MATHEMATICS
Monday October 17 2016
Fourth Exercise Class

1) Verify that $a_n = 3 - \frac{1}{n^2+1}$ approaches $L = 3$ for $n \to +\infty$.

2) Verify that $a_n = (-1)^n$ does not admit limit.

3) Verify whether the sequence $a_n = \left(-\frac{1}{n}\right)_{n=1}^{\infty}$ is bounded from below, bounded from above, or bounded.

3) Verify whether the sequence $a_n = \left(1 - n^2\right)_{n=1}^{\infty}$ is bounded from below, bounded from above, or bounded.

Sol: bounded

4) Study domain, sign and axis-intercepts of the following functions

Sol: bounded from above

- $y = \frac{5\cos x}{1+2\cos x}$

$$D_y = \mathbb{R} \ \left\{\frac{2}{3}\pi + 2\pi i\right\}_{i\in\mathbb{Z}} \cup \left\{2\pi i - \frac{2}{3}\right\}_{i\in\mathbb{Z}}$$
$$S_y : + \to \left[0, \frac{\pi}{2}\right) \cup \left(\frac{2}{3}\pi, \frac{4}{3}\pi\right) \cup \left(\frac{3}{2}\pi, 2\pi\right); - \to \left(\frac{\pi}{2}, \frac{2}{3}\pi\right) \cup \left(\frac{4}{3}\pi, \frac{3}{2}\pi\right)$$
$$I_y : \frac{5}{3} \qquad I_x : \left\{\frac{\pi}{2}, \frac{3}{2}\pi\right\}$$

- $y = \frac{\ln x}{\sqrt{1+\cos x}}$

$$D_y = \mathbb{R}^+ \ \{0\} \cup \{\pi + 2\pi i\}_{i\in\mathbb{Z}}$$
$$S_y : + \to x > 1; \quad - \to x < 1$$
$$I_y : Impossible \qquad I_x : x = 1$$

- $y = \frac{\ln(|\sin x + \cos x|)}{3^{2\sin x} - 4 \cdot 3^{\sin x} + 3}$

$$D_y = \mathbb{R}\ \left\{ \left(\frac{3}{4}\pi, \frac{7}{4}\pi, \frac{\pi}{2}, 0 \right) + 2\pi i \right\}_{i \in \mathbb{Z}}$$

$$S_y : + \rightarrow \left(\pi, \frac{3}{2}\pi \right); \qquad - \rightarrow (0, \pi) \cup \left(\frac{3}{2}\pi, 2\pi \right)$$

$$I_y : y = 0 \qquad I_x : x = 0$$

- $y = \frac{\ln(1 - \sin^2 x)}{x e^{\sin x}}$

$$D_y = \mathbb{R}\ \{0\} \cup \left\{ \frac{\pi}{2} + 2\pi i, \frac{3}{2}\pi + 2\pi i \right\}_{i \in \mathbb{Z}}$$

$$S_y : + \rightarrow x < 0; \qquad - \rightarrow x > 0$$

$$I_y : Impossible; \qquad I_x : \{\pi, 2\pi\}$$

- $y = \frac{\log(1 + x^2)}{\sin x - x}$

$$D_y = \mathbb{R}\ \{0\}$$

$$S_y : + \rightarrow x < 0; \qquad - \rightarrow x > 0$$

$$I_y : Impossible; \qquad I_x : Impossible$$