] = (1-30\%)\times(0.016219+0.016708+0.017096+0.017272+0.017715)$$

$$= (1-30\%)\times(0.08501) = 0.059507$$

Therefore the Net Present Value of the CDS is:

$$NPV = NPV_{Float} - NPV_{Fixed} = 0.059507 - 0.036708 = 0.022799$$

Example of a CDS Pricing from a Default Probability Curve

Example in Thomson Reuters Eikon Excel

Consider a Credit Default Swap, which expires on 15th September 2004. The settlement date is on 23rd September 2001, it starts on 15th September 2002, the spread is 200 basis points, and the recovery rate is 30%.

So we have:

Settle	23SEP01
Start Date	15SEP02
Maturity Date	15SEP04
Spread	200
R	30%
NbDays	146

CdsStructure	CDSTYPE:AMERCDS CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4
CreditStructure	RISKMODEL:CURVE RECOVERY:0.3 NBDAYS:146 ND:DIS
Ratestructure	RM:YC ZCTYPE:DF

RISK-FREE DF CURVE	
19-Sep-02	100.00%
19-Sep-03	96.73%
19-Sep-04	93.57%
19-Sep-05	89.87%

RISK-FREE DF CURVE	
19-Sep-06	86.05%
19-Sep-07	82.23%
19-Sep-08	78.31%
19-Sep-09	74.46%
19-Sep-10	70.75%
19-Sep-11	67.22%
19-Sep-12	63.90%

PROBA CURVE	
19-Sep-02	0%
24-Mar-03	2.26%
23-Sep-03	4.46%
23-Sep-04	8.94%
23-Sep-05	13.02%
25-Sep-06	18.67%
24-Sep-07	23.47%
23-Sep-09	36.96%
24-Sep-12	51.76%

In this example the return value of the `AdCdsNpv()` function is retrieved and then the formula to recalculate the result is applied manually.

Syntax

```
=AdCdsNpv("23SEP01","15SEP02","15SEP04",200,Risk-Free DF curve,Proba Curve,"CDSTYPE:AMERCDS
CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4","RISKMODEL:CURVE RECOVERY:0.3 NBDAYS:146
ND:DIS","RM:YC ZCTYPE:DF")
```

CdsStructure

KEYWORD	SPECIFIES
CDSTYPE:AMERCDS	an American CDS
AOD:YES	that the accrued is paid at the credit event date

CreditStructure

KEYWORD	SPECIFIES
RISKMODEL:CURVE	the credit event probability curve
RECOVERY:XX	the recovery rate value in percentage

Result returned by the function

=`AdCdsNpv()` returns:

NPV	0.021263

Manual Calculation

Fixed leg

$DATE_i$	DFI *	$P(T \le_{Ti})$	$P(T \ge T_i)=1 - P(T \le T_i)$	$DF(T_i)$	$CF_iXP(T \ge T_i)X\ DF(T_i)$
15-Dec-02	50.00	0.010576	0.989424	0.992206	49.085595
15-Mar-03	50.00	0.021517	0.978483	0.984143	48.148333
15-Jun-03	50.00	0.032581	0.967419	0.975901	47.205228
15-Sep-03	50.00	0.043632	0.956368	0.967658	46.271872
15-Dec-03	50.00	0.054755	0.945245	0.959789	45.361758
15-Mar-04	50.00	0.065897	0.934103	0.951932	44.460122
15-Jun-04	50.00	0.077161	0.922839	0.943989	43.557476
15-Jun-04	50.00	0.077161	0.922839	0.943989	43.557476

In the table

TO CALCULATE	USE	EXAMPLE
$P(\tau \le T_i)$	`AdDefaultProba()`	`=AdDefaultProba("15SEP02",Datei,CIR Array, "LAY:H")`
Discount Factor at time T	`AdRate()`	For example on the first cash flow: `DF2 = AdRate("23SEP01", "15DEC02", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.992206`
	`AdInterp()`	Interpolation on the Yield Curve at the date (15DEC02) of the first cash flow for example: `AdInterp("15DEC02";ZCDate;ZCRate;"IM:LIN") = 0.992206`

Hence,

$$NPV_{Fixed} = \sum_{i=1}^{8} CF_i \times DF(T_i) \times P(\tau \geq T_i) = 0.036675$$

Floating leg:

