MATHEMATICS

Monday October 5 2015

Second Exercise Class

1) Given the following sets

$$A = \left\{ x \in \mathbb{R} : \frac{x-1}{x+2} \geq 0 \right\}$$

$$B = \{ x \in \mathbb{R} : x^2 - 4x + 5 \geq 0 \}$$

$$C = \{5, 7\}$$

find accumulation points, interior and exterior points, isolated points.
Moreover determine A^c, A^c, $A \cup B$, $A \cap B$, $A^c \cap C$.

2) For each of the following sets find accumulation points, isolated points, boundary points, interior points, exterior points, infimum, supremum, maximum, minimum

a) $A = \left\{ x; x = \frac{n-2}{2n}, \quad n \in \mathbb{N} - \{0\} \right\}$

b) $B = (0, 1] \cup \{2, 3, 4\} \cup [5, 6)$

c) $C = \{ x \in \mathbb{R}; -x^2 + 6x - 8 \geq 0 \}$

d) $D = \left\{ x \in \mathbb{R};\ x = \frac{|3-2n|}{n+2},\ n \in \mathbb{N} \right\}$

e) $E = \left\{ \frac{1}{n} \right\}_{n \geq 1} \cup \left\{ \frac{n+1}{n} \right\}_{n \geq 1}$

3) Given the quadrilateral of vertices $A(4; 3)$, $B(12; 9)$, $C(13; 16)$, $D(5; 10)$, verify that it is a parallelogram.

4) Given the points $A(-1; 2)$, $B(3; -1)$, $C(2; 4)$, determine the equations of the sides of the triangle defined by these points.

5) Given the triangle ABC of vertices $A(1;2)$, $B(6;2)$, $C(3;8)$, determine the equations of its heights.

6) Given the family of lines

$$(k+1)x - 2y + 3 = 0$$

determine k such that (three different exercises follow)

a) the line is parallel to the line $y - 1 = 0$;

b) the line is perpendicular to the line $x - 3y = 0$

c) the line passes through the point $(2; -1)$.

7) Given the family of lines

$$x + (a+2)y - 1 = 0$$

determine a such that (three different exercises follow)

a) the line is parallel to the x axis;

b) the line is perpendicular to the y axis;

c) the line passes through the origin.

8) In each of the following conditions determine an expression for the linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ that satisfies the indicated conditions

a) the graph of f intersects the x axis in $(3;0)$ and the y axis in $(0;-2)$:

b) the inverse of f is the function $f^{-1}(x) = 2x - 3$

c) the graph of f intersects the y axis in $(0;-2)$ and does not intersect the x axis;

d) f is an odd function and $f(1) = -\frac{1}{3}$.

9) Determine an expression for the quadratic function $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that the graph of f intersects the x axis in $(-2; 0)$ and in $(1; 0)$ and the y axis in $(0; 2)$.

9) When possible, determine the inverse function of the following functions. When not possible, restrict the domain in a suitable way and invert the restricted function

a) $f(x) = 3x + 5$

b) $f(x) = \sqrt{x + 1}$

c) $f(x) = (x + 1)^2$