

**MATHEMATICS**  
**Friday April 22 2016**  
**Seventh Exercise Class**

1) Given  $A = \begin{pmatrix} -3 & 1 \\ -3 & 8 \end{pmatrix}$  and  $x = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ , establish if  $x$  is an eigenvector of  $A$ .

2) Given  $A = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$  and  $x = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ , establish if  $x$  is an eigenvector of  $A$ . Moreover show that 7 is an eigenvalue of  $A$  and determine a correspondent eigenvector.

3) Given  $A = \begin{pmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{pmatrix}$  and  $x = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ , establish if  $x$  is an eigenvector of  $A$ .

4) For which values of  $\theta$  does the following matrix admits Real eigenvalues?

$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

5) Find eigenvalues and (when possible) eigenvectors of the following matrices

a)

$$\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix}$$

6) Find eigenvalues and (when possible) eigenvectors of the following matrices

a)

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

c)

$$\begin{pmatrix} 5 & 3 & -3 \\ 0 & -4 & 9 \\ 0 & 6 & -1 \end{pmatrix}$$

d)

$$\begin{pmatrix} 5 & 0 & 0 \\ 5 & 6 & -1 \\ 5 & 1 & 4 \end{pmatrix}$$

e)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

7) Determine the value for the parameter  $h$  such that the following matrix  $A$  admits eigenvalue  $\lambda = 1$ . For this value of  $h$  determine the eigenvalues of  $A$ .

$$A = \begin{pmatrix} h & 1 & 0 \\ 1-h & 0 & 2 \\ 1 & 1 & h \end{pmatrix}$$