Firm supply: cost minimization Excluded section “revealed cost minimization”

Optimal input combination: allows to produce output at the lowest cost

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

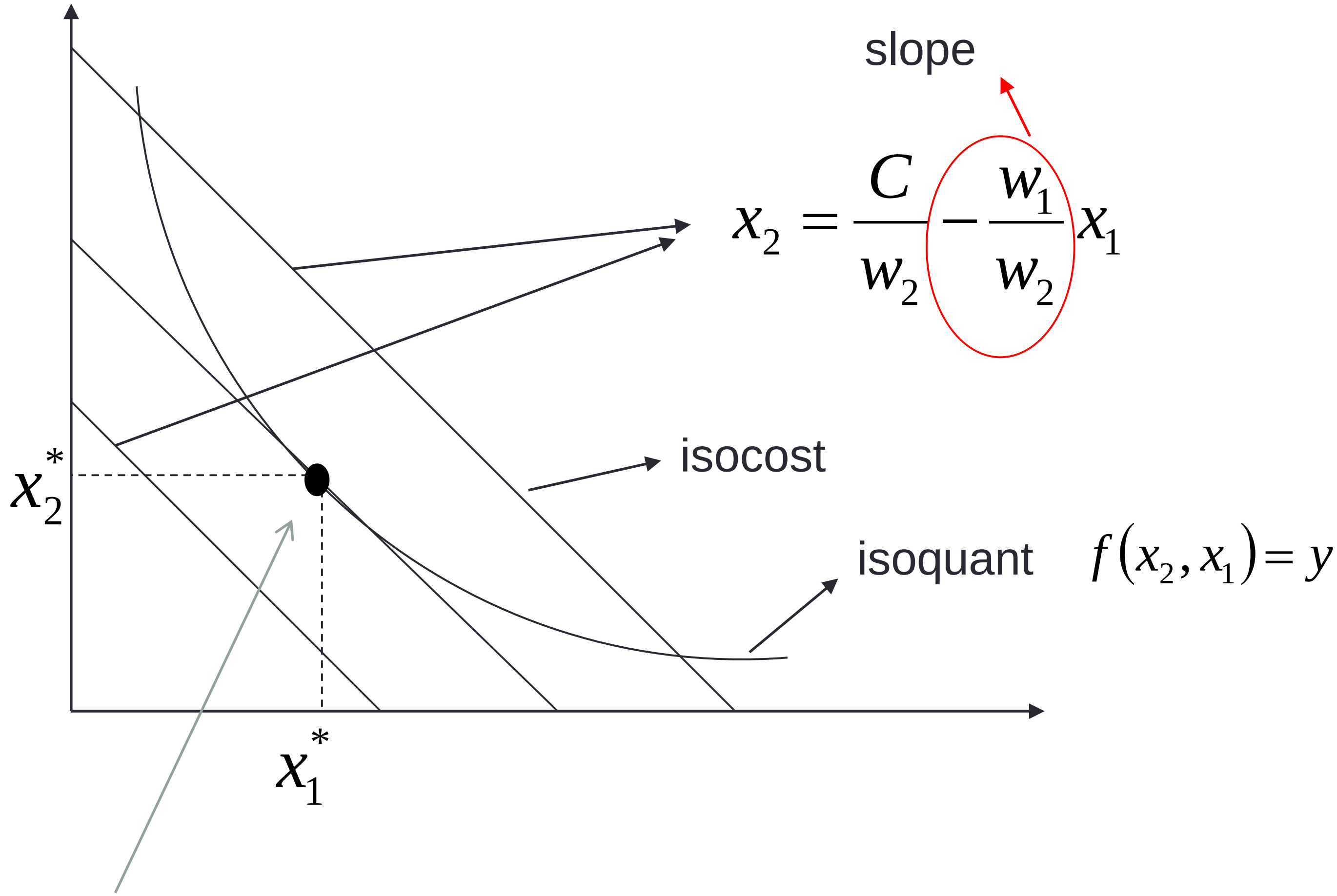
$$f(x_1, x_2) = y$$

Find the all input combinations allowing the specific cost *C*:

$$w_1 x_1 + w_2 x_2 = C$$

Isocost lines:

$$x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1$$

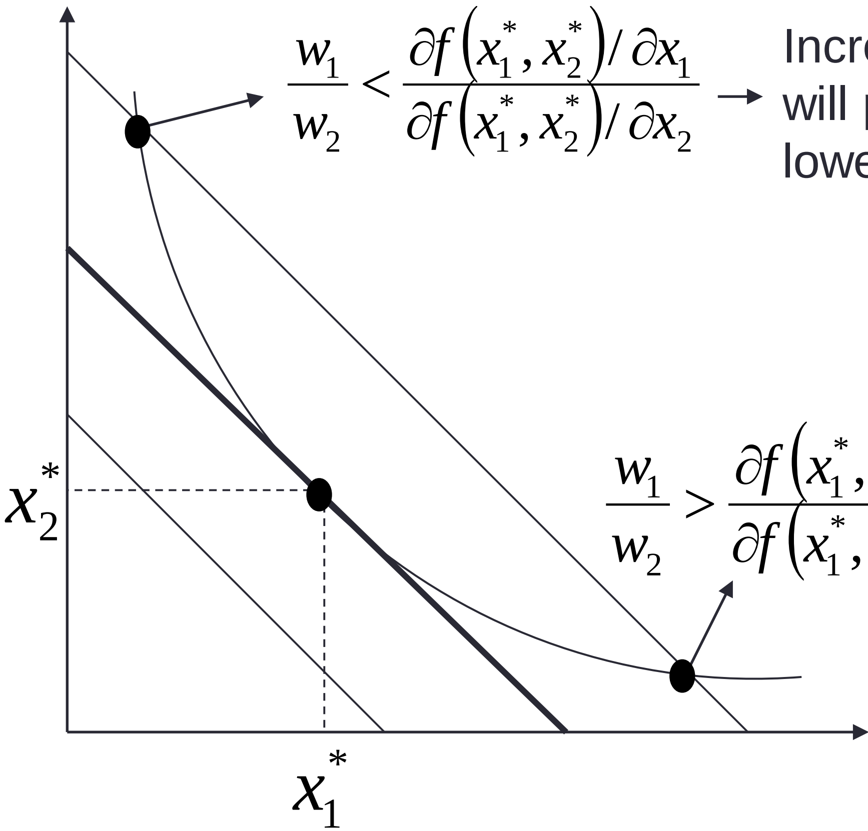


The input combinations allowing the lowest cost to produce output *y* si

$$\left(x_1^*, x_2^*\right)$$

tangency:

$$-\frac{w_1}{w_2} = TRS = -\frac{\partial f\left(x_1^*, x_2^*\right)/\partial x_1}{\partial f\left(x_1^*, x_2^*\right)/\partial x_2}$$



$$\frac{w_1}{w_2} < \frac{\partial f\left(x_1^*, x_2^*\right)/\partial x_1}{\partial f\left(x_1^*, x_2^*\right)/\partial x_2} \rightarrow$$

Increasing x1 and decreasing x2 will produce the same output at a lower cost

$$\frac{w_1}{w_2} > \frac{\partial f\left(x_1^*, x_2^*\right)/\partial x_1}{\partial f\left(x_1^*, x_2^*\right)/\partial x_2} \rightarrow$$

Increasing x2 and decreasing x1 will produce the same output at a lower cost

Formally

$$L = w_1 x_1 + w_2 x_2 - \lambda\left(f\left(x_1, x_2\right) - y\right)$$

FOCs

$$w_1 - \lambda \frac{\partial f\left(x_1, x_2\right)}{\partial x_1} = 0$$

$$w_2 - \lambda \frac{\partial f\left(x_1, x_2\right)}{\partial x_2} = 0$$

$$f\left(x_1, x_2\right) - y = 0$$

$$\frac{w_1}{w_2} = \frac{\partial f\left(x_1^*, x_2^*\right)/\,\partial x_1}{\partial f\left(x_1^*, x_2^*\right)/\,\partial x_2}$$

