

TECHNOLOGY AND SINGLE FIRM

FIRM'S BEHAVIOR

Input: factors of production in the production cycle
(labor, land, financial/physical capital)

Output what the firm produces by using the
inputs

technological constraints: combinations of inputs that
allow to produce a specific level of quantity

technology is the set of the possible/feasible
production plans

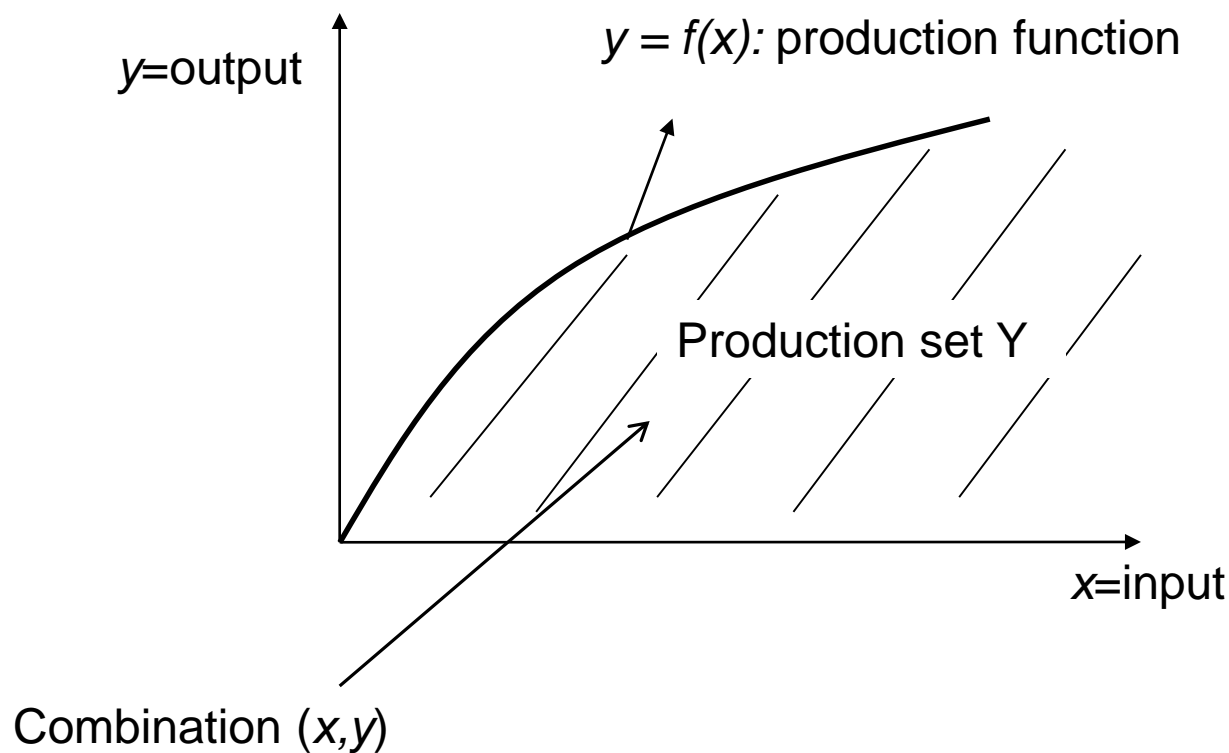
TIME FACTOR

Short run= at least one factor of production is fixed

- Land, fixed land size, number of machines, some works with fixed-period contract

Long run= all factors of production are variable

PRODUCTION SET : INPUT/ OUTPUT COMBINATIONS TECHNICALLY FEASIBLE



2-input production function

$$y = f(x_1, x_2)$$

Isoquant: the set of all possible input combinations x_1, x_2 just sufficient to produce a given amount of output y

