Single firm and profit maximization (ch 19, except 19.11)

The production plan (use of inputs) maximizes the profit

Competitive market: input/output price are given

$$\pi = \sum_{i=1}^{n} p_i y_i - \sum_{j=1}^{m} w_j x_j$$

n output *i=1,…,n*

m inputs *j=1,…,m*

Short Run: profits may be negative even producing zero

↓

Fixed cost, fixed (ex ante) inputs/investment (ex. early rent payment of the office)

Long Run: all inputs are flexible, profit may be zero

↓

Only variable costs depending on the produced quantity (electricity, no use no cost)

$$\max_{x_1, x_2} \pi = py - w_1 x_1 - w_2 x_2$$

$$y \le f\left(x_1, x_2\right)$$

Feasible production plan

$$\max_{x_1, x_2} \pi = pf\left(x_1, x_2\right) - w_1 x_1 - w_2 x_2$$

Optimal conditions:

$$\frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{w_1}{p}$$

MP of each= *relative price* of the input

$$\frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{w_2}{p}$$

Solve the system:

$$\begin{cases} \dfrac{\partial f(x_1, x_2)}{\partial x_1} = \dfrac{w_1}{p} \\[2ex] \dfrac{\partial f(x_1, x_2)}{\partial x_2} = \dfrac{w_2}{p} \end{cases} \Rightarrow \begin{array}{l} x_1^* = x_1(w_1, w_2, p) \\ x_2^* = x_2(w_1, w_2, p) \end{array}$$