TRS

The lowest cost to produce output when the prices of the inputs are w_1 w_2 can be represented by the **cost function**:

$$c\left(w_1, w_2, y\right)$$

From the FOC **conditional factor demand functions** (or **derived factor demands**)

$$x_1^*(w_1, w_2, y) \qquad\qquad x_2^*(w_1, w_2, y)$$

$$c(w_1, w_2, y) = w_1 x_1^*(w_1, w_2, y) + w_2 x_2^*(w_1, w_2, y)$$

cost function: minimum cost to produce output *y*

Long run and short run cost functions

Short run cost function

$$c_s\left(y,\bar{x}_2\right)=\min_{x_1} w_1 x_1 + w_2\bar{x}_2$$

$$t.c.$$

$$f\left(x_1,\bar{x}_2\right)=y$$

Short-run factor demand

$$x_1 = x_1^s\left(w_1, w_2, \bar{x}_2, y\right) \longrightarrow$$

Short-run demand function of factor x1 minimizing the cost

$$x_2 = \bar{x}_2$$

ex. $\bar{x}_2$ = office size, x_1 workers, at given prices and output (depending on the building size)

Short run cost function:

$$c_s\left(y, \bar{x}_2\right) = w_1 x_1\left(w_1, w_2, \bar{x}_2, y\right) + w_2 \bar{x}_2$$

Long-run cost function:

$$c(y) = \min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

s.t.

$$f(x_1, x_2) = y$$

Long-run factors demands:

$$x_1(w_1, w_2, y) \qquad\qquad x_2(w_1, w_2, y)$$

Returns to scale, another perspective

•ConsRTS: costs proportionally increase with the output

Intuition: to double output the firm needs to double each level of input, but this implies doubling the costs

Cost of 1 unit

$$c(w_1, w_2, 1)$$

The lowest cost of producing y units is:

$$c(w_1, w_2, 1)y$$

ConstRTS: liner cost function (in the output)

- IncRTS: costs increase but by **less than proportionally** with the output

Cost function not linear in the output

- DecRTS: costs increase more than proportionally with the output

Not linear cost function

It depends on *y*

$$c(w_1, w_2, 1)y$$

Average cost function-AC (per unit cost)