Cash flows dates calculation for the floating leg are:

$$n = \mathrm{int}(\frac{Maturity - StartDate}{NbDays}) = \mathrm{int}(\frac{15SEP04 - 15SEP02}{146}) = 5$$

$$dt = \frac{15SEP04 - 15SEP02}{5} = 146$$

$$NodeDate_i = StartDate + i \times dt = 15SEP02 + i \times 146$$ for i=1,…,5

Hence we have the following table:

DATE$_J$	CONTINGENT PAYMENT	DF(T_J)*	P(T≤T_J)	P(T≥T_J)=1 - P(T≤T_J)	DF(TJ)X[P(T≥T_{J-1}) - P(T≥T_J)]
15-Sep-02	0.70	0.70	0	1	
08-Feb-03	0.70	0.70	0.017262	0.982738	0.017043
04-Jul-03	0.70	0.70	0.034863	0.965137	0.017147
27-Nov-03	0.70	0.70	0.052551	0.947449	0.017004
21-Apr-04	0.70	0.70	0.070427	0.929573	0.016959
15-Sep-04	0.70	0.70	0.088425	0.911575	0.016847

In the table:

TO CALCULATE	USE	EXAMPLE
Discount Factor at time T	`AdRate()`	`DF2=AdRate("23SEP01", "08FEB03", ZcDate:ZcRate, "RM:YC RATETYPE:CMP ZCTYPE:DF") = 0.987278`
	`AdInterp()`	Interpolation on the Yield Curve at the at the date (08FEB03): `=AdInterp("08FEB03";ZCDate;ZCRate;"IM:LIN") = 0.987278`

Hence,

$$NPV_{Float} = (1-R) \times \sum_{j=1}^{m} DF(T_j) \times \left[P(\tau \geq T_{j-1}) - P(\tau > T_j)\right] = (1-30\%) \times \left(0.017043 + 0.017147 + 0.0170004 + 0.016959 + 0.016847\right)$$

$$= (1-30\%) \times \left(0.08500\right) = 0.059500$$

Therefore the Net Present Value of the CDS is:

$$NPV = NPV_{Float} - NPV_{Fixed} = 0.059500 - 0.036675 = 0.022825$$

CDS BIG BANG IMPACT ON PRICING WITH ADFIN

The Adfin pricing library replicates the methodology used by the CDS pricer published on www.cdsmodel.com. For example, note that the accrual on default of the protection leg is calculated under assumptions that permit an exact integration, whereas the old JPM method approximated the integral.

There are three important concepts to define: the Par Spread, the Upfront Payment, and the Conventional Spread.

- The Par spread is the spread at which the CDS is traded on the market, meaning the CDS that gives an NPV equal to 0 for the default probability curve and for all tenors
- The upfront payment (in %) is the amount of cash to be paid to the protection seller at the settlement of the deal for a CDS with a fixed coupon (100 or 500). It can be either positive or negative
- The conventional spread is the spread that makes the NPV equal to the upfront with a fixed coupon payment. Unlike the par spread, each tenor has its own default probability curve. This calculation is standardized using a flat hazard rate and the ISDA Zero Coupon rate.

How to match the ISDA/Markit calculators

- Build your zero coupon curve using AdTermStructure()
- Change keywords in CDSStructure to reflect new contract specifications
- Accrued has been added to AdCdsNpv() as a fourth output

In the inputs, you have to enter the market data for discount factors. For example, this data can be obtained from https://www.markit.com/. For example, on the trade date of 04-Mar-2010, we obtain the following inputs:

M	1M,M	0.2281%
M	2M,M	0.2394%
M	3M,M	0.2519%
M	6M,M	0.3832%
M	9M,M	0.5976%
M	1Y	0.8344%
S	2Y	1.0368%
S	3Y	1.6305%
S	4Y	2.1380%
S	5Y	2.5436%
S	6Y	2.8870%
S	7Y	3.1487%
S	8Y	3.3545%

S	9Y	3.5250%
S	10Y	3.6656%
S	12Y	3.9006%
S	15Y	4.1415%
S	20Y	4.3115%
S	25Y	4.3950%
S	30Y	4.4205%

Using the `AdTermStrucure()` function, we get the following curve (please refer to the *Adfin Term Structure Calculation Guide* for more information):

08-Mar-10	1
08-Apr-10	0.9998036
10-May-10	0.9995812
08-Jun-10	0.9993567
08-Sep-10	0.9980453
08-Dec-10	0.9954557
08-Mar-11	0.9916111
08-Mar-12	0.979499
08-Mar-13	0.9521254
10-Mar-14	0.9173192
09-Mar-15	0.8790974
08-Mar-16	0.8383507
08-Mar-17	0.7982538