Short Run

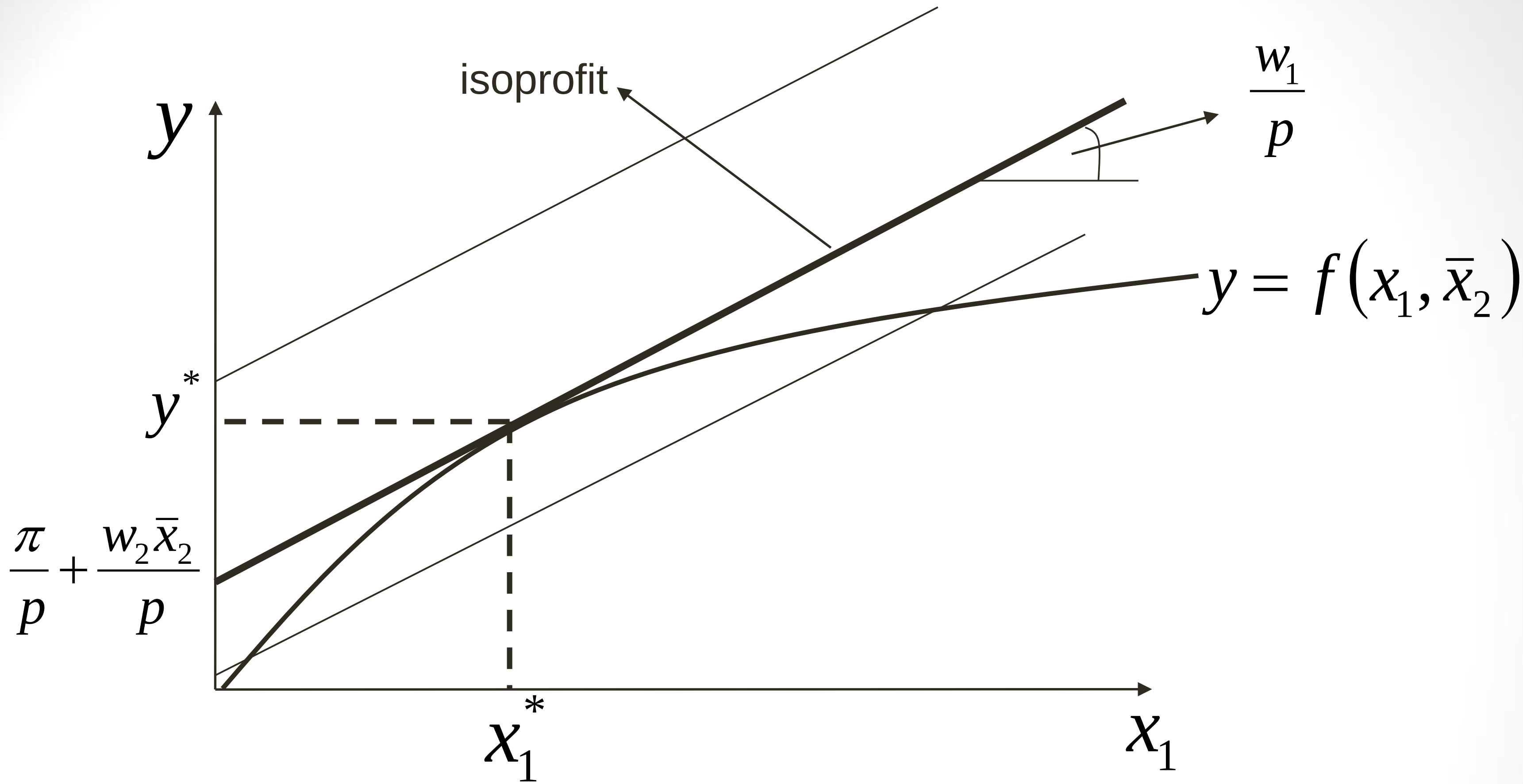
$$y = f\left(x_1, \bar{x}_2\right)$$

$$\max_{x_1} \pi = py - w_1 x_1 - w_2 \bar{x}_2$$

From the profit function we obtain the **isoprofit lines**

$$y = \frac{\pi}{p} + \frac{w_2}{p} \bar{x}_2 + \frac{w_1}{p} x_1$$

Isoprofit line represents the all input/output combinations associated to a constant level of profit



Vertical intercept denotes **profit + fixed costs**. Lines only differ with respect to profits Higher intercept higher profits

Higher profits at north west

Optimal condition:

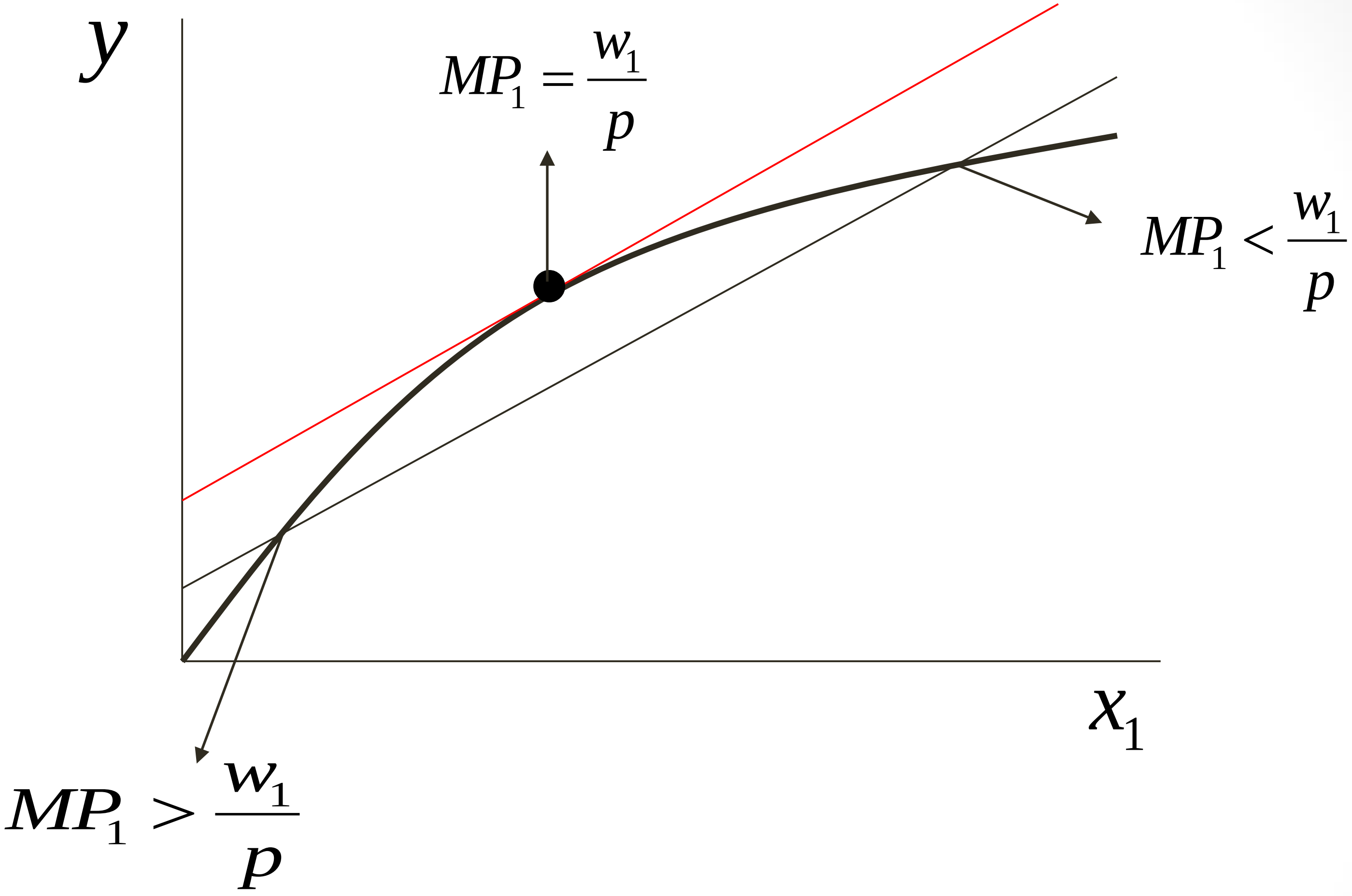
$$MP_1 = \frac{\partial f\left(x_1, \bar{x}_2\right)}{\partial x_1} = \frac{w_1}{p}$$

