University of Rome Tor Vergata
Undergraduate Degree (B.A.) in Global Governance

Course Syllabus

Applied Quantitative Analysis

12 CFU

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Course description

The Course is divided into two modules.

I. Calculus in one Variable (Prof. Moskovitz), 6 CFU
II. Introduction to differential calculus and its applications (Prof. Porretta), 6 CFU

I Module: In this course we will treat the essentials of calculus in one variable both thoroughly and with dispatch, giving the student the preparation needed to continue the study of this subject in several variables and differential equations, as well as apply it to economic problems in one variable. Calculus is fundamental and so is of importance to any area with a quantitative aspect. It is also one of the very big intellectual ideas of western civilization.

The course will address two basic questions and their many ramifications and consequences. First, how does one find the tangent line to a point on a smooth (plane) curve. Second, how does one find the area under such a curve.

Our study consists of 5 chapters. As the student will see, the separation cannot always be made strict since each of these flows into the other because of the Fundamental Theorem of Calculus. Numerous examples and exercises will be interspersed so that the student can take an activist role as well as gauge his or her own progress. Concerning prerequisites, a basic knowledge of College Algebra and Trigonometry should suffice.

1. Introduction: We will give a brief introduction to the real numbers where calculus takes place. We will also explain what the rational numbers are and their place within the real numbers. We then turn to functions of one real variable and their graphs, domain and range, Key examples include polynomials and trigonometric functions. Then follows radian measure, periodicity, roots and intercepts and the concepts of an even and odd function as well as the inverse of a function. We will study vectors and their inner products in R^2. Finally, we will give an informal definition of limits and continuity illustrated with key examples.

2. Differential Calculus: Here we will learn how to calculate derivatives and how they linearly approximate a function near a point. We will also work with higher derivatives and understand what a differential equation is, solving some simple ones. We will then apply these things to graphing functions, solving related rate and minimum maximum problems, L'Hopital’s rule for calculating limits and Newton's method, an iterative procedure for finding roots.

3. Integral Calculus: We will deal with the second question asked at the beginning. Namely, how to calculate the area under a plane curve and its consequences. We will develop Riemann integration for functions defined on a closed interval, and prove the Fundamental Theorem of Calculus in several forms, followed by integration by parts and the change of variable formula, calculation of areas and volumes of surfaces of revolution, and finally the definition and properties of the log and exponential functions. We will then use the exponential function to solve various rate of growth problems, such as compound interest as well as a number of differential equations.

4. Taylor's Theorem and Power Series: Then we study the approximation of important functions nearby a point (and sometimes uniformly) by polynomials, providing an important generalization of the linear approximations of the first chapter.

5. Polar Coordinates and improper integrals: We give a brief introduction to the relationship of rectangular coordinates in the plane to polar coordinates. In particular, we find the element of area in polar coordinates and apply this to calculating the area under a normal Gaussian distribution which is important in probability and statistics.

II Module

Topics

Real numbers and elementary functions (polynomials, exponentials, logarithms and trigonometric functions).
Discrete models: sequences, limits, cluster points, recurrence and discrete time growth models, infinite sums.
Binomials, discrete probability. Continuity and differentiability: derivatives, Taylor expansions.
Optimizations: local and global maxima and minima, Fermat's rule and necessary conditions, convexity and sufficient conditions.
Newton's method.
Integration: areas, antiderivatives and fundamental theorem of calculus.
Differential equations: models from social sciences and economics.
Multivariable calculus: partial derivation, optimization.

Learning objectives

Students are expected to get acquainted with the main concepts and tools required to understand and to handle basic mathematical models arising in economics and in other applications.
Learning skills: Comprehension and practice with basic differential calculus.

Required readings

Materials that will be provided by the Professors.

Exam: Oral and Written