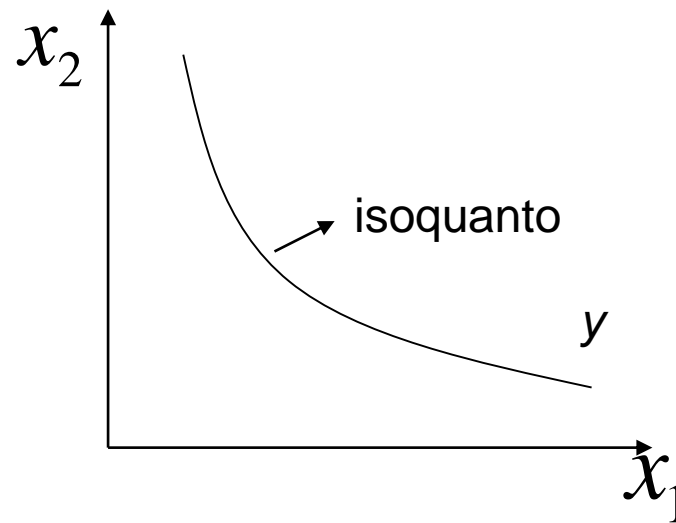


TABLE 1 A PRODUCTION FUNCTION AND TOTAL COST: HUNGRY HELEN'S COOKIE FACTORY

Number of Workers	Output (quantity of cookies produced per hour)	Marginal Product of Labor	Cost of Factory	Cost of Workers	Total Cost of Inputs (cost of factory + cost of workers)
0	0		\$30	\$ 0	\$30
		50			
1	50		30	10	40
		40			
2	90		30	20	50
		30			
3	120		30	30	60
		20			
4	140		30	40	70
		10			
5	150		30	50	80

THE PRODUCTION FUNCTION

Marginal Product

- The *marginal product* of any input in the production process is the increase in output that arises from an additional unit of that input

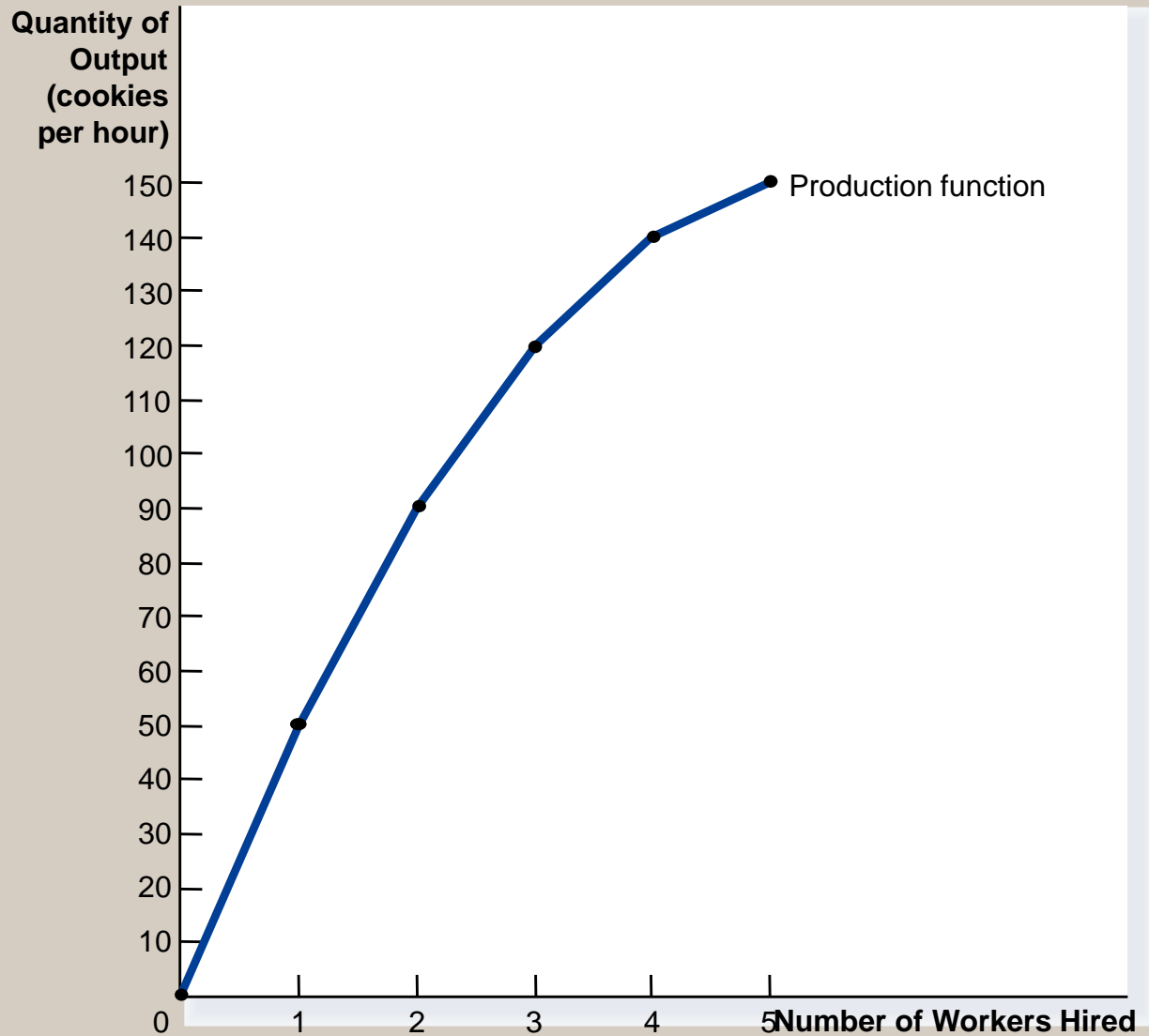
$$MP \text{ of input } 1 = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