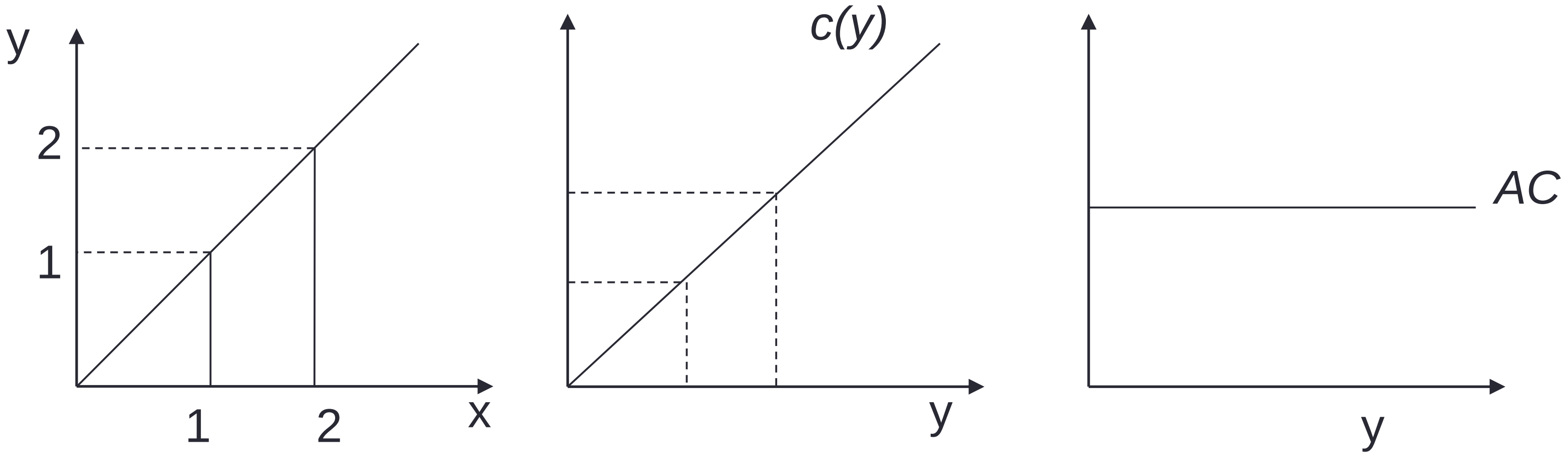
$$AC(y) = \frac{c(w_1, w_2, y)}{y}$$

With Constant Return to scale

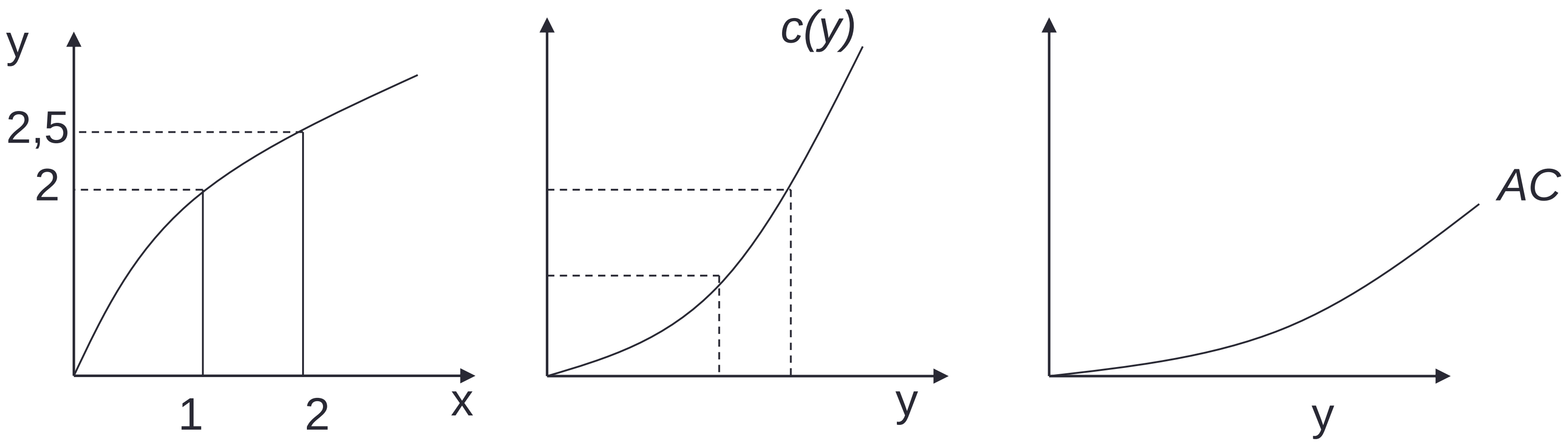
$$c(w_1, w_2, y) = c(w_1, w_2, 1)y$$

$$AC(y) = \frac{c(w_1, w_2, 1)y}{y} = c(w_1, w_2, 1)$$

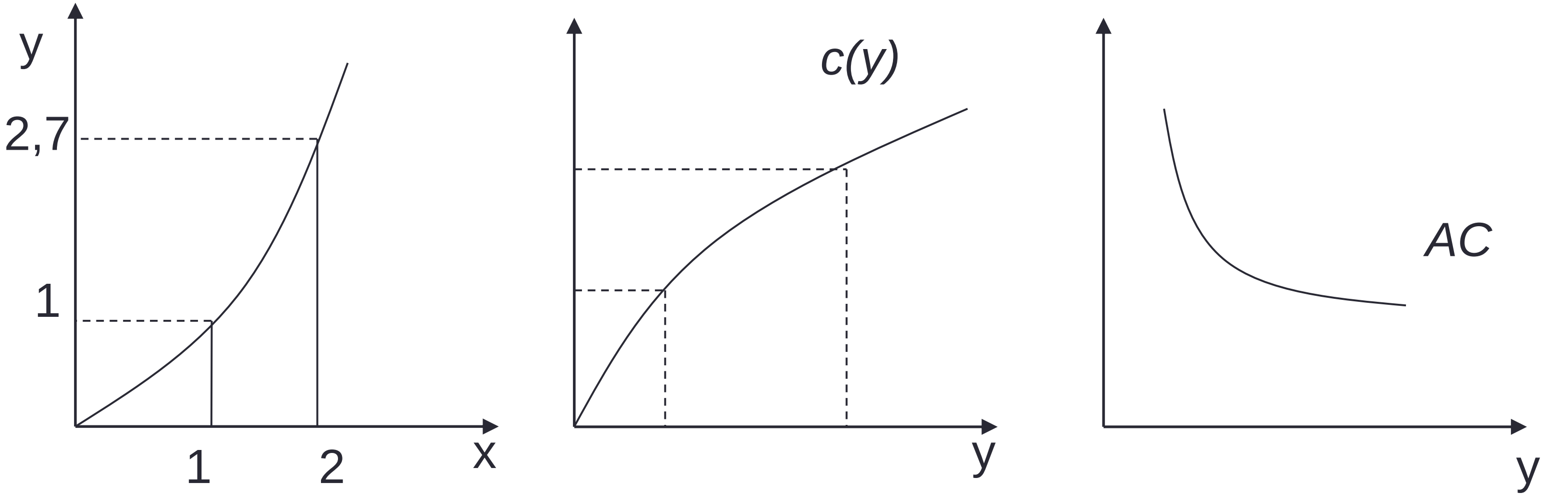
The cost per unit is constant, no matter the produced output



ConstRTS



DecRTS



IncrRTS

Cobb-Douglas:

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

$$x_1^a x_2^b = y$$

$$x_2 = \left(y x_1^{-a}\right)^{1/b}$$

$$\min_{x_1} w_1 x_1 + w_2 \left(y x_1^{-a}\right)^{1/b}$$

$$L = w_1 x_1 + w_2 x_2 - \lambda\left(x_1^a x_2^b - y\right)$$

FOCs

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = 0 \\ \dfrac{\partial L}{\partial x_2} = 0 \\ \dfrac{\partial L}{\partial \lambda} = 0 \end{cases} \Rightarrow \begin{aligned} w_1 &= \lambda a x_1^{a-1} x_2^b \\ w_2 &= \lambda b x_1^a x_2^{b-1} \\ x_1^a x_2^b - y &= 0 \end{aligned}$$

$$w_1 x_1 = \lambda a x_1^a x_2^b = \lambda a y$$

$$w_2 x_2 = \lambda b x_1^a x_2^b = \lambda b y$$