08-Mar-18	0.7592775
08-Mar-19	0.7212892
09-Mar-20	0.684675
08-Mar-22	0.6145853
10-Mar-25	0.5212228
08-Mar-30	0.4037828
08-Mar-35	0.3146851
08-Mar-40	0.2493768

Features of the CDS:

Trade Date	04-Mar-10
Last Coupon Date	20-Dec-09
Maturity Date	20-Mar-15
Coupon (bps)	500
Recovery	40%
Notional	1,000,000.00

Conversion Examples

Converting a conventional spread of 501 to an upfront

Build the credit structure using `AdCreditStructure()`

04-Mar-10	0
20-Mar-15	0.346876128

To obtain the upfront, use the `AdCdsNpv()` function:

Syntax

```
=AdCdsNpv("04mar10","20dec09","20mar15",500,IsdaCurve,CreditStructure, "FRCD:40167 CLDR:WEEKEND
CFADJ:Yes CFADJ:42082:NO DMC:F AOD:YES MDILD:YES AODMT:Exact STEPIN:1 CDSTYPE:AMERCDS
CASHSETTLEDELAY:3WD LFIXED FRQ:4 CCM:MMAO ALIMIT:NO ACC:AO PX:C","RISKMODEL:CURVE IMPROBA:CFT
NBDAYS:1 RECOVERY:0.4","RM:YC ZCTYPE DF IM:LOG CLDRADJ:CLDR')
```

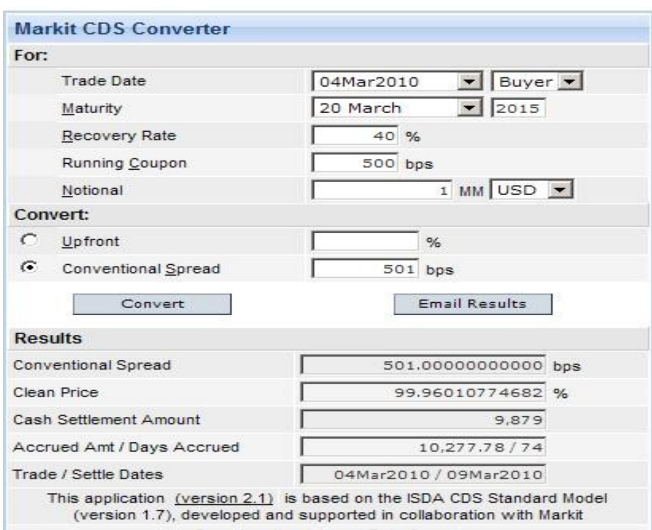
Add or change keywords in the CDS Structure to reflect new contract specifications:

KEYWORD	COMMENT
AODMT:EXACT	Specifies the calculation of accrual on default
MDILD:YES	Specifies that the end date is included in the contract
STEPIN:1	Specifies that T+1 is used to determine accrual dates
CASHSETTLEDELAY:3WD	To discount premium and accrued accordingly
CFADJ:YES CFADJ:[MaturityDate]-1:NO	Specifies that the maturity date must not be adjusted

Finally, we obtain the upfront:

Upfront	0.0398923%

For the sake of comparison, here is the corresponding Markit Calculator result:



Upfront% = 1 – Clean Price% = 1 – 99.96010774682% = 0.0398923%

Converting an upfront to a conventional spread

We build the CDS structure using `AdCreditStructure()` as follows:

FRCD:40167 CLDR:WEEKEND CFADJ:YES CFADJ:42082:NO DMC:F AOD:YES MDILD:YES AODMT:EXACT STEPIN:1 CDSTYPE:AMERCDS CASHSETTLEDELAY:3WD LFIXED FRQ:4 CCM:MMA0 ALIMIT:NO ACC:A0 PX:C UPFRONT:-0.1983

We obtain:

04-Mar-10	0
20-Mar-15	0.065458395

To calculate the conventional spread, we use the `AdCdsSpread()` function:

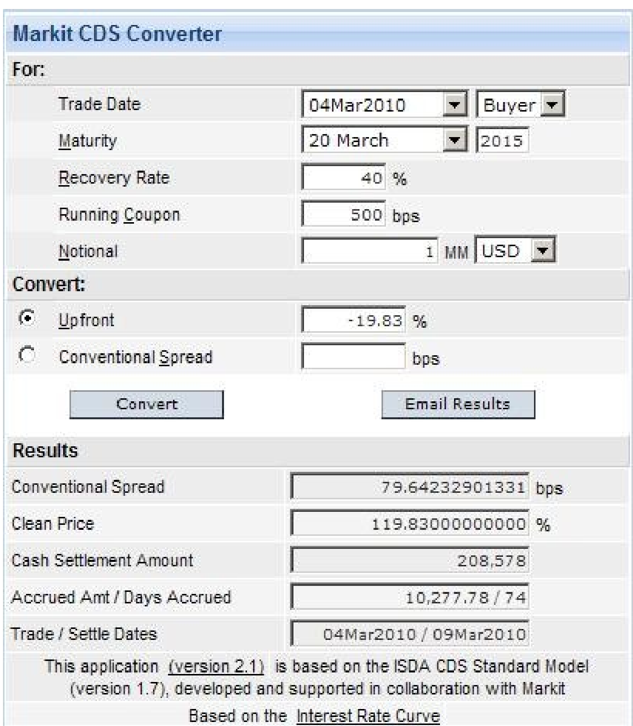
Syntax

```
=AdCdsSpread('04mar10","20dec009","20mar15",0,IsdaCurve,CreditStructure, "FRCD:40167 CLDR:WEEKEND
CFADJ:YES CFADJ:42082:NO DMC:F AOD:YES MDILD:YES AODMT:EXACT STEPIN:1 CDSTYPE:AMERCDS
CASHSETTLEDELAY:3WD LFIXED FRQ:4 CCM:MMAO ALIMIT:NO ACC:AO PX:C","RISKMODEL:CURVE IMPROBA:CFI
NBDAYS:1 RECOVERY:0.4","RM:YC ZCTYPE:DF IM"LOG CLDRADJ:CLDR")
```

Finally, we obtain the conventional spread:

Conventional Spread	79.64232901331

Here is the corresponding Markit calculator:



Conventional spread = 79.64232901331

Pricing a CDS

The accrual on default is an integral. Its value can be approximated numerically, or calculated exactly with some assumptions. Adfin takes the latter approach.

There are several ways of integrating the accrual on default of the fee leg and the contingent leg.

Most financial software products approximate all integrals in this calculation numerically. The ISDA xll library (which is based on a JPM model) uses the exact integral described here.

We have the following formula for the fixed (also known as premium) leg:

$$\textbf{Premium Leg} = S_N \sum_{i=0}^{N} \Delta_i DF_i s_i - S_N \sum_{i=0}^{N-1} \int_{T_i}^{T_{i+1}} DF(l) \Delta_i \frac{l - T_i}{T_{i+1} - T_i} ds(l)$$

WHERE	DENOTES
DF_i	Discount factor
S_N	Fixed coupon/spread for the CDS of maturity TN
Δ_i	Year fraction between T_i and T_{i+1}
S_i	Probability that the reference entity survives time T_i, observed at time 0

The hazard rate is assumed to be a piecewise constant function composed of individual hazard rates λ_k. The forward rate curve is assumed to be deterministic, and also piecewise constant composed of the rates f_k. Under these assumptions, the integral for accrual on default can be written as follows:

$$\textbf{Accrual on Default} = S_N \sum_{i=0}^{N-1} \frac{\lambda_i \Delta_i}{T_{i+1} - T_i} \sum_{k=0}^{n(i)} \left[\left((T'_k - T_i) + \frac{1}{\lambda_k + f_k} \right) \frac{s_k DF_k}{\lambda_k + f_k} - \left((T'_{k+1} - T_i) + \frac{1}{\lambda_k + f_k} \right) \frac{s_{k+1} DF_{k+1}}{\lambda_k + f_k} \right]$$

We have the following formulas for the contingent leg:

American case: payment at default time

$$\textbf{Protection Leg} = -LGD \int_0^{T_N} DF(l) ds(l)$$

, with LGD=(1–recovery)

European case: payment at maturity time

$$\textbf{Protection Leg} = -LGD \times DF_N (1 - s_N)$$

, with LGD=(1–recovery)

We again assume that the hazard rate and forward rate are piecewise constant functions, as defined for the fixed leg. The American integral can then be calculated exactly as follows:

$$\textbf{Protection Leg} = \sum_{i=0}^{N} \frac{\lambda_i}{\lambda_i + f_i} \left(1 - e^{-(\lambda_i + f_i)(T_{i+1} - T_i)} \right) s_i DF_i$$

CREDIT LINKED NOTE PRICING

Since a CLN is a bond plus a CDS, its evaluation can be viewed as the sum of a classical bond pricing and a CDS pricing detailed upwards.