Optimal input/output combination is at the tangency point between the production function and the isoprofit.

CONSIDER…

$$MP_1 > \frac{w_1}{p}; MP_1 < \frac{w_1}{p}$$

Are optimal (why not)?

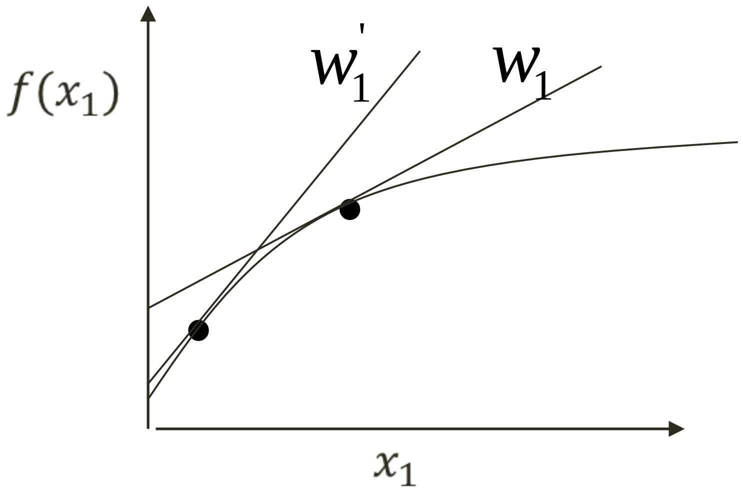

y
$MP_1 = \frac{w_1}{p}$
$MP_1 < \frac{w_1}{p}$
x_1
$MP_1 > \frac{w_1}{p}$

$$MP_1 > \frac{w_1}{p} \longrightarrow$$ It is possible to increase profit by increasing the level of input

$$MP_1 < \frac{w_1}{p} \longrightarrow$$ It is possible to increase profit by reducing the level of input

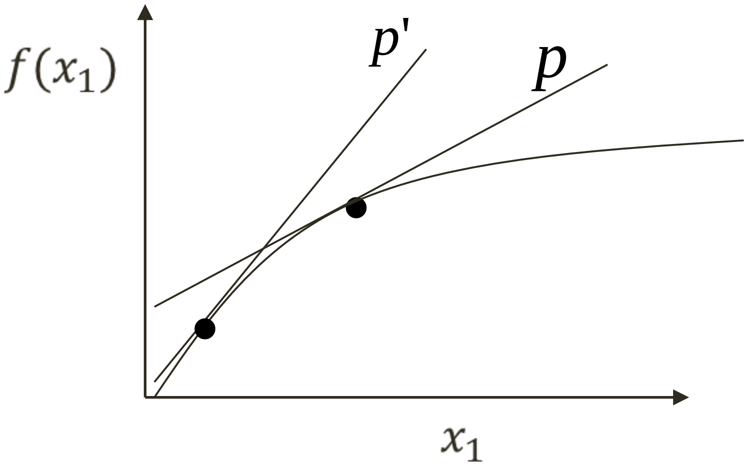
Comparative statics

if w_1 increases, then the optimal quantity of 1 decreases. **Factor demand is downward sloping**



$$w'_1 > w_1$$

If p decreases, the demand of factor x_1 decreases



$p' < p$

If demand of factor decreases, since factor 2 is fixed, then the supply of output *y* decreases. **Supply curve is upward sloping**