$$MP_1 = \frac{\partial f(x_1, x_2)}{\partial x_1}$$

Diminishing Marginal Product

- *Diminishing marginal product* is the property whereby the marginal product of an input declines as the quantity of the input increases
 - Example: As more and more workers are hired, each additional worker contributes less and less to production because the firm has a limited amount of equipment.
 - It is a short run phenomenon (being all other input fixed, ex land-fixed- and workers)

FIGURE 2 HUNGRY HELEN'S PRODUCTION FUNCTION



Diminishing Marginal Product

- The **slope** of the **production function** measures the **marginal product** of an input, (i.e. worker)
- When the marginal product declines, the production function becomes **flatter**.

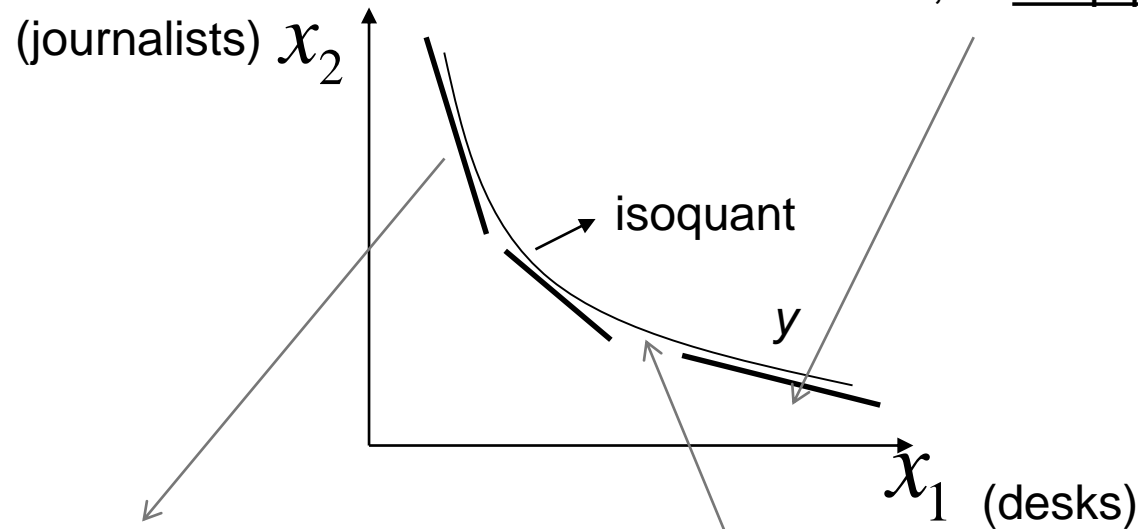
TECHNICAL RATE OF SUBSTITUTION (TRS)

$$TRS = \frac{dx_2}{dx_1} = - \frac{\partial f(.) / \partial x_1}{\partial f(.) / \partial x_2}$$