SPREAD CALCULATION

The spread calculation is a solver on the NPV calculation. The first solver used is Newton; when Newton fails, we use dichotomy.

Example in Thomson Reuters Eikon Excel

We use the same feature of the American CDS described upwards but with a new NPV equals to 0.02.

Syntax

```
=AdCdsSpread("23SEP01","15FEB02","15SEP04",0.02,Risk-Free DF curve,CIR_Array,"CDSTYPE:AMERCDS CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4","RISKMODEL:CIR RECOVERY:0.3 NBDAYS:146 ND:DIS","RM:YC ZCTYPE:DF")
```

Result returned by the function

=`AdCdsSpread()` returns:

Spread	215.2515

Manual Calculation

As explained above, the spread is calculated by solving the following problem:

We use the `AdCdsNpv()` function for our calculation.

Syntax

```
=AdCdsNpv("23SEP01","15FEB02","15SEP04",XX,Risk-Free DF curve,CIR_Array,"CDSTYPE:AMERCDS CLDR:NULL LFLOAT AOD:NO LFIXED FRQ:4","RISKMODEL:CIR RECOVERY:0.3 NBDAYS:146 ND:DIS","RM:YC ZCTYPE:DF")
```

For the moment we assume that XX (unknown spread) is 0. In the example, the value of XX is the one, which enables to return a NPV of the CDS equal to 0.02.

By using the Excel solver, we have:

Spread	215.2515

COLLATERALIZED DEBT OBLIGATIONS

A collateralized debt obligation (CDO) is similar to a CDS; in exchange for a set of periodic payments, one buys protection against losses due to default.

The difference between the two is that a CDO groups a large number of reference instruments (bonds, loans, CDSs, etc.) into a single debt instrument. A CDO is sold in slices of different risks. Each slice is known as a tranche.

Adfin Analytics functions apply to several types of CDOs: cash CDOs, synthetic CDOs, and single-tranche CDOs such as indexes (CDX, iTraxx, etc.).

The following sections discuss CDOs in detail:

CDO PRICING

The pricing model for a CDO is a multifactor copula model. Adfin Analytics calculates the distribution of the default loss conditionally on the factors of the model. The factors are random variables which describe the behavior of the credit market.

The basic formula that describes the model is

$$x_i = a_i M + \sqrt{1 + a_i^2} W_i \tag{1}$$

where x_i is a random variable. This basic formula defines a correlation structure between the x_i, which here depend partially on a single common factor M. M and W_i are also random, with independent, zero-mean, unit variance distributions. The coefficients a_i are constrained:

$$a_i \in [-1, 1]$$

Input factors

Adfin Analytics needs several input factors, which should be determined before calculation:

- The default probability of each issuer in the portfolio. The copula model decouples the individual distributions from the correlation structure. Adfin Analytics assumes that the marginal default probabilities are known (most likely derived from the credit default swap market).
- The correlation values (a_i).
- Discount factors or zero-bond prices for known maturities.
- Recovery rates. These can be determined from data published by rating agencies.
- The type of copula you want to use: Gaussian or Student's t. The Gaussian copula is the model described above, but more generically Adfin Analytics permits up to three common factors. The Student's t copula in Adfin Analytics is an extension of the Gaussian copula, again for up to three common factors. The underlying vector $(Z_1, Z_2 \ldots Z_N)$ is assumed to follow a Student's t-distribution with μ degrees of freedom. $Z_i = \sqrt{G} x_i$, where G follows an inverse Gamma distribution with $\mu / 2$ parameters.

- Expected Loss: Both the spread and contingent payment legs of a CDO strongly depend on the expected global losses in the portfolio. The total loss is divided into discrete time steps $(t_1, t_2 \ldots t_K)$, and Adfin Analytics calculates the probability at each step using a recursive approach.

Default correlation model

Let us assume that we have a debt portfolio of N loans or bonds. Define

τ_i	the default time of the ith company or issuer
$F_i(t)$	the probability that default of the ith company occurs before the date t.