$$x_1 = \lambda \frac{ay}{w_1}$$

$$x_2 = \lambda \frac{by}{w_2}$$

$$\left(\frac{\lambda a y}{w_1}\right)^{a}\left(\frac{\lambda b y}{w_2}\right)^{b} = y$$

$$\lambda = \left(a^{-a} b^{-b} w_1^{a} w_2^{b} y^{1-a-b}\right)^{\frac{1}{a+b}}$$

$$x_1(w_1, w_2, y) = \left(\frac{a}{b}\right)^{\frac{a}{a+b}} w_1^{\frac{-b}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$$

$$x_2(w_1, w_2, y) = \left(\frac{a}{b}\right)^{-\frac{a}{a+b}} w_1^{\frac{a}{a+b}} w_2^{\frac{-a}{a+b}} y^{\frac{1}{a+b}}$$

Using the cost function…

$$c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y)$$

$$c(w_1, w_2, y) = \left[\left(\frac{a}{b}\right)^{\frac{b}{a+b}} + \left(\frac{a}{b}\right)^{\frac{-a}{a+b}}\right] w_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$$

constant

Not always linear in *y*
It depends on *a*+*b*

$a+b>1$ → IncrRTS → Costs increase less than linearly with the output

$a+b=1$ → ConsRTS → Costs increase proportionally with the output

$a+b<1$ → DecrRTS → Costs increase more than linearly with the output