Marginal productivities

- The rate at which the firm have to substitute inputs to keep output constant
- Slope of the isoquant
- **Diminishing TRS**
 - The rate of substitution of inputs (in absolute value) decreases in the level of each input if production remains constant

When the use of x_1 is big: technology does not still require a big reduction in x_2 , in favor of one more unit of x_1 , to keep production constant



When the use of x_1 is small: technology implies that the firm needs to exchange (reduce) more inputs x_2 with one more unit of input x_1 to keep production constant

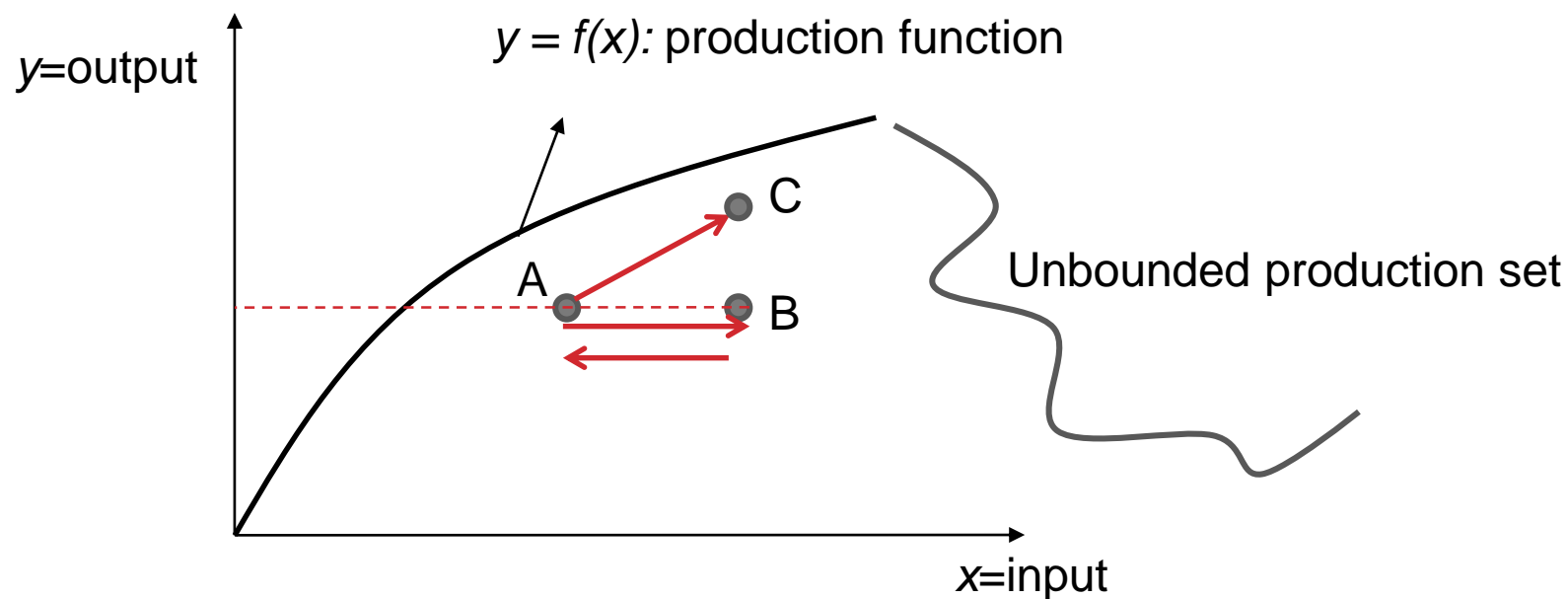
Q: production of newspapers

PROPERTIES OF TECHNOLOGY (PRODUCTION PLANS)

$$Y = \{y : y \leq f(x)\}$$

1. Y non-empty
2. Y closed: Y contains its boundary
3. No “free lunch”: you cannot produce output without using any inputs
4. Monotonicity (o free disposal): an increase in at least one input implies a production at least equal to the quantity produced without this increase. The firm can always throw away inputs if it wants (unbounded set)
5. Convexity: the production set Y is convex. For a given level of output, the set inputs pair able to produce at least that level of output is convex