If Q_i is the cumulative distribution of x_i , then x_i can be mapped to τ_i using a percentile-to-percentile transformation. That is,

$$Q_i(x) = F_i(t) \tag{2}$$

Combining this relation with the basic model for x_i, we get

$$P(\tau_i \leq t / M) = F_W \left\{ \frac{Q_i^{-1}(F_i(t)) - a_i M}{\sqrt{1 - a_i^2}} \right\} \tag{3}$$

This equation can be generalized to many common factors:

$$P(\tau_i \leq t / M_1, M_2, \ldots, M_{n)} = F_W \left\{ \frac{Q_i^{-1}(F_i(t)) - \sum a_{ij} M_j}{\sqrt{1 - \sum a_{ij}^2}} \right\} \tag{4}$$

Finally, an integral over the common factors must be performed to compute the cumulative distribution of the default time τ_i. For that we need the joint probability density of all common factors. Fortunately, we know that the common factors are independent. This fact makes the integration easier.

The user's choice of copula model is important. The standard Gaussian copula model is not very accurate in some circumstances. In the next section, we talk about some other copula models the end-user can choose to price CDO tranches.

Copula Models

Gaussian copula

Consider Equation (1) for all issuers. If we assume that

- M is a standard Gaussian variable, and
- $(W_1, W_2 \ldots W_N)$ is a Gaussian vector,

then the model is called a one-factor Gaussian copula. However, M may be extended to a Gaussian vector of common factors. This model is called a multi-factor Gaussian copula (Equation 4).

Student's *t* copula

This model is an extension of the Gaussian copula. Define the vector $(W_1, W_2 \ldots W_N)$ as follows:

$$W_i = \sqrt{Gx_i} \tag{5}$$

where G follows an inverse Gamma distribution with $\mu / 2$ parameters. The vector $(W_1, W_2 \ldots W_N)$ is assumed to follow a Student's *t* distribution with μ degrees of freedom. Furthermore, we only consider the case of symmetric random variables W_i. Let

$$W_i = \sqrt{Gx_i} \tag{6}$$

The vector $(W_1, W_2 \ldots W_N)$ is assumed to follow a Student's *t* distribution with μ degrees of freedom. Furthermore, we only consider the case of symmetric random variables W_i. Let

$$x_i = a_i M + \sqrt{1 - a_i^2 V_i} \tag{6}$$

If S_μ is the cumulative distribution function of W_i, then the conditional probability that a default of the ith issuer occurs before date t given (M, G) is

$$P(\tau_i \leq t \,|\, M, G) = F_V \left\{ \frac{G^{-1/2} S_\mu^{-1} F_i(t) - a_i M}{\sqrt{1 - a_i^2}} \right\} \tag{7}$$

This probability is integrated over M and G to determine the probability of default. This is a two-factor model.

Double t copula

In this model, the default times are modeled from a latent random vector $(x_1, x_2 \ldots x_N)$ such that

$$x_i = a_i \left(\frac{\mu_m - 2}{\mu_m} \right)^{\frac{1}{2}} M + \sqrt{1 - a_i^2} \left(\frac{\mu_v - 2}{\mu_v} \right)^{\frac{1}{2}} V_i \tag{8}$$

M and V_i are independent. They follow Student's *t*-distributions with μ_m and μ_v degrees of freedom respectively. The probability that the ith company defaults before date t is

$$P(\tau_i \leq t \,|\, M, G) = \left(S_{\mu v} \sqrt{\frac{\mu_v}{\mu_v - 2}} \right) \frac{H^{-1} F_i(t) - a_i M \sqrt{(\mu_m - 2) / \mu}}{\sqrt{1 - a_i^2}} \tag{9}$$

The probability should be integrated over the factor(s) M and V_i, as with the other copula models.

This is the most commonly used type of copula for pricing CDO tranches. Other variants exist, such as the Clayton copula. In the future we may implement other models to enrich the choice for the end-user.

The loss given default

In the "Copula Models" section , we introduced various formulas to compute the probability that a given company defaults before a fixed date t. Anyone who prices CDO tranches is mainly interested in the global loss risk in the debt portfolio.

A CDO tranche $[A;B]$ absorbs any loss greater than A and less than B that occurs in the debt portfolio. The protection buyer makes periodic interest payments on a nominal value to the protection seller.

If a default occurs, the protection seller must pay the amount of the loss to the protection buyer. The protection seller's future interest payments on the nominal value are reduced by the amount he paid to compensate the loss. The fixed leg is paid by the seller of protection. The floating leg is paid by the protection buyer. The issue is to calculate the expected loss, as both the floating and fixed legs strongly depend on its value.

