MATHEMATICS

Monday October 31 2016

Sixth Exercise Class

1) For each of the following functions find, when possible, vertical and horizontal asymptotes:

$$y = \frac{x^3 - x}{x^2 - 4}$$

$$y = f(x) = \log_4\left(\frac{3x}{x-1}\right)$$

$$y = f(x) = \frac{\sqrt{x^2-9} - 1}{x}$$

$$y = f(x) = xe^{\frac{x-1}{x+1}}$$

$$y = f(x) = \sqrt{\frac{x^3-1}{x^2+x-2}}$$

$$y = f(x) = 3x + \sqrt{x^2-1}$$

$$y = f(x) = \frac{e^x}{e^x - 1}$$

$$y = f(x) = x\frac{2\log(x) - 3}{\log x - 2}$$

$$y = f(x) = 3x + \log\left(\frac{5x}{x-2}\right)$$

$$y = f(x) = \ln(\sqrt{2+x^2} + x)$$

$$y = f(x) = xe^{-\frac{1}{|x|}}$$

$$y = f(x) = \frac{1+\cos x}{1-\cos x}$$

$$y = f(x) = \frac{\ln x}{\sqrt{1+\cos x}}$$

2) Determine if the following function is continuous on $\mathbb{R}$

$$y = f(x) = \begin{cases} \frac{e^x-1}{x} - 1 & \text{if } x \in (-\infty, 0) \\ \frac{\ln(1+x)}{x^2+1} & \text{if } x \in [0, 1] \\ x^2 - 1 & \text{if } x \in (1, +\infty) \end{cases}$$

3) State where the following functions are continuous on their domain. Classify (if any) the points in which they are discontinuous.

$$y = f(x) = \begin{cases} \frac{1-\cos^3 x}{x(e^x-1)} & \text{if } x < 0 \\ \log(\sqrt{x}+1) & \text{if } x \geq 0 \end{cases}$$

$$y = f(x) = \begin{cases} \left(\frac{2}{2+x}\right)^{\frac{1}{x}} & \text{if } -2 < x < 0 \\ \frac{1}{x+e} & \text{if } x \geq 0 \end{cases}$$

$$y = f(x) = \frac{x+3}{3x^2+x^3}$$

$$y = f(x) = \frac{\log(1+x^2)}{\sqrt{3-x}}$$

$$y = f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

$$y = f(x) = \begin{cases} e^{\frac{x}{\sqrt{-1+x^2}}} & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ \frac{e^{\cos(x+1)-1}-e^{x+1}}{x+1} & \text{if } x > -1 \end{cases}$$

4) Determine $k \in \mathbb{R}$ such that

$$y = f(x) = \begin{cases} 2x^2+4x & \text{if } x \geq 1 \\ -x+k & \text{if } x < 1 \end{cases}$$

is continuous in $\mathbb{R}$.

5) Determine $a, b \in \mathbb{R}$ such that

$$y = f(x) = \begin{cases} \log(1+x) & \text{if } -1 < x \geq 0 \\ a\sin x + b\cos x & \text{if } 0 < x < \frac{\pi}{2} \\ x & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

is continuous in $\mathbb{R}$.

6) Verify that the function

$$y = f(x) = e^{-\frac{1}{x^2}}$$

is extendable continuously.