MATHEMATICS

Monday November 23 2015

Eighth Exercise Class

1) Verify if each of the following functions satisfy, in the indicated interval, Rolle's theorem conditions, and, if so, find the points of the interval that satisfy the theorem.

$$y = f(x) = x^2 - 4x + 1, \ I = [0, 4]$$

$$y = f(x) = |x^2 - 2x|, \ I = [1 - \sqrt{2}, 1 + \sqrt{2}]$$

$$y = f(x) = \sqrt{9 - x^2}, \ I[-2, 2]$$

2) Verify Rolle's Theorem for the function $y = \begin{cases} x & \text{if } 1 \leq x \leq 3 \\ -x + 5 & \text{if } 3 < x \leq 4 \end{cases}$.

3) Find values for a and b such that the function

$$y = \begin{cases} \sqrt{|x-1|} & [0, 1] \\ ax^2 + bx + 1 & [-2, 0] \end{cases}$$

satisfies the Rolle's theorem conditions in the interval $[-2, 1]$. Find then the points whose existence is guaranteed by Rolle's theorem.

4) Verify if each of the following functions satisfy, in the indicated interval, Lagrange's theorem conditions, and, if so, find the points of the interval that satisfy the theorem.

$$y = f(x) = x^3 - x^2 + 2, \ I = [-1, 2]$$

$$y = f(x) = 2 + \sqrt[3]{x^4}, \ I = [-1, 4]$$

$$y = f(x) = 2e^x - x, \ I[0, 2]$$

5) Prove, applying Lagrange's theorem to the function $y = f(x) = e^x$, that

$$e^x \geq 1 + x \quad \forall x$$

6) Prove, applying Lagrange's theorem, that the following inequality

$$|\cos b - \cos a| \leq |b - a|$$

holds $\forall a, b \in \mathbb{R}$, with $a \neq b$.

7) Calculate the following limits through de L'Hopital rule

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{2}}$$

$$\lim_{x\to 0} \frac{x^2}{1-\cos x}$$

$$\lim_{x\to \infty} \frac{\log(e^{x^2}+1)}{3x^2}$$

$$\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$$

8) Calculate the Taylor polynomial of the following functions around the specified points and stopping at the specified order:

$$y = f(x) = \log(2-x) \quad (x_0 = 2,\ n = 3)$$

$$y = f(x) = \sqrt{1+\sin x} \quad (x_0 = 0,\ n = 3)$$

9) Calculate the order of the following functions around the specified points:

$$y = f(x) = e^{-x\cos x} + \sin x - \cos x \text{ at } x = 0$$

$$y = f(x) = (x^2-4)\sin(2-x) \text{ at } x = 2$$