Assume that the recovery rate of the ith company is R_i and that its nominal value is M_i. If this company defaults on date t, there is a loss equal to $(1–R_i)M_i$. The loss function at time t can be written as

$$(1-R_i)M_i 1_{\tau_i \le t}$$

The aggregate portfolio loss at time t is

$$L(t) = \sum_i (1-R_i)M_i 1_{\tau_i \le t} \tag{10}$$

In tranche $[A;B]$, expected loss absorbed at time t is

$$l_{[A,B]}(t) = E\left\{\left[\min(L(t),B) - A\right]^+\right\} \tag{11}$$

Now, if there is a tenor structure $\{t_0, t_1, \ldots t_k, \ldots t_K\}$, at time t_k, the floating leg cash flow is

$$\{l(t_k) - l(t_{k-1})\}$$

and the fixed leg cashflow is

$$\delta_k r\left\{B - A - \frac{1}{2}[\{l(t_k) - l(t_{k-1})\}]\right\}$$

Here δ_k is the accrual factor and r is the rate or the premium. The present value of the CDO is

Recursive approach

As the previous formula shows, we must calculate $l(t_k)$ for all $k \in [0\ldots$ Adfin takes a recursive approach.

The boundaries of the loss interval are 0 (zero loss) and $\max_i(l_i)$. To set up the recursive calculation, we must first choose an appropriate discretization step for this interval. Sidenius argues for a discretization method with little impact on the computational effort of the integration. His method also depends on a sensitivity threshold, which is arbitrarily chosen. Adfin adopts the same method, but instead of a sensitivity threshold sets a maximum iteration number for the loop.

Every company or obligor of the pool has a possible loss equal to $(1–R_i)M_i$.

Denote by $l_{\min}$ the minimum loss, $\min_i(1–R_i)M_i$. It is not to be confused with the minimum of the loss interval, which is 0.

Suppose we know the maximum iteration number S. Choose an obligor k from the pool and loop over i from 1 to S:

STEP	ACTION
1	Compute $$u_i = \frac{l_{\min}}{i}$$
2	Calculate the difference δ_{ki} between $(1–R_k)M_k$ and $$E\left[\frac{(1-R_k)M_k}{u_i}\right] * u_i$$ where $E[.]$ refers to the integer part (floor) of the result.

The only thing we need to keep from this loop is the value

$$E\left[\frac{(1-R_k)M_k}{u_i}\right],$$ which corresponds to the smallest difference δ_{ki}:

$$v_k = E\left[\frac{(1-R_k)M_k}{u_i}\right] = \mathrm{Argmin}_i\, \delta_{ki}$$

After checking each obligor in the pool, we have N values of v_k. The loss discretization step is

$$v = \frac{\sum (1-R_k)M_k}{\sum v_k}$$

Suppose that no debt instrument exists in a collateral portfolio. Then there is no loss in any bin defined by the discretization points, and the probability of loss is zero at every point. We add a single company to this pool with a possible loss equal to $(1–R_i)M_i$, and update the probability of loss at all discretization points. This step is repeated until we add the last company to the pool, and we have the probability of loss at all discrete points. This procedure requires the probability of default for each company, which was computed in previous sections.

Let $p^i(l,t|M)$ denote the probability that a loss l occurs before time t, conditional on the common factor vector M, in a reference portfolio of i companies. Assume that we know the loss probabilities $p^i(l,t|M)$ for all l.

Adding one company introduces one credit with a conditional default probability $q_{i+1}(t|M)$ (the Section "CDO Pricing" describes how to evaluate these terms for the various models). The loss distribution for the new reference portfolio of $i+1$ companies is given by

$$p^{i+1}(0,t/M) = p^i(0,t/M)(1-q_{i+1}(t/M)) \qquad (12) \qquad (12)$$
$$p^{i+1}(l,t/M) = p^i(l,t/M)(1-q_{i+1}(t/M)) + p^i(l-1,t/M)q_{i+1}(t/M)$$

for all $l \neq 1$.

Starting with the degenerate case of no credit (i=0 and $p^0(0,t|M)$=1), the following recursion determines the loss distribution of the reference portfolio with N credits at all points:

$p^N(l,t/M)$ for $l = 0,\ldots$

The unconditional loss distribution is

$$p(l,t) = \int_{-\infty}^{+\infty} p^N(l,t/M)g(M)dM \tag{13}$$

Finally, the equation for the expected loss (11) can be evaluated. This approach can also be used to compute the sensitivities of CDO tranches.