The change in the price of factor x_2 does not affect the optimal choice because of the short run

$\downarrow$

The slope of the isoprofit does not change, optimal choice x_1 dos not change then y does not change.

$\downarrow$

Only profit changes

- In perfect competition and constant returns to scale, equilibrium profit in the long run is zero

Intuition…

Is it possible NO constRetSc+competitive market+zero profit?

- What if profit in the long run was positive?

The firm could have the incentive to expand somehow the production plan (price is given), but..

• …too big to become inefficient<u>, this inefficiency contradicts the ConRS for all the possible leveles of outputs</u>

• ..if big and but efficient then it could obtain a dominant position (price maker), but this contradicts the assumption of <u>**competitive market .**</u>

Also:

With positive profit in the short run, more firms could get in the market (free entry)..

then aggregate supply could shift up and then the equilibrium price should decrease by inducing reduction the profit to zero

Lets' find the firm's supply in the long run with the Cobb-Douglas production function

$$f\left(x_1, x_2\right) = x_1^a x_2^b$$

First order conditions

$$p\frac{\partial f\left(x_1, x_2\right)}{\partial x_1} = w_1 \Rightarrow pax_1^{a-1}x_2^b - w_1 = 0$$

$$p\frac{\partial f\left(x_1, x_2\right)}{\partial x_2} = w_2 \Rightarrow pbx_1^a x_2^{b-1} - w_2 = 0$$

$$pax_1^a x_2^b - w_1 x_1 = 0$$

$$pbx_1^a x_2^b - w_2 x_2 = 0$$

Given $y = x_1^a x_2^b$

$$pay = w_1 x_1$$

$$pby = w_2 x_2$$

Solving wrt x_1 x_2

$$x_1^* = \frac{pay}{w_1}$$

$$x_2^* = \frac{pby}{w_2}$$

By substituting into the production function..

$$y = \left(\frac{pay}{w_1}\right)^a \left(\frac{pby}{w_2}\right)^b$$

$$y = \left(\frac{pa}{w_1}\right)^a \left(\frac{pb}{w_2}\right)^b y^{a+b}$$

Firm's supply is:

$$y = \left(\frac{pa}{w_1}\right)^{\frac{a}{1-a-b}} \left(\frac{pb}{w_2}\right)^{\frac{b}{1-a-b}}$$

Given:

$$f(tx_1, tx_2) = (tx_1)^a (tx_2)^b = t^{a+b} x_1^a x_2^b$$

$$tf(x_1, x_2) = t(x_1^a x_2^b)$$

with *a+b=1*, constant return to scale because

$$t(x_1^a x_2^b) = t^{a+b}(x_1^a x_2^b) \Rightarrow tf(x_1, x_2) = f(tx_1, tx_2)$$

with *a+b>1*, increasing return to scale because

$$t(x_1^a x_2^b) < t^{a+b}(x_1^a x_2^b) \Rightarrow tf(x_1, x_2) < f(tx_1, tx_2)$$

with *a+b<1*, decreasing return to scale because

$$t(x_1^a x_2^b) > t^{a+b}(x_1^a x_2^b) \Rightarrow tf(x_1, x_2) > f(tx_1, tx_2)$$

With Constant Return to scale the supply is not well defined

$$y=\left(\frac{pa}{w_1}\right)^{\frac{a}{1-a-b}}\left(\frac{pb}{w_2}\right)^{\frac{b}{1-a-b}}=\left(\frac{pa}{w_1}\right)^{\infty}\left(\frac{pb}{w_2}\right)^{\infty}=\infty\times\infty$$

As long as input/output prices are consistent with the zero profit

(Remind we are in the long run)

a firm with Cobb-Douglas technology is indifferent about its level of supply (ant level is a solution)