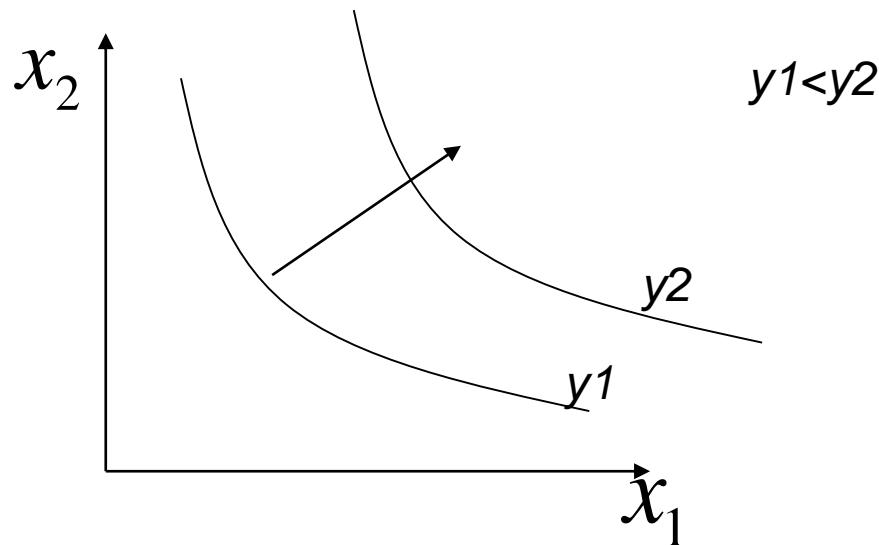
Monotonicity



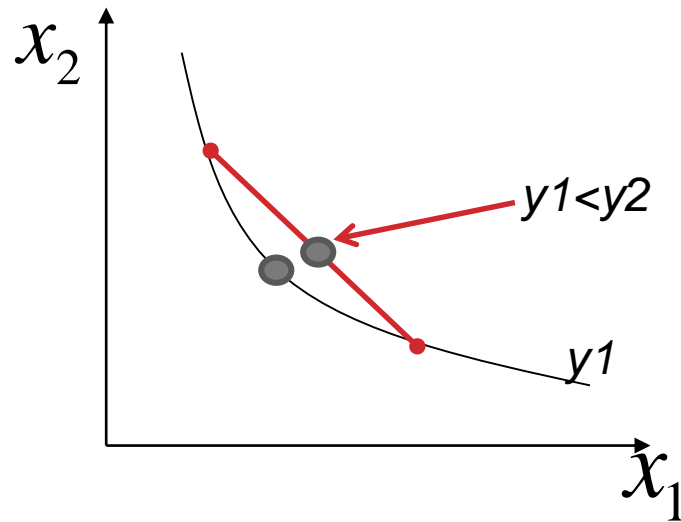
- A to C: more input more output
- A to B: more input same output
- B to A: it is possible to throw away inputs without costs

MONOTONICITY (...AGAIN):

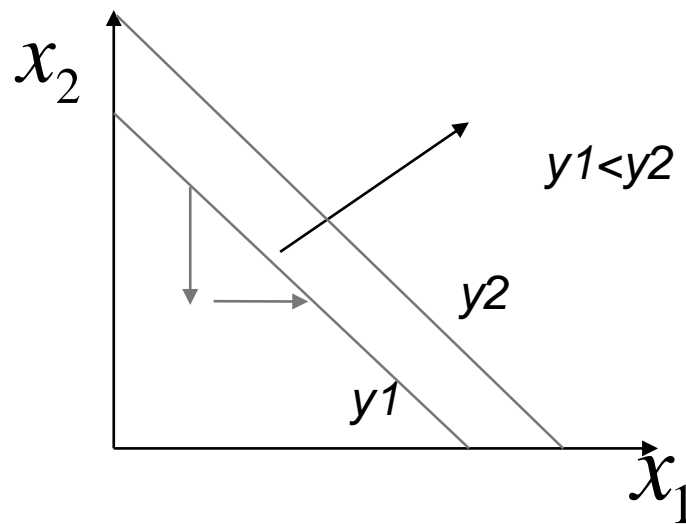
1. Isoquants far from the origin measure higher levels of output
2. *MP* of one input is positive: a higher input, keeping the other constant, always increases the output.



