

## MATHEMATICS - SECOND EXERCISE LESSON

MONDAY, OCTOBER 3 2016

1) Given the following set

$$X = (1, 2) \cup \left\{ \frac{1}{n} \right\}_{n \geq 1}$$

find accumulation points, boundary points, interior and exterior points, isolated points, infimum, supremum, max and min.

2) Write the equations of the lines that contain the sides of the quadrilateral  $ABCD$ , where  $A(-3; 3)$ ,  $A(-3; -1)$ ,  $A(2; -2)$ ,  $A(2; 2)$ . Verify that the quadrilateral is a parallelogram.

3) Determine for which values of  $k$  the line of equation  $kx + (k + 1)y + 2 = 0$  is

- a) parallel to the  $x$ -axis;
- b) parallel to the  $y$ -axis;
- c) parallel to the line of equation  $x - 2y = 0$ ;
- d) perpendicular to the line of equation  $4x - 2y + 1 = 0$ .

4) Given the line of equation

$$x + (a + 2)y - 1 = 0$$

determine  $a$  such that the line

- a) parallel to the  $x$ -axis;
- b) parallel to the  $y$ -axis;
- c) passes through the origin.

5) Find the domain of the functions below and express it in set notation

- $f(x) = \sqrt{x}$
- $f(x) = \frac{1}{3x-2}$
- $f(x) = x^3 - 1$

6) Let  $A = \{0, \pi, 5, 7\}$  and  $f : A \rightarrow \mathbb{R}$ . Determine the image of

- $f(x) = x + 3$
- $f(x) = x^2 + 2$

7) Find if the following functions are injective, surjective, bijective and invertible.

- $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x - 3$
- $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2 - 3$

8) Given the functions  $f(x) = \frac{1}{1-x}$  and  $g(x) = \sqrt{x-1}$  find  $f \circ f(x)$ ,  $g \circ g(x)$ ,  $f \circ g(x)$ ,  $g \circ f(x)$ , and the correspondent domains.

9) In each case replace the dots with the missing function. Determine the domain of each composite function

- a)  $f(x) = \dots$     $g(x) = x + 4$     $f \circ g(x) = x$ ;
- b)  $f(x) = \sqrt{x}$     $g(x) = \dots$     $f \circ g(x) = |x|$ ;
- c)  $f(x) = \frac{x+1}{x}$     $g(x) = \dots$     $f \circ g(x) = x$ .

10) Determine, if it exists, the inverse function  $f^{-1}$  of the following functions and the correspondent domains

- a)  $y = f(x) = 2 - 3x$ ;
- b)  $y = \frac{x}{x+3}$ ;
- c)  $f(x) = \frac{3}{4x-4}$
- d)  $f(x) = \frac{x+1}{x+2}$

11) Verify that the function defined by  $y = f(x) = x^2 - 4x + 9$  is not invertible. Identify a suitable restriction of the domain in which the function is invertible and identify the inverse function.

12) Prove that the functions

$$f(x) = x^2 - x + 1, \quad x \geq \frac{1}{2} \quad \text{and} \quad g(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}, \quad x \geq \frac{3}{4}$$

are inverse of each other.