In a Cash CDO, the investor makes an initial investment which can never be negative. In a synthetic CDO, no initial investment is made. The value of the tranche can be negative or positive. The synthetic CDO is actually constructed using a portfolio of CDSs. The premiums are usually invested in safe bonds or assets. In contrast, a cash CDO's debt portfolio is completely transferred to the vehicle that creates tranches for market investors.

The structures of the two CDOs are different, but their pricing methods are similar.

IMPLIED CORRELATION

CDO tranches are also quoted in terms of implied correlations. Two such correlations exist: compound correlation and base correlation.

If we know the spread and the market value of a CDO tranche, we may be interested by an implicit quotation (in the same way that implied volatility provides an implicit measure of an option's worth). The usual solution is a compound correlation, which assumes that all names in the CDO have the same correlation with each common factor. This unique correlation is fictitious, thus 'implied'. A solver is needed to retrieve this value.

The base correlation is the fictitious constant correlation that sets the total value of all tranches, up to the attachment or detachment point of the tranche of interest, equal to zero.

Both correlations should be interpreted carefully. The compound correlation is not monotonic (a smile might be observed, for example). Base correlations may skew upward.

EXAMPLE USING THOMSON REUTERS EIKON EXCEL

Features of the CDO

CalcDate	01-Oct-07
StartDate	03-Oct-07
Maturity	5Y
Entities	125
Notional	1000000

Deal Spread	47.5
CDS Spread	30

CdoStructure: ATTPOINT:0 DETPOINT:0.06 CFADJ:NO DMC:N FRQ:4

KEYWORD	COMMENT
ATTPOINT:0	Specifies the attachment point to be 0%
DETPOINT:0.06	Specifies the detachment point to be 6%

The compound correlation is set to 32%.

Our example uses 2 different models: the curve model and the Poisson model.

The curve model

RiskStructure: RISKMODEL:MULTI:125:CURVE COPULA:GAUSSIAN

KEYWORD	COMMENT
RISKMODEL:MULTI:125: CURVE	The model is a multifactor copula with 125 entities.
COPULA:GAUSSIAN	Specifies that we use the Gaussian copula

RateStructure: RM:YC ZCTYPE:DF RATETYPE:CMP IM:LIN

SOURCE	VALUE
CreditStructure	RISKMODEL:CURVE NBDAYS:5 ND:DIS IM:LIN INSTTYPE:CDS RECOVERY:0.4
RateStructure	RM:YC ZCTYPE:DF IM:LIN ND:DIS

Using the `AdCreditStructure()` function, we get the following array of default probabilities from 125 CDSs, each with a spread of 30:

01/10/2007	**20/12/2012**
0	0.02587075
0	0.02587075

…	…
0	0.02587075

The `AdCdoNpv()` function then gives the market value.

Syntax

```
=AdCdoNpv("01oct07","03oct07","20dec12",47.5,Zero coupon curve,default proba
array,notional,RecoveryRate,CompoundCorrelation,CDOStructure,RiskStructure,RateStructure)*Notional
```

And the ratio of Payoff to the Default Present Value (that is, the NPV for a spread set to 0) is given by

```
=AdCdoNpv("01oct07","03oct07","20dec12",0,Zero coupon curve,default proba
array,notional,RecoveryRate,CompoundCorrelation,CDOStructure,RiskStructure,RateStructure)*Notional
```

Here are the results:

Fair Value/Market Value	Payoff/Default PV	Premium Value
1,292,147.58	1,438,951.26	146,803.68

The premium value is obtained as DefaultPV – FairValue.

The Poisson model

We consider a unique intensity computed by the following formula:

$$\lambda = \frac{S_i}{1 - R_i}$$

$$e^{-\lambda t_i} = 1 - P_{Default}$$

$$\Rightarrow \lambda = -\frac{\ln(1 - P_{Default})}{t_i}$$

Intensity	**0.005027387**

RiskStructure: RISKMODEL:MULTI:125:`POISSON COPULA:GAUSSIAN

KEYWORD	COMMENT
RISKMODEL:MULTI:125: POISSON	Use a Poisson model with 125 entities.

The remaining steps are identical to the curve model, with the following results:

Fair Value/Market Value	Value of PayOff/Default PV	Premium Value
1,294,672.69	1,441,442.70	146,770.02

CDO REFERENCES

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