CONVEXITY



$$f(x_1, x_2) = x_1 + x_2$$



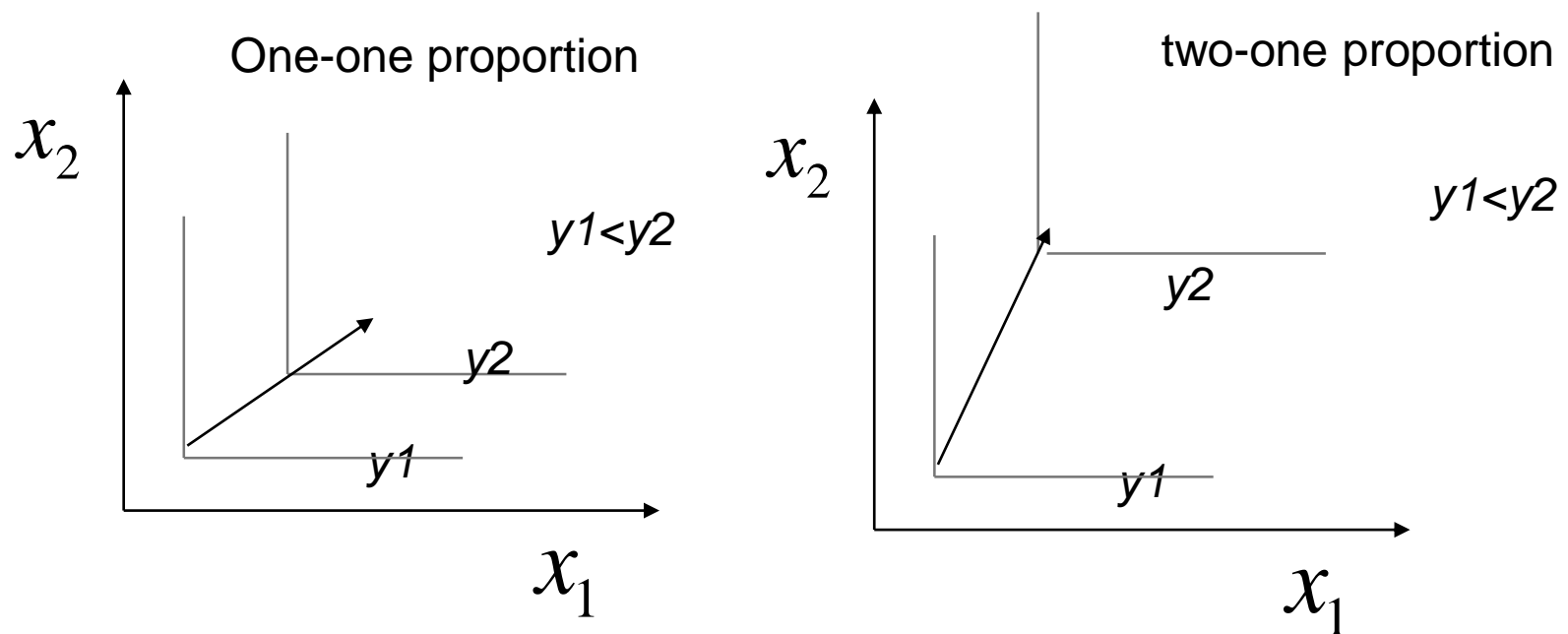
Technology with **perfect substitutes inputs**

