

**MATHEMATICS**  
**Friday April 29 2016**  
**Eighth Exercise Class**

1) Identify the points on the plane  $0xy$  in which the following functions are defined

- a)  $z = f(x, y) = x^2 - y^2$
- b)  $z = f(x, y) = 1 + \sqrt{-(y-x)^2}$
- c)  $z = f(x, y) = \log(x+y)$
- d)  $z = f(x, y) = \log(xy)$
- e)  $z = f(x, y) = \log(x^2 + y)$
- f)  $z = f(x, y) = \frac{y+x-1}{y(x-1)}$

2) Identify the level curves of the following functions

- a)  $z = f(x, y) = x^2 + y^2$
- b)  $z = f(x, y) = x + y$
- c)  $z = f(x, y) = \frac{y}{x^2}$
- d)  $z = f(x, y) = y - x$

3) Determine the Gradient of the following functions

- a)  $z = f(x, y) = x^2 - y^2$
- b)  $z = f(x, y) = x^2 + y^3 - 3xy$
- c)  $z = f(x, y) = \sin(xy)$
- d)  $z = f(x, y) = \sqrt{x^2 + y^2}$
- e)  $z = f(x, y) = \log \sqrt{x^2 + y^2}$
- f)  $z = f(x, y) = x^y$

4) Determine the Second partial derivatives of the following functions

a)  $z = f(x, y) = x^2y^3 - x^3y^2$

b)  $z = f(x, y) = x^2 - y^3 + 3xy$

c)  $z = f(x, y) = \log(x^2 + y)$

5) Identify the stationary points of the following functions

a)  $z = f(x, y) = x^2 - y^2$

b)  $z = f(x, y) = \sqrt{x^2 + y^2}$

c)  $z = f(x, y) = (x - 1)^2 + (y - 1)^3 - 4xy + 4x + 8y + 8$

d)  $z = f(x, y) = xy - x - y + \frac{8}{x-1} + \frac{1}{y-1} + 1$

e)  $z = f(x, y) = (x - 1) \log(xy)$

6) Find critical points of the following function and determine whether they are maximum, minimum or saddle points.

a)  $z = f(x, y) = x^2 + y^2$

b)  $z = f(x, y) = xy$

c)  $z = f(x, y) = x^3 + 3xy^2 - 15x - 12y$

d)  $z = f(x, y) = \sqrt{x^2 + y^2}$

e)  $z = f(x, y) = (x - 2) \log(xy)$