

MATHEMATICS

Thursday March 2 2017

First exercise class

- 1) Given in \mathbb{R}^3 the vectors $\vec{u} = (1, 0, -1)$, $\vec{v} = (2, 1, 3)$, $\vec{w} = (-1, -2, -3)$,
 - calculate the combination $2\vec{u} - 3\vec{v} + 5\vec{w}$.
 - The inner product $\vec{u} \cdot \vec{v}$,
 - The combination $\vec{u} \cdot \vec{v} \times \vec{w} - \vec{w} \cdot \vec{u} \times \vec{v}$.
 - The vector $\frac{\vec{u}}{||\vec{u}||}$,
 - The vector $(\vec{u} \cdot \vec{v})\vec{w}$.
- 2) Given in \mathbb{R}^4 the vectors $\vec{u} = (1, -1, 0, 2)$, $\vec{v} = (2, 0, -3, 4)$, $\vec{w} = (0, -1, 2, 3)$, $\vec{z} = (3, -2, 1, -1)$, calculate
 - $\vec{u} - \vec{v}$,
 - The length of each vector,
 - $3\vec{u} + \vec{v} - 2\vec{w} + \vec{z}$,
 - The length of $\vec{w} - \vec{z}$.
- 3) Given in \mathbb{R}^3 the vector $\vec{u} = (2, 0, -1)$, $\vec{v} = (4, -6, -1)$, $\vec{w} = (1, 3, 2)$, find a , b , and c in \mathbb{R} such that:
 $av + bv + cw = 0$.
- 4) Given the vector $\vec{v} = (1, -3)$ and $\vec{w} = (k, 1 - k)$, determine values for k such that \vec{v} and \vec{w} are orthogonal.
- 5) Given in \mathbb{R}^3 the vectors $\vec{v} = (1, -1, 0)$, $\vec{w} = (0, 2, -1)$, $\vec{u} = (2, 0, 1)$, $\vec{z} = (4, 5, 6)$, find a , b , c in \mathbb{R} such that $a\vec{v} + b\vec{w} + c\vec{u} = \vec{z}$.
- 6) Given the following vectors, for each find the corresponding versor:
 - $\vec{v} = (\alpha, -\alpha)$,
 - $\vec{w} = (3\alpha, 2\alpha, \alpha)$,
 - $\vec{u} = (1, \alpha, 1 - \alpha)$,
 - $\vec{z} = (\alpha, -\alpha, \alpha, -\alpha)$.
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- 7) Given in \mathbb{R}^3 the vector $\vec{v} = (1, 3, -2)$, determine the vectors parallel to \vec{v} and having length 3.
- 8) Given in \mathbb{R}^2 the vector $\vec{v} = (3, -2)$, determine for which k the vector $\vec{w} = (k, 1 - k)$ is orthogonal to \vec{v} .
- 9) Determine the parametric equation and the Cartesian equation of the line on a plane:
 - Passing through the point $A(2, -1)$, $B(-1, 3)$. Does the point $Q(1, -1)$ belong to the line?
 - Passing through the point $C(1, 2)$ and parallel to the vector $\vec{OP} = (3, -1)$.
 - Of Cartesian equation $y = 4x - 7$.
 - Passing through $A(2, -1)$ and perpendicular to the line $r: x - 3y = 1$.
- 10) Determine the parametric equation and the Cartesian equation of the line on a space:

- Passing through the point $A(-1, 2, -3)$, $B(3, 0, -1)$. Does the point $Q(-1, 2, -3)$ belong to the line?
- Passing through the point $C(3, 5, 4)$ and parallel to the vector $\overrightarrow{OP} = (1, -1, 2)$.
- Of Cartesian equation $\begin{cases} 2x + y - z = 0 \\ z = x - 3y \end{cases}$

11) Determine the reciprocal position of the lines r and r' of parametric equations:

$$r: \begin{cases} x = t - 1 \\ y = 2 - t, \\ z = 3 \end{cases} \quad r': \begin{cases} x = s - 2 \\ y = 3s \\ z = s + 1 \end{cases}$$

12) Determine the parametric equations of the line r passing through the point $A(1, 0, 1)$ and $B(0, -2, 1)$ and the line r' passing through $C(-2, 1, 0)$ and $D(-2, 0, 0)$. Establish if r and r' lie on the same plane. If so, find a Cartesian equation of the plane containing r and r' .

13) Consider the lines of Cartesian equations

$$r: \begin{cases} x - y = 0 \\ 2x - 3y + z = 1 \end{cases} \quad s: \begin{cases} z - 2y = 0 \\ x = 1 \end{cases}$$

- After verification that the two lines intersect, determine the Cartesian equation of the line passing through $P(0, 0, 2)$ and intersect r and s ;
- Determine the Cartesian equation of the plane passing through $C(2, -1, 0)$ and perpendicular to r ;
- Determine the Cartesian equation of the line passing through the point $P(1, 0, 1)$ and perpendicular to the plane of the two lines r and s .

14) Consider the line r of equation

$$r: \begin{cases} x = 1 - 2t \\ y = t \\ z = t + 2 \end{cases}$$

and the family of planes $\pi_k: kx + 2y - kz = 1$, where k is a real number. Determine for which k the plane π_k is parallel to r .

15) Establish if the vector $\vec{v}_1 = (2, 3, 0)$ and $\vec{v}_2 = (4, 0, 5)$ in \mathbb{R}^3 are linearly independent.

16) Study the linear dependence or independence of the vectors

$$\vec{v}_1 = (-1, 8, 1), \vec{v}_2 = (-1, 3, 2), \vec{v}_3 = (2, 4, -2).$$

If they are linearly dependent, when possible, express

\vec{v}_1 as a linear combination of \vec{v}_2 and \vec{v}_3 ,

\vec{v}_2 as a linear combination of \vec{v}_1 and \vec{v}_3 ,

\vec{v}_3 as a linear combination of \vec{v}_1 and \vec{v}_2 .