- blue and black pen in the production of articles
- *Slides* and *ipad* in the production of some lectures
-

$$f(x_1, x_2) = \min\{x_1, x_2\}$$

Isoquants for the case of **fixed proportion inputs**

- pilots and airplane in the production/delivery of the «flight» (2/1)
- carpenter/hammer.... (1/1)



Well behaved isoquants with Cobb-Douglas production function

$$f(x_1, x_2) = Ax_1^a x_2^b$$

A : scale of production, how much output we get if we use one unit of each input

a e b measure how the output responds to the change in the inputs

Returns to scales: how the output changes if we change the level of inputs

Constant returns to scale if

$$f(tx_1, tx_2) = tf(x_1, x_2) \forall t \geq 0$$

Increasing return to scale if

$$f(tx_1, tx_2) > tf(x_1, x_2) \forall t > 1$$

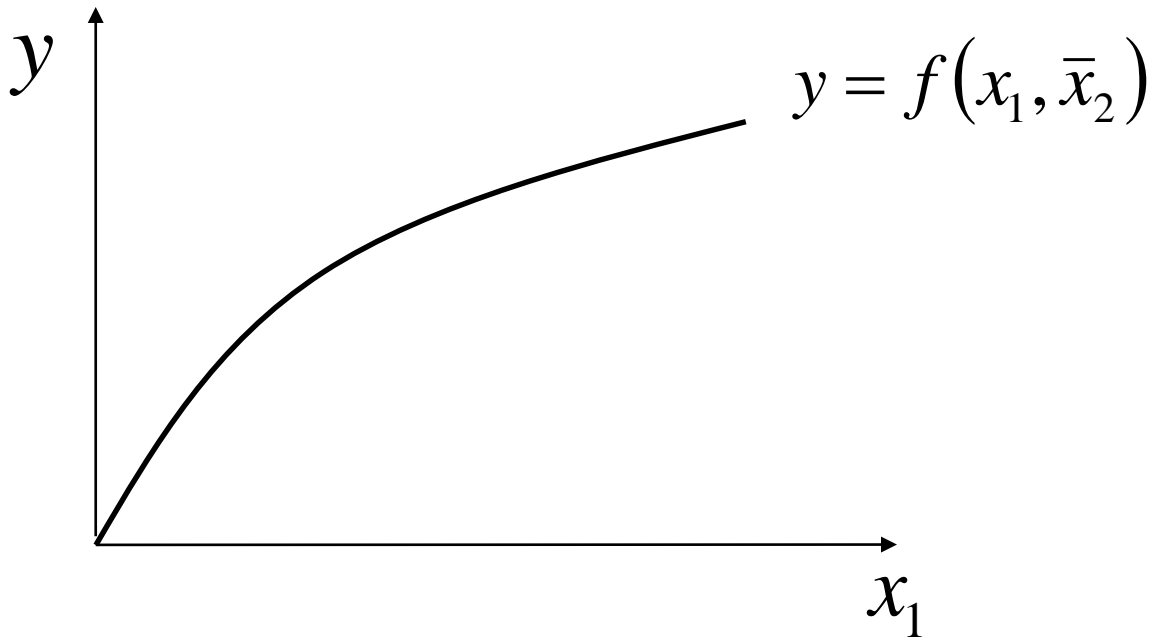
- oil pipeline: if you double the diameter of the pipe, you use double material, but the cross section diameter of the pipe capacity is able to pump more than twice as much oil
- House tasks: liter of water to cock

decreasing return to scale if

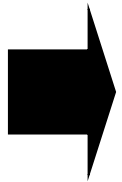
$$f(tx_1, tx_2) < tf(x_1, x_2) \forall t > 1$$

- Football: number of strikers put on the field during the match, doubling the strikers does not necessarily imply doubling the own goals (sometimes the opposite!!)
- Production mistake: input are not efficiently working together (better to be back to the previous structure)

Short run phenomenon: some input are working already at their maximal capacity (you cannot fix the productive problem by changing some factor)



Given the diminishing marginal product, production function is flatter and flatter when x_1 increases



In the short run (when one input is constant) the **decreasing returns to scale are equivalent** to the **decreasing marginal productivity**