

**MATHEMATICS**  
**Friday April 8 2016**  
**Fifth Exercise Class**

1) Given the following set of vectors  $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$  where  $\vec{a}_1 = (1, -1, \alpha)$ ,  $\vec{a}_2 = (2, -2, 1)$ ,  $\vec{a}_3 = (\alpha, -\frac{1}{2}, \frac{1}{4})$ , study the rank of the set of vectors on varying the parameter  $\alpha$ .

2) For which values of  $a$  and  $b$  the following matrix has rank 1? and for which values of  $a$  and  $b$  the following matrix has rank 0?

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

3) Determine the rank of the following matrices

a)

$$\begin{pmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 2 & -6 \\ 1 & -1 & 3 & -3 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 5 \\ 0 & -5 & 3 \end{pmatrix}$$

4) Determine the rank of the following matrices on varying the parameter  $\alpha$

a)

$$\begin{pmatrix} 1 & \alpha & 3 \\ 2 & 2 & 4 \\ 1 & 1 & \alpha \end{pmatrix}$$

b)

$$\begin{pmatrix} \frac{1}{2} & \alpha \\ \frac{1}{2} & 1 \\ \alpha & 3 \end{pmatrix}$$

c)

$$\begin{pmatrix} \alpha & 1 & \alpha - 1 \\ 1 & 1 & 0 \\ 1 & \alpha & 1 - \alpha \end{pmatrix}$$

5) Study the following systems

a)

$$\begin{cases} x - z = 0 \\ 2y - 2w = 1 \\ y - z = -1 \\ 4y - 4z - w = 0 \end{cases}$$

b)

$$\begin{cases} x - z = 0 \\ 2y - 2w = 1 \\ y - z = -1 \\ 4y - 4z - w = 0 \end{cases}$$

c)

$$\begin{cases} x - y + 3z = 2 \\ -2x + 2y + z = 5 \\ 3x - 3y + 9z = 6 \end{cases}$$

d)

$$\begin{cases} 2x + 3y - 2z = 3 \\ x - 2y + 3z = 2 \\ 4x - y + 4z = 7 \end{cases}$$

6) Study the following systems on varying the parameters  $\lambda$  and  $\gamma$ :

a)

$$\begin{cases} 3x + y = \lambda x \\ x + 3y = \lambda y \end{cases}$$

b)

$$\begin{cases} x - y + z = \lambda \\ x + y + z = 2 \\ -2x + 2y - 2z = -2 \end{cases}$$

c)

$$\begin{cases} x - y + z = \lambda \\ x + y - 2z = 2 \\ x + 2y - 2z = -2 \end{cases}$$

d)

$$\begin{cases} x - y + z = \lambda \\ x + y + z = 2 \\ \gamma x + 2y - 2z = -2 \end{cases}$$

e)

$$\begin{cases} (2 - \lambda)x - y + 2z = 0 \\ -x - \lambda y = 0 \\ y - \lambda z = 0 \end{cases}$$