

MATHEMATICS
Friday March 11 2016
Second Exercise Class

1) Given the matrices

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 1 \\ 2 & -1 & -1 \\ 1 & 2 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 3 & 1 \\ -4 & 2 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

- calculate $A + B$
- calculate A^T
- verify that $(A + B)^T = A^T + B^T$
- calculate $C = 4A$

2) Recognize if it is possible to realize the product between the indicated matrices, and when possible calculate it:

- $A = \begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & -1 & 3 & 5 \\ 1 & -3 & 2 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 0 \\ 4 & -5 \end{pmatrix}$

- $A = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 1 & 3 \\ 4 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 1 \end{pmatrix}$

- $A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ 1 & 1 \end{pmatrix}$

- $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

- $A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$

3) Given the matrices $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$, calculate AB and BA . Is it $AB = BA$?

4) Given the matrices $A = \begin{pmatrix} 2 & -1 \end{pmatrix}$, $B = \begin{pmatrix} \beta & 2 \\ 1 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$, calculate β such that $A \cdot B \cdot C = 0$

5) Given the matrices $A = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$, find a matrix $X = \begin{pmatrix} x \\ y \end{pmatrix}$ such that $A \cdot X = B$

6) Evaluate k such that the matrix $A = \begin{pmatrix} 1 & 0 \\ 2 & k \end{pmatrix}$ satisfies $A^2 = A$ (a matrix that satisfies such property is called IDEMPOTENT)

7) For which value $t \in \mathbb{R}$ the following matrix is symmetric?

$$A = \begin{pmatrix} 0 & 3t-2 & -1 \\ t^2 & 3 & t^2+4 \\ -1 & 4t & 1 \end{pmatrix}$$

8) Given the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ -1 & 5 & -1 & 3 \\ 4 & 3 & 2 & -2 \\ 2 & 0 & 1 & 8 \end{pmatrix}$$

write the minors M_{11} , M_{23} , M_{33} , M_{42} .

9) Calculate the determinant of the following matrices

- $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \end{pmatrix}$
- $B = \begin{pmatrix} -11 & 3 \\ 2 & 0 \end{pmatrix}$
- $C = \begin{pmatrix} 2 & 3 & -2 \\ 1 & -2 & 0 \\ 0 & -1 & 2 \end{pmatrix}$
- $D = \begin{pmatrix} 2 & -2 & -2 \\ 1 & 1 & 0 \\ -3 & 4 & 0 \end{pmatrix}$
- $E = \begin{pmatrix} 7 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 4 & -3 \end{pmatrix}$
- $F = \begin{pmatrix} 1 & -4 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 5 \end{pmatrix}$
- $G = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$$\bullet \quad H = \begin{pmatrix} 1 & 2 & 2 & 3 \\ 4 & 1 & -1 & 2 \\ 3 & 6 & 6 & 8 \\ 3 & 2 & 1 & 3 \end{pmatrix}$$

10) Given the matrix

$$D = \begin{pmatrix} t & 0 & 4 & -1 \\ 0 & 2 & 1 & 3 \\ -2 & 1 & 0 & t-1 \\ 4 & 0 & 0 & 1 \end{pmatrix}$$

for which value of t the determinant of D is zero?

11) Calculate the inverse matrix of the following matrices

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 & 3 \\ 1 & 1 & 2 \\ 2 & 0 & 7 \end{pmatrix}$$

12) For which values of α and β the matrices

$$A = \begin{pmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} \beta & 1 & 0 \\ 1 & \beta & 1 \\ 0 & 1 & \beta \end{pmatrix}$$

are invertible?